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Oscillation Criteria for Fourth Order Nonlinear Neutral Delay Dynamic Equations on Time Scales

Said R. Grace
Department of Engineering Mathematics,
Faculty of Engineering, Cairo University,
Orman, Giza 12221, Egypt
E-mail: saidgrace@yahoo.com

John R. Graef
Department of Mathematics,
University of Tennessee at Chattanooga,
Chattanooga, TN 37403, USA.
E-mail: John-Graef@utc.edu

Abstract

The authors establish some new criteria for the oscillation of solutions of fourth order nonlinear neutral delay dynamic equations of the form

$$\left(a(t)\,(x(t) - p(t)\,x\,(\tau(t)))^{\Delta\Delta\Delta}\right)^{\Delta} + q(t)\,x^{\lambda}\,(g(t)) = 0$$

on a time scale T, where $\lambda > 0$ is the ratio of odd positive integers, and a, p, and q are real-valued positive rd-continuous functions defined on T.

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1. Introduction

In this paper we examine the oscillatory behavior of solutions of the fourth order nonlinear neutral delay dynamic equation

$$\left(a(t)\,(x(t) - p(t)\,x\,(\tau(t)))^{\Delta\Delta\Delta}\right)^{\Delta} + q(t)\,x^{\lambda}\,(g(t)) = 0 \tag{1.1}$$

on an arbitrary time scale $T \subseteq \mathbb{R}$ with $Sup\, T = \infty$. We will use the notation that $[t_0, \infty)_T = [t_0, \infty) \cap T$, and for convenience when we write $t \geq t_0$ we mean that $t \in [t_0, \infty)_T$. We will assume that

(i) $\lambda > 0$ is the ratio of odd positive integers;

(ii) $a : T \to (0, \infty)$ is a real valued rd-continuous function with $a^{\Delta}(t) \geq 0$ on T and

$$\int_{t_0}^{\infty} \frac{1}{a(s)} \Delta s = \infty; \tag{1.2}$$

(iii) $p, q : T \to (0, \infty)$ are real valued rd-continuous functions and $0 < p(t) \leq 1$ on T;

(iv) $g, \tau : T \to T$ are real valued rd-continuous functions such that $g(t) \leq t$, $\tau(t) \leq t$, $\tau^{\Delta}(t) > 0$, $g^{\Delta}(t) > 0$, and $\lim_{t\to\infty} g(t) = \lim_{t\to\infty} \tau(t) = \infty$;

(v) $\xi(t) = \tau^{-1} \circ g(t) \leq t$, $\xi^{\Delta}(t) \geq 0$ on T, and $\lim_{t\to\infty} \xi(t) = \infty$.

Standard notation for times scale calculus will be used such as that which can be found in the monographs by Bohner and Peterson [1,2]. We recall that a solution x of equation (1.1) is said to be *nonoscillatory* if there exists $t_0 \in T$ such that $x(t)\, x\,(\sigma(t)) > 0$ for all $t \in [t_0, \infty)_T$; otherwise, it is said to be *oscillatory*. Equation (1.1) is said to be oscillatory if all its solutions are oscillatory.

Recently, there has been an increasing interest in investigating the oscillatory behavior of first, second, and even third order dynamic equations on time-scales; for example, for second order equations, see [1,2,4,5,7], and for results on the oscillatory and asymptotic behavior of solutions of third order equations, see [3,6] and the references contained therein. It appears that very little is known regarding the oscillation of fourth order nonlinear neutral delay equations on time scales and our purpose here is to establish some new oscillation criteria for such equations.

2. Main Results

Before presenting our main results, we begin with some lemmas that will be needed in their proofs. We let

$$y(t) = x(t) - p(t)\, x\,(\tau(t)) \quad \text{for} \quad t \in [t_0, \infty)_T \tag{2.1}$$

so that equation (1.1) can be written as

$$\left(a(t)\, y^{\Delta\Delta\Delta}(t)\right)^{\Delta} + q(t)\, x^{\lambda}\,(g(t)) = 0. \tag{2.2}$$

Now $a^{\Delta}(t) \geq 0$ so if x is a positive solution of (1.1) with $y^{\Delta\Delta\Delta} > 0$, we have

$$a^{\Delta}(t)\, y^{\Delta\Delta\Delta}(t) + a^{\sigma}(t)\, y^{\Delta^4}(t) = \left(a(t)\, y^{\Delta\Delta\Delta}(t)\right)^{\Delta} < 0.$$

This implies

$$y^{\Delta^4}(t) < 0 \quad \text{for} \quad t \in [t_0, \infty) \cap T.$$

Lemma 2.1. Let condition (1.2) hold and assume that x is an eventually positive solution of equation (1.1). Then there are only three possible cases for the behavior of y for sufficiently large t:

(I) $y(t) > 0$ and $y^{\Delta^i}(t) > 0$ for $i = 1, 2, 3$; and $y^{\Delta^4}(t) < 0$;

(II) $y(t) > 0$ $y^{\Delta}(t) > 0$, $y^{\Delta\Delta}(t) < 0$, and $y^{\Delta\Delta\Delta} > 0$; and $y^{\Delta^4} < 0$;

(III) $y(t) < 0$, $y^{\Delta}(t) > 0$, $y^{\Delta\Delta}(t) < 0$, and $y^{\Delta\Delta\Delta} > 0$; and $y^{\Delta^4} < 0$.

In [1, Section 1.6], the generalized Taylor monomials $\{h_n(t, s)\}_{n=0}^{\infty}$ are defined recursively by

$$h_0(t, s) = 1 \quad \text{and} \quad h_{n+1}(t, s) = \int_s^t h_n(\tau, s)\, \Delta\tau, \quad t, s \in T, \ n \geq 1.$$

The following lemma is due to Erbe et al. [3, Lemma 4].

Lemma 2.2. Assume that $z(t)$ satisfies

$$z(t) > 0, \ z^{\Delta}(t) > 0, \ z^{\Delta\Delta}(t) > 0, \quad \text{and} \quad z^{\Delta\Delta\Delta}(t) \leq 0 \quad \text{for} \quad t \geq t_0.$$

Then

$$\liminf_{t\to\infty} \frac{t\, z(t)}{h_2(t, t_0)\, z^{\Delta}(t)} \geq 1. \tag{2.3}$$

Our first oscillation result for equation (1.1) is the following.

Theorem 2.3. Let $\lambda \leq 1$ and conditions (i)–(iv) and (1.2) hold. If

$$\limsup_{t\to\infty} \frac{1}{a(g(t))} \int_{g(t)}^t q(s)\, h_3^{\lambda}(g(s), t_0)\, \Delta s > \begin{cases} 1, & \text{if } \lambda = 1, \\ 0, & \text{if } \lambda < 1, \end{cases} \tag{2.4}$$

$$\limsup_{t\to\infty} \int_{g(t)}^t \frac{1}{a(u)} \int_u^t (g(t) - g(s))^{\lambda} q(s) g^{\lambda}(s) \Delta s\, \Delta u > \begin{cases} 1, & \text{if } \lambda = 1, \\ 0, & \text{if } \lambda < 1, \end{cases} \tag{2.5}$$

and

$$\limsup_{t\to\infty} \frac{1}{a(\xi(t))} \int_{\xi(t)}^t q(s)\, h_3^{\lambda}(\xi(t), \xi(s))\, \Delta s > \begin{cases} 1, & \text{if } \lambda = 1, \\ 0, & \text{if } \lambda < 1, \end{cases} \tag{2.6}$$

then equation (1.1) is oscillatory.

Proof. Let $x(t)$ be a nonoscillatory solution of equation (1.1), say $x(t) > 0, x(\tau(t)) > 0$, and $x(g(t)) > 0$ for $t \in [t_1, \infty)_T$ and let y be defined as in (2.1). By Lemma 2.1, there are three possible cases for y. Suppose y satisfies Case (I). Then there exist a constant c_1, $0 < c_1 < 1$, and $t_2 \geq t_1$ such that

$$y^{\Delta\Delta}(t) \geq c_1 t y^{\Delta\Delta\Delta}(t) \quad \text{for} \quad t \geq t_2. \tag{2.7}$$

From (2.3) with $z(t) = y^{\Delta}(t)$, for any constant c_2 with $0 < c_2 < 1$, we have

$$y^{\Delta}(t) \geq c_2 \frac{h_2(t,t_0)}{t} y^{\Delta\Delta}(t), \tag{2.8}$$

and so substituting (2.7) into (2.8) gives

$$y^{\Delta}(t) \geq c_1 c_2 h_2(t,t_0) y^{\Delta\Delta\Delta}(t) \quad \text{for} \quad t \geq t_2.$$

Integrating from t_2 to t, we obtain

$$y(t) \geq c h_3(t,t_0) y^{\Delta\Delta\Delta}(t) \quad \text{for} \quad t \geq t_2,$$

where $c = c_1 c_2$. Now, there exists $t_3 \geq t_2$ such that $g(t) \geq t_2$ for $t \geq t_3$, so

$$y(g(t)) \geq c h_3 \left(g(t), t_0\right) y^{\Delta\Delta\Delta}(g(t)) \quad \text{for} \quad t \geq t_3. \tag{2.9}$$

Clearly,

$$x(t) \geq x(t) - p(t)\, x\left(\tau(t)\right) = y(t) \quad \text{for} \quad t \geq t_3. \tag{2.10}$$

Using (2.9) and (2.10) in equation (2.2), we have

$$\left(a(t)\, w(t)\right)^{\Delta} + c^{\lambda}\, q(t) h_3^{\lambda}\left(g(t), t_0\right) w^{\lambda}\left(g(t)\right) \leq 0, \tag{2.11}$$

where $w(t) = y^{\Delta\Delta\Delta}(t)$ for $t \geq t_3$. Integrating (2.11) for $t \geq g(t) \geq t_2$ yields

$$\begin{aligned} a\left(g(t)\right) w\left(g(t)\right) &\geq c^{\lambda} \int_{g(t)}^{t} q(s) h_3^{\lambda}\left(g(s), t_0\right) w^{\lambda}\left(g(s)\right) \Delta s \\ &\geq c^{\lambda} w^{\lambda}\left(g(t)\right) \int_{g(t)}^{t} q(s) h_3^{\lambda}\left(g(s), t_0\right) \Delta s, \end{aligned}$$

or

$$w^{1-\lambda}\left(g(t)\right) \geq \frac{c^{\lambda}}{a\left(g(t)\right)} \int_{g(t)}^{t} q(s) h_3^{\lambda}\left(g(s), t_0\right) \Delta s. \tag{2.12}$$

Now take the lim sup as $t \to \infty$ of both sides of the above inequality. If $\lambda = 1$, the contradiction is obvious. If $\lambda < 1$, then the left hand side of (2.12) is positive and must decrease to zero (to prevent a contradiction to the positivity of $x(t)$). This contradicts (2.4), so Case (I) can not hold.

Next, assume that y satisfies Case (II). There exist a constant k, $0 < k < 1$, and $t_2 \geq t_1$ such that

$$y\left(g(t)\right) \geq k\, g(t) y^{\Delta}\left(g(t)\right) \quad \text{for} \quad t \geq t_2. \tag{2.13}$$

Using (2.10) and (2.13) in equation (2.2) gives

$$\left(a(t)\, z^{\Delta\Delta}(t)\right)^{\Delta} + k^{\lambda} g^{\lambda}(t) q(t) z^{\lambda}\left(g(t)\right) \leq 0 \quad \text{for} \quad t \geq t_2, \tag{2.14}$$

where $z(t) = y^{\Delta}(t)$ satisfies $z(t) > 0$, $z^{\Delta}(t) < 0$, and $z^{\Delta\Delta}(t) > 0$ for $t \geq t_2$. Integrating (2.14) for $t \geq u \geq t_2$, we have

$$z^{\Delta\Delta}(u) \geq \frac{k^{\lambda}}{a(u)} \int_u^t q(s) g^{\lambda}(s) z^{\lambda}\left(g(s)\right) \Delta s. \tag{2.15}$$

We also have

$$-z\left(g(s)\right) \leq z\left(g(t)\right) - z\left(g(s)\right) = \int_{g(s)}^{g(t)} z^{\Delta}(\tau)\, \Delta\tau \leq \left(g(t) - g(s)\right) z^{\Delta}\left(g(t)\right). \tag{2.16}$$

Substituting (2.16) into (2.15) gives

$$z^{\Delta\Delta}(u) \geq \frac{k^{\lambda}}{a(u)} \int_u^t \left(g(t) - g(s)\right)^{\lambda} q(s) g^{\lambda}(s) \left(-z^{\Delta}\left(g(t)\right)\right)^{\lambda} \Delta s.$$

Integrating this inequality from $g(t)$ to t, we have

$$\begin{aligned} -z^{\Delta}\left(g(t)\right) &\geq z(t) - z^{\Delta}\left(g(t)\right) \\ &\geq k^{\lambda} \left(\int_{g(t)}^t \frac{1}{a(u)} \int_u^t \left(g(t) - g(s)\right)^{\lambda} q(s) g^{\lambda}(s) \Delta s\, \Delta u \right) \left(-z^{\Delta}\left(g(t)\right)\right)^{\lambda}, \end{aligned}$$

or

$$\left(-z^{\Delta}\left(g(t)\right)\right)^{1-\lambda} \geq k^{\lambda} \int_{g(t)}^t \frac{1}{a(u)} \int_u^t \left(g(t) - g(s)\right)^{\lambda} q(s) g^{\lambda}(s) \Delta s\, \Delta u.$$

Taking lim sup as $t \to \infty$ of both sides of the above inequality, we obtain a contradiction to condition (2.5).

Finally, we assume that y satisfies Case (III). Setting $z(t) = -y(t)$, we see that

$$x\left(g(t)\right) \geq z\left(\tau^{-1} \circ g(t)\right) \quad \text{for} \quad t \geq t_1. \tag{2.17}$$

Using (2.17) in equation (2.2), we have

$$\left(a(t)\, z^{\Delta\Delta\Delta}(t)\right)^{\Delta} \geq q(t) z^{\lambda}\left(\xi(t)\right) \quad \text{for} \quad t \geq t_1. \tag{2.18}$$

Clearly z satisfies

$$z^{\Delta}(t) < 0,\ z^{\Delta\Delta}(t) > 0,\ z^{\Delta\Delta\Delta}(t) < 0, \text{ and } z^{\Delta^4}(t) > 0 \quad \text{for} \quad t \geq t_1.$$

Now

$$-z^{\Delta\Delta}(v) + z^{\Delta\Delta}(u) = \int_u^v (-z^{\Delta\Delta\Delta}(s)) \Delta s,$$

so

$$z^{\Delta\Delta}(u) \geq h_1(v, u)\left(-z^{\Delta\Delta\Delta}(v)\right).$$

Integrating twice, we obtain

$$z(u) \geq h_3(v,\ u)\left(-z^{\Delta\Delta\Delta}(v)\right) \quad \text{for} \quad v \geq u \geq t_1. \tag{2.19}$$

Setting $u = \xi(s)$ and $v = \xi(t)$ in (2.19) gives

$$z\left(\xi(s)\right) \geq h_3\left(\xi(t), \xi(s)\right)\left(-z^{\Delta\Delta\Delta}\left(\xi(t)\right)\right) \quad \text{for} \quad t \geq s \geq t_1. \tag{2.20}$$

Integrating inequality (2.18) from $\xi(t) \geq t_1$ to t and using (2.20), we obtain

$$a\left(\xi(t)\right) w\left(\xi(t)\right) \geq \int_{\xi(t)}^{t} q(s) h_3^{\lambda}\left(\xi(t), \xi(s)\right) w^{\lambda}\left(\xi(s)\right) \Delta s,$$

or

$$w^{1-\lambda}\left(\xi(t)\right) \geq \frac{1}{a\left(\xi(t)\right)} \int_{\xi(t)}^{t} q(s) h_3^{\lambda}\left(\xi(t), \xi(s)\right) \Delta s, \tag{2.21}$$

where $w(t) = -z^{\Delta\Delta\Delta}(t)$ for $t \geq t_1$. Taking lim sup as $t \to \infty$ of both sides of (2.21) yields a contradiction to condition (2.6). This completes the proof of the theorem. ■

Remark 2.4. What might be useful consequences of Theorem 2.3 are that under the covering assumptions, the conditions (2.4), (2.5), and (2.6) eliminate Cases (I), (II), and (III), respectively, of Lemma 2.1 from occurring.

Our second oscillation result for equation (1.1) is the following.

Theorem 2.5. Let $\lambda \leq 1$ and conditions (i)–(v) and (1.2) hold. If, in addition to condition (2.6), we have

$$\limsup_{t\to\infty} \left[h_k(g(t), t_0) h_{3-k}(t,\ g(t))\right]^{\lambda} \left(\frac{1}{a(t)} \int_t^{\infty} q(s)\Delta s\right) > \begin{cases} 1, & \text{if } \lambda = 1, \\ 0, & \text{if } \lambda < 1, \end{cases} \tag{2.22}$$

for $k \in \{1,\ 3\}$, then equation (1.1) is oscillatory.

Proof. Proof. Let $x(t)$ be a nonoscillatory solution of equation (1.1), say $x(t) > 0$ for $t \in [t_1, \infty)_{\mathbb{T}}$. As in the proof of Theorem 2.3, we see that y satisfies one of the three cases in Lemma 2.1. Suppose y satisfies either Case (I) or Case (II). Notice that if $k = 1$ in Kigurardze's Lemma, then Case (II) holds, and if $k = 3$, Case (I) holds. Now if $k = 1$,

$$y(v) \geq y(v) - y(u_1) = \int_{u_1}^{v} y^{\Delta}(s)\Delta s \geq h_1(v, u_1) y^{\Delta}(v). \tag{2.23}$$

Also,

$$-y^{\Delta\Delta}(u) \geq y^{\Delta\Delta}(v) - y^{\Delta\Delta}(u) \geq \int_u^v y^{\Delta\Delta\Delta}(s)\Delta s \geq h_1(v, u) y^{\Delta\Delta\Delta}(v).$$

An integrating then gives

$$y^{\Delta}(u) \geq h_2(v,u)y^{\Delta\Delta\Delta}(v). \tag{2.24}$$

If $k = 3$, then

$$y^{\Delta\Delta}(v) \geq y^{\Delta\Delta}(v) - y^{\Delta\Delta}(u_1) = \int_{u_1}^{v} y^{\Delta\Delta\Delta}(s)\Delta s \geq h_1(v,u_1)y^{\Delta\Delta\Delta}(v),$$

and integrating twice, we have

$$y(v) \geq h_3(v,u_1)y^{\Delta\Delta\Delta}(v). \tag{2.25}$$

Combining (2.23) and (2.25) gives

$$y(v) \geq h_k(v,u_1)y^{\Delta^k}(v) \quad \text{for} \quad k \in \{1,3\} \quad \text{and} \quad v \geq u_1 \geq t_1.$$

Thus, for $g(t) \geq t_1$, we have

$$y(g(t)) \geq h_k(g(t),t_1)y^{\Delta^k}(g(t)) \quad \text{for} \quad k \in \{1,3\} \quad \text{and} \quad t \geq g(t) \geq t_1. \tag{2.26}$$

Also, from (2.24) with $u = g(t) \geq t_1$, we obtain

$$y^{\Delta^k}(g(t)) \geq h_{3-k}(v,\ u)\, y^{\Delta\Delta\Delta}(v) \quad \text{for} \quad k \in \{1,3\} \quad \text{and} \quad v \geq u \geq t_1, \tag{2.27}$$

Combining (2.26) and (2.27), we have

$$y\,(g(t)) \geq h_k\,(g(t),t_1)\, h_{3-k}\,(t,\ g(t))\, y^{\Delta\Delta\Delta}(t) \quad \text{for} \quad k \in \{1,3\} \quad \text{and} \quad t \geq g(t) \geq t_1. \tag{2.28}$$

Using (2.10) in equation (2.2), integrating from t to u, and letting $u \to \infty$, we obtain

$$y^{\Delta\Delta\Delta}\,(t) \geq \left(\frac{1}{a(t)}\int_t^{\infty} q(s)\Delta s\right) y^{\lambda}\,(g(t)) \quad \text{for} \quad t \geq t_1. \tag{2.29}$$

From (2.28) and (2.29), we obtain

$$w^{1-\lambda}(t) \geq [h_k\,(g(t),t_1)\, h_{3-k}\,(t,\ g(t))]^{\lambda} \left(\frac{1}{a(t)}\int_t^{\infty} q(s)\Delta s\right) \quad \text{for} \quad t \geq t_1,$$

where $w(t) = y^{\Delta\Delta\Delta}(t)$. Taking lim sup as $t \to \infty$ of both sides of the above inequality as $t \to \infty$, we obtain a contradiction to condition (2.22).

Finally, for Case (III), the proof is similar to that of the corresponding case in Theorem 2.3; we omit the details. This completes the proof of our theorem. ■

For the special case of equation (1.1) with $p(t) = 0$, namely, the equation

$$\left(a(t)x^{\Delta\Delta\Delta}(t)\right)^{\Delta} + q(t)x^{\lambda}\,(g(t)) = 0, \tag{2.30}$$

we can obtain the following result.

Theorem 2.6. Let $\lambda \leq 1$ and conditions (i)–(v) and (1.2) hold. If, in addition to condition (2.4), either (2.5) or (2.22) holds, then equation (2.30) is oscillatory.

Finally, we consider the case where $p(t)$ satisfies $-1 \leq p(t) \leq 0$ for $t \geq t_0$, i.e., we consider the equation

$$\left(a(t)\left(x(t)+r(t)\,x\left(\tau(t)\right)\right)^{\Delta\Delta\Delta}\right)^{\Delta}+q(t)\,x^{\lambda}\left(g(t)\right)=0, \tag{2.31}$$

where $0 \leq r(t) \leq 1$ and establish the following result.

Theorem 2.7. Let $\lambda \leq 1$, conditions (i)–(v) and (1.2) hold. If either

$$\limsup_{t\to\infty}\left[h_k\left(g(t),t_0\right)h_{3-k}\left(t,\ g(t)\right)\right]^{\lambda}\left(\frac{1}{a(t)}\int_t^{\infty}\left(1-r(g(s))\right)^{\lambda}q(s)\Delta s\right) > \begin{cases}1, & \text{if } \lambda=1,\\ 0, & \text{if } \lambda<1,\end{cases}$$

for $k \in \{1, 3\}$, or,

$$\limsup_{t\to\infty}\frac{1}{a\left(g(t)\right)}\int_{g(t)}^{t}q(s)\left[1-r\left(g(s)\right)h_3\left(g(s),t_0\right)\right]^{\lambda}\Delta s > \begin{cases}1, & \text{if } \lambda=1,\\ 0, & \text{if } \lambda<1,\end{cases}$$

and

$$\limsup_{t\to\infty}\int_{g(t)}^{t}\frac{1}{a(u)}\int_u^t\left[\left(1-r\left(g(s)\right)\right)\left(g(t)-g(s)\right)\right]^{\lambda}q(s)g^{\lambda}(s)\Delta s\Delta u > \begin{cases}1, & \text{if } \lambda=1,\\ 0, & \text{if } \lambda<1,\end{cases}$$

then equation (2.31) is oscillatory.

Proof. Let $x(t)$ be a nonoscillatory solution of equation (2.31), say $x(t) > 0$ for $t \in [t_1, \infty)_T$. Setting

$$y(t)=x(t)+r(t)x(\tau(t)),$$

equation (2.31) becomes

$$\left(a(t)y^{\Delta\Delta\Delta}(t)\right)^{\Delta}+q(t)x^{\lambda}\left(g(t)\right)=0. \tag{2.32}$$

From Lemma 2.1, we see that y satisfies either Case (I) or Case (II), and in both cases we have $y^{\Delta}(t) > 0$ for $t \geq t_1$. Now,

$$\begin{aligned}x(t)=y(t)-r(t)x(\tau(t)) &= y(t)-r(t)\left[y(\tau(t))-r(\tau(t))x(\tau\circ\tau(t))\right]\\ &\geq y(t)-r(t)y(\tau(t))\\ &\geq \left(1-r(t)\right)y(t)\end{aligned} \tag{2.33}$$

for $t \geq t_1$. Using (2.33) in equation (2.32), we obtain

$$\left(a(t)y^{\Delta\Delta\Delta}(t)\right)^{\Delta} + q(t)\,(1 - r\,(g(t)))^{\lambda}\, y^{\lambda}\,(g(t)) \leq 0$$

for $t \geq t_1$. The remainder of the proof is similar to that of Theorems 2.3 and 2.5 and we omit the details. ■

Concluding Remarks

Our results here are presented in high degree of generality. Notice that they are not applicable to equations of type (1.1) with $g(t) = t$. This means that the delays in equation (1.1) clearly generate the oscillations. There is no restriction on the type of time scales to which our results apply. It will be of interest to study the oscillatory behavior of solutions of equation (1.1) if $\lambda > 1$ or for $p(t) > 1$ or $p(t) < -1$.

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