Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 7, Number 2 (2011), pp. 191-199

http://www.ripublication.com/gjpam.htm

A Fixed Point Theorem in Menger Space

Renu Chugh and Tamanna

Department of Mathematics, M.D. University, Rohtak-124001, India
E-mail: chughrenu@yahoo.com, tamannakadian@yahoo.co.in

Abstract

The aim of this paper is to prove a common fixed point theorem in Menger space by studying the relationship between the continuity and reciprocal continuity.

Mathematics Subject Classification: 47H10, 54H25.

Keywords: Fixed point, Menger space, Compatible maps, R-weakly commuting maps, Reciprocal continuity.

Introduction

There have been a number of generalizations of metric space. One such generalization is Menger space introduced in 1942 by Menger [10] who used distribution functions instead of nonnegative real numbers as values of the metric. Schweizer and Sklar [16] studied this concept and then the important development of Menger space theory was due to Sehgal and Bharucha-Reid [13].Sessa [14] introduced weakly commuting maps in metric spaces. Jungck [6] enlarged this concept to compatible maps. The notion of compatible maps in Menger spaces has been introduced by Mishra [11]. Recently, Singh and Jain [15] generalized the results of Mishra [11] using the concept of weak compatibility and compatibility of pair of self maps. The aim of this paper is to prove a common fixed point theorem in Menger space by studying the relationship between the continuity and reciprocal continuity. We also give an example to illustrate our main theorem.

Preliminaries

In this section, we recall some definitions and known results in Menger space.

For more details, we refer the readers to [1- 5,7,8,9,12,17].

Definition 2.1: A binary operation $*$: [0, 1] × [0, 1] → [0, 1] is a continuous t-norm if $*$ satisfying the following conditions:

a. $*$ is commutative and associative;
b. $*$ is continuous;
c. $a * 1 = a$ for all $a \in [0, 1]$;
d. $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, and $a, b, c, d \in [0, 1]$.

Examples: (i) $a * b = ab$; (ii) $a * b = \max\{a + b-1,0\}$; (iii) $a * b = \min\{a, b\}$.

Definition 2.2: (Schweizer and Sklar [16]):- A probabilistic metric space (PM-space) is a pair (X, F), where X is a set and F is a function defined on X x X into the set of distribution function such that if x, y and z are the points of X, then

(i) $F(x, y; 0) = 0$,
(ii) $F(x, y; t) = 1$ iff $x = y$,
(iii) $F(x, y; t) = F(y, x; t)$,
(iv) if $F(x, y; s) = 1$ and $F(y, z; t)=1$, then $F(x, z; s + t) = 1$for all $x, y, z \in X$, s, t > 0.

For each x and y in X and for each real number $t \geq 0$, F(x, y; t) is to be thought of as the probability that the distance from x to y is less than t. Ofcourse, any metric space (X, d) may be regarded as a PM-space.

Proposition 2.3: (Sehgal and Bharucha-Reid [13]) Let (X, d) be a metric space. Then the metric d induces a distribution function F defined by
$F(x, y; t) = H(t - d(x, y))$ for all $x, y \in X$ and $t > 0$. If t-norm $*$ is $a * b = \min\{a, b\}$ for all $a, b \in [0, 1]$ then (X, F, $*$) is a Menger space. Further, (X, F, $*$) is a complete Menger space if (X, d) is complete.

Definition 2.4: A Menger PM–space is a triplet (X, F, $*$) where (X, F) is a PM-space and $*$ is a t-norm with the following condition:
(v) $F(x, z; s + t) \geq F(x, y; s) * F(y, z; t)$ for all x, y, z in X and s, t > 0 .

This inequality is known as Menger's triangle inequality.

Example 2.5: Let X = R, $a * b = \min\{a, b\}$ for all a, b in [0, 1] and

Define, $F(x, y ;t) = \begin{cases} H(t) & \text{if } x = y \\ 1 & \text{if } x \neq y \end{cases}$ where $H(t) = \begin{cases} 0 & \text{if } t \leq 0 \\ t & \text{if } 0<t<1 \\ 1 & \text{if } t \geq 1 \end{cases}$

Then (X, F, $*$) is a Menger-space.

In 1960, Schweizer and Sklar [19] gave a new implication to theory of PM-spaces by introducing the concept of neighborhoods and convergence in PM-spaces as follows.

Definition 2.6: Let (X, F, $*$) be a Menger space. A sequence $\{x_n\}$ in X is said to be

(i) convergent to a point $x \in X$ if for every $\varepsilon > 0$, $1 > \lambda > 0$ and there exists an integer $N = N(\varepsilon, \lambda)$ such that $x_n \in N(\varepsilon, \lambda)$ for all $n \geq N$, i.e., $F(x_n, x, \varepsilon) > 1 - \lambda$ for all $n \geq N$.

(ii) Cauchy if for every $\varepsilon > 0$ and $1 > \lambda > 0$, there exists an integer $N = N(\varepsilon, \lambda)$ such that $F(x_n, x_m, \varepsilon) > 1\ \lambda$ for all $n, m \geq N$. $\lim_{n \to \infty} F(x_{n+p,}\ x_n, t) = 1$ for all $t > 0$ and $p > 0$.

(iii) complete if every Cauchy sequence in X converges to a point in X.

In 1991, Mishra introduced the concept of compatible mapping in PM-space akin to concept of compatibility in metric space introduced by Jungck[18].

Definition 2.7: Self mappings f and g of a Menger space (X,F, $*$) are said to be compatible if and only if $F(fgx_n, gfx_n, t) \to 1$, for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $fx_n, gx_n \to x$ for some x in X.

Definition 2.8: Two self mappings f and g of a Menger space (X, F, $*$) are called weakly commuting if $F(fgx, gfx, t) \geq F(fx, gx, t)$ for all x in X and $t > 0$.

Definition 2.9: Two self mappings f and g of a Menger space (X, F, $*$) are called pointwise R-weakly commuting if there exists $R > 0$ such that

$F(fgx, gfx, t) \geq F(fx, gx, t/R)$ for all x in X and $t > 0$.

Remark 2.10: Clearly, pointwise R-weakly commutativity implies weak commutativity only when $R \leq 1$.

Remark 2.11: It is obvious that f and g can fail to be pointwise R-weakly commuting if there is some x in X such that fx = gx but $fgx \neq gfx$, that is, only if they possess a coincidence point at which they do not commute. This means that a contractive type mapping pair can not possess a common fixed point without being pointwise R-weakly commuting since a common fixed point is also a coincidence point at which the mappings commute and since contractive conditions exclude the possibility of two types of coincidence points, and compatible mappings are necessarily pointwise R-weakly commuting since compatible mappings commute at coincidence points. However, pointwise R-weakly commuting mappings need not to be compatible as shown in the example.

Example 2.12: Let X = [2, 20) with the usual metric d and define

$F(x, y, t) = \frac{t}{t + |x - y|}$ for all $x, y \in X$ and $t > 0$. Clearly (X, F, $*$) is a complete Menger space where $*$ is defined by $a * b = ab$ for all $a, b \in [0, 1]$. Let f and g be self mappings of X defined as

$$fx=\begin{cases}2, & x=2\, or\, x>5\\ 8, & 2<x\le 5\end{cases} \quad \text{and} \quad gx=\begin{cases}2, & x=2\\ 12+x, & 2<x\le 5\\ x-3, & x>5\end{cases}$$

It can be verified that f and g are pointwise R-weakly commuting mappings but not compatible. Also, neither f nor g is continuous, not even at their coincidence points.

Definition 2.13: Two self maps f and g of a Menger space (X, F, $*$) are called reciprocally continuous on X if $\lim_{n\to\infty} fgx_n = fx$ and $\lim_{n\to\infty} gfx_n = gx$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = x$ for some x in X.

Remark 2.14: If f and g are both continuous, then they are obviously reciprocally continuous but the converse is not true. Moreover, in the setting of common fixed point theorems for pointwise R-weakly commuting mappings satisfying contractive conditions, continuity of one of the mappings f or g implies their reciprocal continuity but not conversely.

Lemma 2.15: (Singh and Jain [15]) Let $\{x_n\}$ be a sequence in a Menger space (X, F, $*$) with continuous t-norm $*$ and $t * t \geq t$. If there exists a constant $k \in (0, 1)$ such that $F(x_n, x_{n+1}; kt) \geq F(x_{n-1}, x_n; t)$ for all $t > 0$ and n = 1, 2..., then $\{x_n\}$ is a Cauchy sequence in X.

Lemma 2.16: (Singh and Jain [15]) Let (X, F, $*$) be a Menger space. If there exists k $\in (0, 1)$ such that $F(x, y; kt) \geq F(x, y; t)$ for all x, y$\in$ X and t > 0, then x = y.

Main Results

If A, B, S and T are self mappings of Menger space (X, F, $*$) in the sequel we shall denote $G(x, y, t) = F(Sx, Ax, t) * F(Ty, By, t) * F(Sx, Ty, t) * F(Ty, Ax, \alpha t) * F(Sx, By, (2-\alpha)t)$ for all x, y $\in$ X, $\alpha \in (0, 2)$ and $t > 0$.

We shall need the following lemma for proof of our main Theorem:

Lemma 3.1: Let (X, F, $*$) be a complete Menger space with $a * a \geq a$ for all $a \in [0, 1]$. Let (A, S) and (B, T) be pointwise R-weakly commuting pairs of self mappings of X such that

a. $AX \subseteq TX$, $BX \subseteq SX$;
b. there exists $k \in (0, 1)$ such that $F(Ax, By, kt) \geq G(x, y, t)$ for all x, y $\in$X, $\alpha \in (0, 2)$ and $t > 0$.

Then the continuity of one of the mappings in compatible pair (A, S) or (B, T) on (X, F, $*$) implies their reciprocal continuity.

Proof: Suppose that A and S are compatible and S is continuous. We claim that A and S are reciprocally continuous. Let $\{x_n\}$ be a sequence such that $Ax_n \to z$ and $Sx_n \to z$ for some z in X as $n \to \infty$. Since S is continuous, we have $SAx_n \to Sz$ and $SSx_n \to Sz$ as $n \to \infty$ and since (A, S) is compatible, we also have $\lim_{n \to \infty} F(ASx_n, SAx_n, t) = 1$. This implies that $\lim_{n \to \infty} F(ASx_n, Sz, t) = 1$, that is, $ASx_n \to Sz$ as $n \to \infty$. By (a), for each n, there exists y_n in X such that $ASx_n = Ty_n$. Thus we have $SSx_n \to Sz$, $SAx_n \to Sz$, $ASx_n \to Sz$ and $Ty_n \to Sz$ as $n \to \infty$ whenever $ASx_n = Ty_n$.

We claim that $By_n \to Sz$ as $n \to \infty$. If not, then there exists a subsequence $\{By_m\}$ of $\{By_n\}$ such that for given $t > 0$, there exists a number $\varepsilon > 0$ and a positive integer n_0 such that for all $m > n_0$

$$F(By_m, Sz, t) \leq \varepsilon \text{ and } F(ASx_m, By_m, t) \leq \varepsilon$$

and so $F(ASx_m, By_m, kt) \geq G(Sx_m, y_m, t) = G(Ty_m, By_m, t) = F(ASx_m, By_m, t)$ which is a contraction. Hence $By_n \to Sz$ as $n \to \infty$.

Now, the inequality $F(Az, By_n, kt) \geq F(z, y_n, t)$

On letting $n \to \infty$, implies $F(Az, Sz, kt) \geq F(Az, Sz, t)$.

It follows that $Az = Sz$. Thus, $SAx_n \to Sz$ and $ASx_n \to Sz = Az$ as $n \to \infty$.

Therefore A and S are reciprocally continuous on X. If the pair (B, T) is assumed to be compatible and T is continuous, the proof is similar.

Theorem 3.2: Let (X, F, $*$) be a complete Menger space with $a * a \geq a$ for all $a \in [0, 1]$. Let (A, S) and (B, T) be pointwise R-weakly commuting pairs of self mappings of X such that

a. $AX \subseteq TX$, $BX \subseteq SX$;
b. there exists $k \in (0, 1)$ such that $F(Ax, By, kt) \geq G(x, y, t)$ for all $x, y \in X$, $\alpha \in (0, 2)$ and $t > 0$.

If one of the mappings in compatible pair (A, S) or (B, T) is continuous, then A, B, S and T have a unique common fixed point.

Proof: Suppose that (A, S) are compatible and S is continuous. Then, by lemma3.1, A and S are reciprocally continuous. Let x_0 be any point in X. From condition (a), there exists $x_1, x_2 \in X$ such that $Ax_0 = Tx_1 = y_0$ and $Bx_1 = Sx_2 = y_1$. Inductively, we can construct sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$Ax_{2n} = Tx_{2n+1} = y_{2n}$ and $Bx_{2n+1} = Sx_{2n+2} = y_{2n+1}$ for n = 0, 1, 2, ...

Putting $x = x_{2n}$, $y = x_{2n+1}$ for $t > 0$ and $\alpha = 1 - q$ with $q \in (0, 1)$ in (b), we have

$F(Ax_{2n}, Bx_{2n+1}, kt) \geq F(Sx_{2n}, Ax_{2n}, t) * F(Tx_{2n+1}, Bx_{2n+1}, t) * F(Sx_{2n}, Tx_{2n+1}, t)$

$* F(Tx_{2n+1}, Ax_{2n}, (1 - q)t) * F(Sx_{2n}, Bx_{2n+1}, (1 + q)t),$

$F(y_{2n}, y_{2n+1}, kt) \geq F(y_{2n-1}, y_{2n}, t) * F(y_{2n}, y_{2n+1}, t) * F(y_{2n-1}, y_{2n}, t)$

$* F(y_{2n-1}, y_{2n+1}, (1 + q)t)$

$\geq F(y_{2n-1}, y_{2n}, t) * F(y_{2n}, y_{2n+1}, t) * F(y_{2n-1}, y_{2n}, t) * F(y_{2n}, y_{2n+1}, qt)$

$\geq F(y_{2n-1}, y_{2n}, t) * F(y_{2n}, y_{2n+1}, t) * F(y_{2n}, y_{2n+1}, qt).$

Since t-norm $*$ is continuous, letting $q \to 1$, we have

$F(y_{2n}, y_{2n+1}, kt) \geq F(y_{2n-1}, y_{2n}, t) * F(y_{2n}, y_{2n+1}, t) * F(y_{2n}, y_{2n+1}, t)$

$\geq F(y_{2n-1}, y_{2n}, t) * F(y_{2n}, y_{2n+1}, t).$

It follows that, $F(y_{2n}, y_{2n+1}, kt) \geq F(y_{2n-1}, y_{2n}, t) * F(y_{2n}, y_{2n+1}, t)$.

Similarly, $F(y_{2n+1}, y_{2n+2}, kt) \geq F(y_{2n}, y_{2n+1}, t) * F(y_{2n+1}, y_{2n+2}, t)$. Therefore, for all n even or odd, we have $F(y_n, y_{n+1}, kt) \geq F(y_{n-1}, y_n, t) * F(y_n, y_{n+1}, t)$.

Consequently, $F(y_n, y_{n+1}, t) \geq F(y_{n-1}, y_n, k^{-1}t) * F(y_n, y_{n+1}, k^{-1}t)$.

By a simple induction, we have $F(y_n, y_{n+1}, t) \geq F(y_{n-1}, y_n, k^{-1}t) * F(y_n, y_{n+1}, k^{-m}t)$.

Since $F(y_n, y_{n+1}, k^{-m}t) \to 1$ as $m \to \infty$, it follows that $F(y_n, y_{n+1}, kt) \geq F(y_{n-1}, y_n, t)$ for all $n \in N$ and $t > 0$. Therefore, by lemma 2.15, $\{y_n\}$ is a Cauchy sequence.

Since X is complete, then there exists a point z in X such that $y_n \to z$ as $n \to \infty$. Moreover, $y_{2n} = Ax_{2n} = Tx_{2n+1} \to z$ and $y_{2n+1} = Bx_{2n+1} = Sx_{2n+2} \to z$.

Since A and S are compatible and reciprocally continuous mappings, then

$ASx_{2n} \to Az$ and $SAx_{2n} \to Sz$. Compatibility of A and S yields

$\lim_{n \to \infty} F(ASx_{2n}, SAx_{2n}, t) = 1$, that is, $F(Az, Sz, t) = 1$. Hence $Az = Sz$.

Since $AX \subset TX$, there exists a point w in X such that $Az = Tw$. Using (b), with $\alpha = 1$, we have

$F(Az, Bw, kt) \geq F(Sz, Az, t) * F(Tw, Bw, t) * F(Sz, Tw, t) * F(Tw, Az, t)$

$* F(Sz, Bw, t)$

$= F(Az, Az, t) * F(Az, Bw, t) * F(Az, Az, t) * F(Az, Az, t) * F(Az, Bw, t)$

$\geq F(Az, Bw, t).$

that is, $Az = Bw$. Thus, $Az = Sz = Bw = Tw$. Since A and S are pointwise

R-weakly commuting mappings, there exists $R > 0$ such that

$F(ASz, SAz, t) \geq F(Az, Sz, t/R) = 1.$

that is, $ASz = SAz$ and $AAz = ASz = SAz = SSz$.

Similarly, since B and T are pointwise R-weakly commuting mappings, we have

BBw = BTw = TBw = TTw.

Now, putting x = Az, y = w with α = 1 in (b), we have

F(AAz, Bw, kt) = F(AAz, Az, kt) ≥ F(SAz, AAz, t) ∗ F(Tw, Bw, t)

∗ F(SAz, Tw, t) ∗ F(Tw, AAz, t) ∗ F(SAz, Bw, t)

= F(AAz, AAz, t) ∗ F(Az, Az, t) ∗ F(AAz, Az, t) ∗ F(Az, AAz, t) ∗ F(AAz, Az, t)

≥ F(AAz, Az, t).

that is, Az = AAz and Az = AAz = SAz. Thus, Az is a common fixed point of A and S. Similarly, by using (b), we find that Bw(= Az) is a common fixed point of B and T. Thus, Az is a common fixed point of A, B, S and T.

Uniqueness, suppose that, Aw(≠ Az) is another common fixed point of A, B, S and T. Then, using (b) with α = 1, we have

F(AAz, BAw, kt) = F(Az, Aw, kt) ≥ F(SAz, AAz, t) ∗ F(TAw, BAw, t)

∗ F(SAz, TAw, t) ∗ F(TAw, AAz, t) ∗ F(SAz, BAw, t)

=F(Az, Az, t) ∗ F(Aw, Aw, t) ∗ F(Az, Aw, t) ∗ F(Aw, Az, t) ∗ F(Az, Aw, t)

≥ F(Az, Aw, t).

that is, Az = Aw. Thus, Az is a unique common fixed point of A, B, S and T.

Remark 3.3: In the view of Proposition 1, a ∗ b = min {a, b} then the condition in the above Theorem becomes there exists k ∈ (0, 1) such that
F(Ax, By, kt) ≥ min
$$\{F(Sx,Ax,t), F(Ty,By,t), F(Sx,Ty,t), F(Ty,Ax,\alpha t), F(Sx,By,(2-\alpha)t)\}$$
for all x, y ∈ X, α ∈ (0, 2) and t > 0.

If we take S = T = I_X (the identy mapping on X) in Theorem3.2 , we have the following result:

Corollary 3.4: Let (X, F, ∗) be a complete Menger space with a ∗ a ≥ a for all a ∈ [0, 1], and A, B be self mappings of X. If there exists k ∈ (0, 1) such that F(Ax, By, kt) ≥ F(x, Ax, t) ∗ F(y, By, t) ∗ F(x, y, t) ∗ F(y, Ax, αt) ∗ F(x, By, (2 − α)t) for all x, y ∈ X, α ∈ (0, 2) and t > 0.

If A and B are reciprocally continuous mappings then A and B have a unique common fixed point.

Now, we prove the projection of Theorem3.2 from complete Menger space to complete metric space:

Theorem 3.5: Let A, B, S and T be self mappings on a complete metric space (X, d) satisfying (a) of Theorem3.2. If there exists k ∈ (0, 1) such that

$$d(Ax, By) \leq k \max\left\{\begin{matrix} d(Sx,Ax), d(Ty,By), d(Sx,Ty), \\ [d(Ty,Ax)+d(Sx,By)]/2 \end{matrix}\right\}$$

for all x, y $\in$ X, then A, B, S and T have a unique common fixed point in X.

Proof: The proof follows from Theorem3.2 considering the induced probabilistic metric space (X, M, $*$), where a $*$ b = min {a, b} and F(x, y, t) = $\frac{t}{t+d(x,y)}$.

Example 3.6: Let X = R^+ with the metric d defined by d(x, y) = |x − y| and define F(x, y, t) = $\frac{t}{t+d(x,y)}$ for all x, y $\in$X, t > 0. Clearly (X, M, $*$) is a complete probabilistic metric space. Let A, B, S and T be self mappings on X defined as

$$Ax=\begin{cases}0, & x=0\\ 1, & x>0\end{cases},\quad Bx=\begin{cases}0, & x=0 \text{ or } x>6\\ 2, & 0<x\le 6\end{cases},\quad Sx=\begin{cases}0, & x=0\\ 2, & x>0\end{cases} \text{ and } Tx=\begin{cases}0, & x=0\\ 4, & 0<x\le 6\\ x-6, & x>6\end{cases}.$$

Then A, B, S and T satisfy all the conditions of Theorem3.2 with k $\in$(0, 1) and have a unique common fixed point x = 0. Clearly A and S are reciprocally continuous compatible mappings. However A and S are not continuous, not even at the common fixed point. The mappings B and T are noncompatible because suppose that $\{x_n\}$ be a sequence defined as $x_n = 6+\frac{1}{n}$, n ≥ 1, then $Bx_n = 0$, $Tx_n \to 0$, $TBx_n = 0$ and $BTx_n = 2$, hence, B and T are noncompatible but pointwise R-weakly commuting since they commute at their coincidence points.

References

[1] G. Constantin, I. Istratescu, Elements of Probabilistic Analysis, Ed. Acad. Bucure¸sti and Kluwer Acad. Publ., 1989.

[2] O. Hadzic, Common fixed point theorems for families of mapping in complete metric space, Math. Japon. 29 (1984), 127-134.

[3] T. L. Hicks, Fixed point theory in probabilistic metric spaces, Rev. Res. Novi Sad, 13 (1983), 63-72.

[4] I. Istratescu, On some fixed point theorems in generalized Menger spaces, Boll. Un. Mat. Ital., 5 (13-A) (1976), 95-100.

[5] I. Istratescu, On generalized complete probabilistic metric spaces, Rev. Roum. Math. Pures Appl., XXV (1980), 1243-1247.

[6] G. Jungck, Commuting mappings and fixed points, Amer. Math. Monthly, 83 (1976), 261-263.

[7] G. Jungck, Compatible mappings and common fixed points, Internat. J. Math. Sci., 9 (1986), 771-779.

[8] G. Jungck, B. E. Rhoades, Some fixed point theorems for compatible maps, Internat. J. Math. & Math. Sci., 3 (1993), 417-428.

[9] S. Kutukcu, D. Turkoglu, C. Yildiz, Common fixed points of compatible maps of type (β) on fuzzy metric spaces, Commun. Korean Math. Soc.,in press.
[10] K. Menger, Statistical metric, Proc. Nat. Acad., 28 (1942), 535-537.
[11] S. N. Mishra, Common fixed points of compatible mappings in PMspaces, Math. Japon., 36 (1991), 283-289.
[12] E. Pap, O. Hadzic, R. Mesiar, A fixed point theorem in probabilistic metric spaces and an application, J. Math. Anal. Appl., 202 (1996), 433-449.
[13] V. M. Sehgal, A. T. Bharucha-Reid, Fixed point of contraction mapping on PM spaces, Math. Systems Theory, 6 (1972), 97-100.
[14] S. Sessa, On a weak commutative condition in fixed point consideration, Publ. Inst. Math., 32 (1982), 146-153.
[15] B. Singh, S. Jain, A fixed point theorem in Menger Space through weak compatibility, J. Math. Anal. Appl., 301 (2005), 439-448.
[16] B. Schweizer, A. Sklar, Probabilistic Metric Spaces, North-Holland, Amsterdam,1983.
[17] R. M. Tardiff, Contraction maps on probabilistic metric spaces, J. Math. Anal. Appl., 165 (1992), 517-523.
[18] G. Jungck, Compatible mappings and common fixed points, Internat. J. Math. and Math. Sci., 9(1986), 771-779.
[19] B. Schweizer and A. Sklar, Statistical metric spaces, Pacific J. of Math., 10(1960), 313-324.
[20] P. Balasubramaniam, S. Muralisankar, R.P. Pant, Common fixed points of four mappings in a fuzzy metric space, J. Fuzzy Math. 10(2) (2002), 379-384.
[21] R.P. Pant, K. Jha, A remark on common fixed points of four mappings in a fuzzy metric space, J. Fuzzy Math. 12(2) (2004), 433-437.
[22] B.E. Rhoades, Contractive definitions and continuity, Contemporary Math. 72 (1988), 233-245.