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# Quantum Algorithm for Timetabling Problem

**Toru Fujimura**

*Chemical Department, Industrial Property Cooperation Center,*
*1-2-15, Kiba, Koto-ku, Tokyo 135-0042, Japan*
*E-mail: tfujimura8@gmail.com*

**Abstract**

A quantum algorithm for the timetabling problem and its example are reported. When $n$ lectures are given at $k$ rooms, an optimal timetable is decided. It is considered this time that the number of persons admitted for each room, the number of participants for each lecture, and connected lectures. A computational complexity of a classical calculation is $k^n$. The computational complexity becomes about $k^{n/2}$ by the quantum algorithm that uses quantum phase inversion gates and quantum inversion about mean gates.

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**Keywords:** Quantum algorithm, timetabling problem, computational complexity.

## 1. Introduction

Deutsch and Jozsa [1–3] started a quantum algorithm for a high-speed process by a parallel computation that uses quantum entangled states. Shor's algorithm for the factorization [2–4], Grover's algorithms for the database search [2,5,6], and so on expand the application range of the quantum computation. A quantum algorithm for the knapsack problem has recently been reported by Fujimura [7]. The timetabling problem [8] is examined this time. Therefore, its result is reported.

## 2. Timetabling Problem

When lectures are given at rooms, an optimal timetable is decided [8]. It is considered that time periods, rooms, and other conditions.

## 3. Quantum Algorithm

When $n$ lectures that have time periods $R_t$ [$0 \leq t \leq n-1$. $t$ is an integer.], numbers of participants $P_t$, and connected lectures are given at $k$ rooms that have numbers of persons admitted $M_u$ [$0 \leq u \leq k-1$. $u$ is an integer.], an optimal timetable is decited. When there is an overlap between the time period of the $i$-th lecture $R_i$ and it of the $j$-th lecture $R_j$, $x_{i,j}$ [$0 \leq i < j \leq n-1$. $i$ and $j$ are integers.] is 1, and when there isn't an overlap between these lectures, it is 0. $\sum_{j=1,i<j}^{n-1} \sum_{i=0}^{n-2} x_{i,j}$ is $m$. When $P_t$ is $M_u$ or less, $y_{t,u}$ is 1, and when $P_t$ is larger than $M_u$, $y_{t,u}$ is 0. When the $i$-th lecture connects the $j$-th lecture, $z_{i,j}$ [$0 \leq i < j \leq n-1$. $i$ and $j$ are integers.] is 0, and when it doesn't conect the $j$-th lecture, $z_{i,j}$ is 1.

First of all, the quantum registers $|a_0>$, $|a_1>$, $\cdots$, $|a_{n-1}>$, and $|b>$ are prepared. When $\alpha$ is a minimum integer that is $log_2 k$ or more, each of $|a_f>$ that $f$ is an integer from 0 to $n-1$ is consisted of $\alpha$ quantum bits [=qubits]. States of $|a_f>$ and $|b>$ are $a_f$ and $b$, respectively.

**Step 1:** Each qubit of $|a_f>$ and $|b>$ is set $|0>$.

**Step 2:** The Hadamard gate $\boxed{\text{H}}$ [2, 3] acts on each qubit of $|a_f>$. It changes them for entangled states. The total states are $(2^\alpha)^n$.

**Step 3:** It is assumed that a quantum gate $(A)$ changes $|b>$ for $|b+1>$ at $a_f \geq k$, or it doesn't change $|b>$ at $a_f < k$. As a target state for $|b>$ is 0, quantum phase inversion gates $(PI)$ and quantum inversion about mean gates $(IM)$ [2,5,6] act on $|b>$. When $\beta$ is a minimum even integer that is $(2^\alpha/k)^{1/2}$ or more, the total number that $(PI)$ and $(IM)$ act on $|b>$ is $\beta$ because they are a couple. Next, an observation gate $(OB)$ observes $|b>$. These actions are repeated sequentially from $|a_0>$ to $|a_{n-1}>$. Therefore, each state of $|a_f>$ is 0, 1, $\cdots$, $k-2$, or $k-1$, and the total states become $k^n$.

**Step 4:** It is assumed that a quantum gate $(B_{i,j})$ [$0 \leq i < j \leq n-1$. $i$ and $j$ are integers.] changes $|b>$ for $|b + x_{i,j} y_{i,a_i} y_{j,a_j} z_{i,j}>$ at $a_i \neq a_j$, or it doesn't change $|b>$ at $a_i = a_j$. These actions are repeated for all combinations of $i$ and $j$. Therefore, combinations that are fitted to conditions are selected. As a target state for $|b>$ is $m$, $(PI)$ and $(IM)$ act on $|b>$. When $\gamma$ is a minimum even integer that is $k^{n/2}$ or more, the total number that $(PI)$ and $(IM)$ act on $|b>$ is $\gamma$. Next, $(OB)$ observes $|a_f>$ and $|b>$. By these actions, $a_f$ and $b$ are obtained. Therefore, when $b$ is $m$, an answer is obtained.

## 4. Numerical Calculation

It is assumed that there are $n = 10$, $R_0 = (9:00 \to 11:50)$, $R_1 = (13:00 \to 15:50)$, $R_2 = (14:30 \to 17:20)$, $R_3 = (13:00 \to 17:20)$, $R_4 = (9:00 \to 10:20)$, $R_5 = (9:00 \to 10:20)$, $R_6 = (10:30 \to 11:50)$, $R_7 = (10:$

30 → 11 : 50), $R_8 = (13 : 00 \to 14 : 20)$, $R_9 = (16 : 00 \to 17 : 20)$, $P_0 = 100$, $P_1 = 50$, $P_2 = 100$, $P_3 = 50$, $P_4 = 60$, $P_5 = 40$, $P_6 = 70$, $P_7 = 30$, $P_8 = 100$, $P_9 = 50$, $M_0 = 100$, $M_1 = 80$, $M_2 = 50$, $k = 3$, $z_{0,8} = z_{1,4} = 0$, and (others of $z_{i,j}$ $[0 \le i < j \le 9]$)= 1. Therefore, it is set that there are $x_{0,4} = x_{0,5} = x_{0,6} = x_{0,7} = x_{1,2} = x_{1,3} = x_{1,8} = x_{2,3} = x_{2,9} = 1$, (others of $x_{i,j}$ $[0 \le i < j \le 9]$) = 0, $m = 9$, $y_{0,1} = y_{2,1} = y_{8,1} = y_{0,2} = y_{2,2} = y_{4,2} = y_{6,2} = y_{8,2} = 0$, and (others of $y_{t,u}$ $[0 \le t \le 9, 0 \le u \le 2]$) = 1.

First of all, $|a_0>, |a_1>, \cdots, |a_9>$, and $|b>$ are prepared. As $log_2 k$ is $log_2 3 \approx 1.6' \le 2 = \alpha$, each of $|a_f>$ that $f$ is an integer from 0 to 9 is consisted of 2 qubits.

**Step 1:** Each qubit of $|a_f>$ and $|b>$ is set $|0>$.

**Step 2:** $\boxed{\text{H}}$ acts on each qubit of $|a_f>$. It changes them for entangled states. The total states are $(2^\alpha)^n = (2^2)^{10}$.

**Step 3:** $(A)$ changes $|b>$ for $|b+1>$ at $a_f \ge 3$, or it doesn't change $|b>$ at $a_f < 3$. As a target state for $|b>$ is 0, $(PI)$ and $(IM)$ act on $|b>$. When $\beta$ is a minimum even integer that is $(2^\alpha/k)^{1/2} = (2^2/3)^{1/2} \approx 1.2 \le 2 = \beta$, the total number that $(PI)$ and $(IM)$ act on $|b>$ is 2. Next, $(OB)$ observes $|b>$. These actions are repeated sequentially from $|a_0>$ to $|a_9>$. Therefore, each state of $|a_f>$ is 0, 1, or 2, and the total states become $k^n = 3^{10}$.

**Step 4:** $(B_{i,j})$ $[0 \le i < j \le 9]$ changes $|b>$ for $|b + x_{i,j} y_{i,a_i} y_{j,a_j} z_{i,j}>$ at $a_i \ne a_j$, or it doesn't change $|b>$ at $a_i = a_j$. These actions are repeated for all combinations of $i$ and $j$. Therefore, combinations that are fitted to conditions are selected. As a target state for $|b>$ is $m = 9$, $(PI)$ and $(IM)$ act on $|b>$. When $\gamma$ is a minimum even integer that is $k^{n/2} = 3^{10/2} = 243 \le 244 = \gamma$, the total number that $(PI)$ and $(IM)$ act on $|b>$ is $\gamma = 244$. Next, $(OB)$ observes $|a_0>, |a_1>, \cdots, |a_9>$, and $|b>$. By these actions, for example, $a_0, a_1, \cdots, a_9$, and $b$ are 0, 1, 0, 2, 1, 2, 1, 2, 0, 1, and 9, respectively. Therefore, an answer is 0, 1, 0, 2, 1, 2, 1, 2, 0, and 1.

## 5. Discussion and Summary

The computational complexity of this quantum algorithm [= $S$] becomes the following. In the order of the actions by the gates, the number of them is $\alpha n$ at $\boxed{\text{H}}$, $n$ at $(A)$, $\beta n = 2n$ at $(PI)$ and $(IM)$, $n$ at $(OB)$, $n(n-1)$ at $(B_{i,j})$ $[0 \le i < j \le n-1]$, $\gamma$ at $(PI)$ and $(IM)$, and 1 at $(OB)$. Therefore, $S$ becomes $n^2 + (\alpha + 3)n + \gamma + 1$. In the example of the section 4, $S$ is 395. The computational complexity of the classical calculation [= $Z$] is $k^n = 3^{10} = 59049$. After all, $S/Z$ becomes about 1/150. When $n$ is large enough, $S$ becomes about $\gamma \approx k^{n/2}$, and $S/Z$ is about $1/k^{n/2}$. For example, as for $k = 3$ and $n = 100$, $S/Z$ is about $1/(7.2' \times 10^{23})$.

Further development of the quantum computation in the future is expected.

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