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Quasiparahyponormal Weighted Composition Operators

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Abstract

In this paper, weighted composition operators of quasiparahyponormal on L^2-spaces are characterized and their various properties are studied.

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1. Introduction

Let (X, Σ, λ) be a σ-finite measure space. A transformation T on (X, Σ) is a Σ-measurable mapping from X onto X such that $\lambda \circ T^{-1}$ is absolutely continuous with respect to λ. A weighted composition operator is linear transformation acting on a set of complex valued Σ measurable functions f of the form

$Wf = wf \circ T$ Where w is a complex valued, Σ measurable function.

In case w=1 a.e., W becomes a composition operator, denoted by C_T.The conditional expectation operator $E(./T^{-1}\Sigma) = E(.).E(f)$ is defined for each non-negative measurable function f or for each $f \in L^P(1 \le P)$, and is uniquely determined by the conditions:

(i) $E(f)$ is $T^{-1}\Sigma$ measurable and

(ii) If B is any $T^{-1}\Sigma$ measurable set for which $\int_B f d\lambda$ converges we have $\int_B f d\lambda = \int_B E(f) d\lambda.$

As an operator on L^P, E is the projection onto the closure of the range of C_T. E is the identity on L^P if and only if $T^{-1}\sum=\sum$.

The Radon-Nikodym derivative of λT^{-1} with respect to λ is denoted by h and that of $\lambda \circ T^{-k}$ with respect to λ is denoted by h_k where T^k is obtained by compositing $T-k$ times. Let w_k denote $w(w\circ T)(w\circ T^2).......(w\circ T^{k-1})$ so that $W^k f = w_k\left(f\circ T^k\right)$.

S.Pannayappan [3], has proved weighted composition operators for the class (M,k) . In view of this we proved the following results.

Definition: Quasiparahyponormal operaor.

An operator T$\in$ L (H) is said to be quasiparahyponormal if $\|TT^*x\|^2 \le \|T^2T^{*2}x\|$ for every x$\in$ H, with $\|x\|$ = 1.

In the previous paper [1], we have proved that an operator T$\in$ L(H) is quasiparahyponormal if and only if $(T^2T^{*2})^2+2\lambda(TT^*)^2+\lambda^2\ge 0$ for every $\lambda>0$.The aim of this paper is to generalize the results of quasiparahyponormal weighted composition operator.

2. Main results

Theorem 2.1: Suppose $T^{-1}\sum=\sum$. Then W is quasiparahyponormal if and only if $\left(w^2\left(h\circ T\right)\right)^2 \le {w_2}^2\left(h_2\circ T^2\right)$a.e.

Proof: We have, $W^k f = w_k\left(f\circ T^k\right)$

and $W^* f = h_k E\left(w_k f\right)\circ T^{-k}$

and so
$$W^{*k}W^k f = W^{*k}\left[w_k\left(f\circ T^k\right)\right]$$
$$= h_k E\left({w_k}^2 f\circ T^k\right)\circ T^{-k}$$
$$= h_k E\left({w_k}^2\right)\circ T^{-k} f$$

Also
$$W^*Wf = hE\left(w^2\right)\circ T^{-1} f$$
$$W^kW^{*K} = {w_k}^2\left(h_k\circ T^k\right)$$

W is quasiparahyponormal $\Leftrightarrow \left(W^2W^{*2}\right)^2+2\lambda\left(WW^*\right)^2+\lambda^2\ge 0$ for every $\lambda>0$

$$\Leftrightarrow \left(w_2^2\left(h_2\circ T^2\right)\right)^2+2\lambda\left(w^2\left(h\circ T\right)\right)^2+\lambda^2\ge 0.a.e$$

Which is equivalent to

$$\left(w^2\left(h\circ T\right)\right)^4 \le w_2^2\left(h\circ T^2\right)^2$$
$$\left(w^2\left(h\circ T\right)\right)^2 \le w_2\left(h\circ T^2\right)$$

Corollary 2.2: Suppose $T^{-1}\sum=\sum$. Then W is M-quasiparahyponormal if and only if $\left(w^2\left(h\circ T\right)\right)^2\le Mw_2\left(h_2\circ T^2\right)$ a.e.

Proof:

W is M-quasiparahyponormal

$$\Leftrightarrow M^2\left(W^2W^{*2}\right)^2+2\lambda\left(WW^*\right)^2+\lambda^2\ge 0 \text{ for every } \lambda>0$$

$$\Leftrightarrow M^2\left(w_2^2\left(h_2\circ T^2\right)\right)^2+2\lambda\left(w^2\left(h\circ T\right)\right)^2+\lambda^2\ge 0.a.e$$

Which is equivalent to

$$\left(w^2\left(h\circ T\right)\right)^2\le M\left(w_2\left(h\circ T^2\right)\right)a.e$$

Theorem2.3: Suppose $T^{-1}\sum=\sum$. Then W^* is quasiparahyponormal if and only if $\left(hE\left(w^2\right)\circ T^{-1}\right)^2\le h_2E\left(w_2{}^2\right)\circ T^{-k}$. a.e.

Proof: Since $T^{-1}\sum=\sum, E$ is an identity and so

$$W^{*k}f=h_k\left(w_kf\right)\circ T^{-k}$$

W^* is quasiparahyponormal

$$\Leftrightarrow\left(W^{*2}W^2\right)^2+2\lambda\left(W^*W\right)^2+\lambda^2\ge 0 \text{ for every } \lambda>0$$

$$\Leftrightarrow\left(h_2E\left(w_2{}^2\right)\circ T^{-2}\right)^2+2\lambda\left(hE\left(w^2\right)\circ T^{-1}\right)^2+\lambda^2\ge 0.a.e$$

Which is equivalent to $\left(hE\left(w^2\right)\circ T^{-1}\right)^2\le h_2E\left(w_2{}^2\right)\circ T^{-2}$

Definition: In [2], for each k, an operator T is k-hyponormal if $\left(TT*\right)^k\le\left(T*T\right)^k$.

Theorem2.4: Suppose $T^{-1}\sum=\sum$. Then W is k-hyponormal if and only if $\left[h\left(E(w^2\right)\circ T^{-1}\right]^k\ge w^{2k}\left(h\circ T\right)^k$ *a.e.*

Proof: Suppose W is k-hyponormal

$$\left(W^*W\right)^k\ge\left(WW*\right)^k$$

$$\left[h\left(E(w^2\right)\circ T^{-1}\right]^k\ge w^{2k}\left(h\circ T\right)^k \text{ a.e.}$$

Note: **T** is M-quasi*paranormal iff $\Leftrightarrow M^2\left(T*^3T^3\right)+2\lambda\left(T^*T\right)^2+\lambda^2\left(T*T\right)\ge 0$ for every $\lambda>0$

Theorem2.5: Suppose $T^{-1}\sum=\sum$. W is M-quasi*paranormal if and only if $\left(hE\left(w^2\right)\circ T^{-1}\right)^4\le M\left(h_3E\left(w_3{}^2\right)\circ T^{-3}\right)\left[hE\left(w^2\right)\circ T^{-1}\right]$

Proof: W is M-quasi*paranormal if and only if $M^2\left(W^{*3}W^3\right)+2\lambda\left(W^*W\right)^2+\lambda^2\left(W^*W\right)\geq 0$

$$M^2\left[h_3E\left(w_3^{\ 2}\right)\circ T^{-3}f\right]+2\lambda\left[hE\left(w^2\right)\circ T^{-1}\right]^2+\lambda^2\left(hE\left(w^2\right)\circ T^{-1}\right)\geq 0 .$$

Which is equivalent to $M\left(h_3E\left(w_3^{\ 2}\right)\circ T^{-3}\right)\left[hE\left(w^2\right)\circ T^{-1}\right]\geq\left(hE\left(w^2\right)\circ T^{-1}\right)^4$

Definition:

An operator T belongs to class A if and only if $\left(T^*|T|^2T\right)^{1/2}\geq T^*T$.

Theorem2.6: Suppose $T^{-1}\Sigma=\Sigma$. If W is of class A if and only if $\left[h_2E\left(w_2^{\ 2}\right)\circ T^{-2}\right]\geq\left[hE(w^2)\circ T^{-1}\right]^2$ *a.e*

Proof: Suppose W is of class A if and only if $\left(W^*|W|^2W\right)^{1/2}\geq W^*W$.

i.e, $\left(W^*|W|^2W\right)\geq\left(W^*W\right)^2$

If and only if

$$\left(W^*\left(W^*W\right)W\right)\geq\left(W^*W\right)^2$$

$$W^{*2}W^2\geq\left(W^*W\right)^2$$

$$\Leftrightarrow\left\langle\left(W^{*2}W^2-\left(W^*W\right)^2\right)f,f\right\rangle\geq 0 \text{ for all } f\in L^2$$

$$\Leftrightarrow\int_E\left(\left[h_2E\left(w_2^{\ 2}\right)\circ T^{-2}\right]-\left[hE(w^2)\circ T^{-1}\right]^2\right)|f|^2\,d\lambda\geq 0 \text{ for every } E\in\Sigma$$

$$\Leftrightarrow\left[h_2E\left(w_2^{\ 2}\right)\circ T^{-2}\right]\geq\left[hE(w^2)\circ T^{-1}\right]^2 \; a.e.$$

Definition:

An operator T is w-hypo normal if $|\tilde{T}|\geq|T|\geq|\tilde{T}^*|$

Theorem2.6: Suppose $T^{-1}\Sigma=\Sigma$. If W is w-hypo normal then W satisfies the inequality $\left(\left[h_2E\left(w_2^{\ 2}\right)\circ T^{-2}\right]^{1/2}-2\lambda\left[hE(w^2)\circ T^{-1}\right]^{1/2}+\lambda^2\right)\geq 0$ for all $\lambda>0$.

Proof: Suppose W is w-hypo normal then W satisfies the inequality

$$\left|W^2\right|-2\lambda|W|+\lambda^2\geq 0 \text{ for all } \lambda>0$$

This implies that $\left(W^{*2}W^2\right)^{1/2}-2\lambda\left(W^*W\right)^{1/2}+\lambda^2\geq 0$ for all $\lambda>0$

$$\left\langle \left(\left(W^{*2}W^{2}\right)^{1/2} - 2\lambda\left(W^{*}W\right)^{1/2} + \lambda^{2} \right) f, f \right\rangle \geq 0 \text{ for all } f \in L^{2}$$

$$\int_{E} \left(\left[h_{2}E\left(w_{2}^{\,2}\right) \circ T^{-2} \right]^{1/2} - 2\lambda\left[hE(w^{2}) \circ T^{-1} \right]^{1/2} + \lambda^{2} \right) |f|^{2} \, d\lambda \geq 0 \text{ for every } E \in \Sigma$$

Therefore if W is w-hypo normal then W satisfies the inequality

$$\left(\left[h_{2}E\left(w_{2}^{\,2}\right) \circ T^{-2} \right]^{1/2} - 2\lambda\left[hE(w^{2}) \circ T^{-1} \right]^{1/2} + \lambda^{2} \right) \geq 0 \text{ for all } \lambda > 0$$

References

[1] S.Panayyapan and A.Radharamani.,"The Class of quasiparahyponormal operator"International journal of mathematical analysis, Preprint.

[2] S.Panayyapan andD.Senthi kumar, "K-Hyponormal composition operator" Acta Ciencia Indica. 607(4),pp, 2002.

[3] S.Panayyapan.," Non Hyponormal weighted composition operators"Indian.J. Pure appl. Math.27(10):979-983pp, 1996.