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Saddle-Node Bifurcation Control for an Odd Non-Linearity Problem

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Abstract

The saddle-node bifurcation may occur in the frequency response curves in the cases of primary and superharmonic resonances of a forced single-degree-of-freedom (SDOF) nonlinear system. The appearance of this discontinuous or catastrophic bifurcation may lead to jump and hysteresis phenomena in the steady-state response, where at a certain interval of the control parameter, two stable attractors exit with an unstable one in between. In this paper, a feedback control law is designed to control the saddle-node bifurcation taking place in the resonance response, thus removing or delaying the occurrence of jump and hysteresis phenomena. The structure of feedback control law is determined by analyzing the eigenvalues of the modulation equations. It is shown that three types of feedback linear, nonlinear or a combination of linear plus nonlinear are adequate for the bifurcation control. Also, it is shown by illustrative examples that the proposed feedback control law is effective for controlling the primary resonance responses.

Keywords: nonlinear system, bifurcation control, saddle-node bifurcation, multiple scales, feedback control.

Introduction

In a forced (SDOF) weakly nonlinear system, primary , sub- and super-harmonic resonances may occur if the linear natural frequency and the frequency of an external excitation satisfy a certain relation. There are some dynamical behaviors of a nonlinear system may be undesirable or unwanted in many applications. When a weakly nonlinear system is under resonance conditions, a small-amplitude excitation may produce a relatively large-amplitude response for the primary resonance. In addition, it is well known that saddle-node bifurcation can occur in the steady-state response of a forced (SDOF) nonlinear system in the cases of primary and superharmonic resonances. This kind of bifurcation can lead to jump and hysteresis phenomena in the frequency response curves. It is thus needed to modify the dynamical behavior via a bifurcation control approach. Dynamical behavior of a nonlinear system can be modified to some desirable dynamical behavior by means of various feedback bifurcation control methods[1,2,3,4]. For example, the bifurcation characteristics of a non-autonomous or autonomous nonlinear system can be modified via a linear and nonlinear feedback control[5], also a nonlinear parametric feedback control is proposed to modify the steady-state resonance responses thus to reduce the amplitude of the response and to eliminate the saddle-node bifurcation [6] and an odd non-linearity problem is treated using MMS I and MMS II modified [7]. Maccari considered the bifurcation control for the forced Zakharov- Kusnetsov (Zk) equation by means of delay feedback linear control terms [8]. One of the representative approaches is a combination of linear and nonlinear feedback controls[9,10,11]. The linear feedback term is designed to modify the associated Jacobian matrix of the system, thus delaying the occurrence of unwanted bifurcations but the nonlinear term is used to suppress subcritical and supercritical bifurcations, hence stabilizing the bifurcations[12,13,14,15].One such example in application is the control of rotating stall and surge in axial flow compressors [16]. However, most of the aforementioned studies dealt with autonomous nonlinear systems and thus the feedback control formula was designed on basis of the corresponding linearized model. But recently, there has been great interest in the research on the subject of bifurcation control for non-autonomous systems. For a non-autonomous nonlinear system, the dynamical behavior of the original system is associated with that of the corresponding averaged equations (an autonomous system), which describe the modulation of both amplitude and phase for the resonance response on a slow time scale. The solutions and their stability of those averaged equations correspond to those of the original non-autonomous system. Hence, it is very natural that a feedback control law is established based on analyzing the modified modulations equations of the controlled system. In this paper, we treat with a nonlinear system which exhibit a saddle-node bifurcation. The saddle-node bifurcation in this nonlinear system is an example of discontinous or catastrophic bifurcation. This type of bifurcation may lead to unbounded motion unless an appropraite control is applied.

In this paper, we extended the work of Ji [5] and took the control feedback term up to quintic terms .The object of the future works is to study some dynamical system by using the perturbation technique and the control method to eliminate or at least reduced the undesirable behavior of this system.

Now we describe how to control one of the important bifurcation that occur in most dynamical systems

The primary resonance

Consider a forced SDOF nonlinearity problem in the form

$$\ddot{x} + \omega_0^2 x + \varepsilon(2\mu\dot{x} + \alpha x^3 + \delta x^5) = P\cos(\Omega t)\,, \qquad (1)$$

where ω_0 is the natural frequency, ε is a small positive parameter, μ is the damping coefficient, α and δ are the coefficients of the nonlinearity terms, P is the amplitude of forcing, and Ω is the excitation frequency. Equation (1) is referred as the uncontrolled system.

The frequency- response curves of the system may exhibit saddle-node bifurcations, which result in undesirable jump and hysteresis phenomena. In this section, a general linear-plus-nonlinear feedback control formula is designed to control the occurrence of saddle-node bifurcations in the primary resonance response. The general feedback control law may be assumed in form

$$\begin{aligned} u = \varepsilon(&2k_{11}x + 2k_{12}\dot{x} + k_{31}x^3 + k_{32}x^2\dot{x} + k_{33}x\dot{x}^2 + k_{34}\dot{x}^3 + k_{51}x^5 + k_{52}x^4\dot{x} + k_{53}x^3\dot{x}^2 \\ &+ k_{54}x^2\dot{x}^3 + k_{55}x\dot{x}^4 + k_{56}\dot{x}^5) \end{aligned} \tag{2}$$

where $2k_{11}$ and $2k_{12}$ are scalar linear feedback gains, k_{3i} $(i = 1 \to 4)$ and $k_{5i}(i = 1 \to 6)$ are the nonlinear feedback gains, and ε is introduced as above. Introducing the general feedback control law into equation (1), then the controlled system takes the form

$$\begin{aligned} \ddot{x} + \omega_0^2 x + \varepsilon(2\mu\dot{x} + \alpha x^3 + \delta x^5) = P\cos(\Omega t) + u = p\cos(\Omega t) + \varepsilon(&2k_{11}x + 2k_{12}\dot{x} + k_{31}x^3 \\ &+ k_{32}x^2\dot{x} + k_{33}x\dot{x}^2 + k_{34}\dot{x}^3 + k_{51}x^5 + k_{52}x^4\dot{x} + k_{53}x^3\dot{x}^2 \\ &+ k_{54}x^2\dot{x}^3 + k_{55}x\dot{x}^4 + k_{56}\dot{x}^5) \end{aligned} \tag{3}$$

To analyze the primary resonance for which $\Omega \approx \omega_0$, the excitation amplitude is ordered as $P = \varepsilon f$, and a detuning parameter σ is introduced such that

$$\Omega = \omega_0 + \varepsilon\sigma. \tag{4}$$

An approximate solution of equation (3) can be obtained by a number of perturbation techniques. The method of multiple scales[17,18] is used and for simplicity, we assume a two scale expansion of the solution

$$x(t;\varepsilon) = x_0(T_0, T_1) + \varepsilon x_1(T_0, T_1) + O(\varepsilon^2), \; T_r = \varepsilon^r t, \qquad r = 0,1 \tag{5}$$

The excitation is expressed in terms of T_0 and T_1 as

$$P\cos(\Omega t) = \varepsilon f \cos(\omega_0 T_0 + \sigma T_1). \tag{6}$$

Substituting equations (5) and (6) into (3) and equating the same power of ε on both sides, we obtain a set of linear partial differential equations:

$$D_0^2 x_0 + \omega_0^2 x_0 = 0,$$

$$D_0^2 x_1 + \omega_0^2 x_1 = -2D_0 D_1 x_0 - 2\mu D_0 x_0 - \alpha x_0^3 - \delta x_0^5 + f\cos(\omega T_0 + \sigma T_1) + 2k_{11}x_0$$

$$+2k_{12}D_0x_0+k_{31}x_0^3+k_{32}x_0^2(D_0x_0)+k_{33}x_0(D_0x_0)^2+k_{34}(D_0x_0)^3$$
$$+k_{51}x_0^5+k_{52}x_0^4(D_0x_0)+k_{53}x_0^3(D_0x_0)^2+k_{54}x_0^2(D_0x_0)^3$$
$$+k_{55}x_0(D_0x_0)^4+k_{56}(D_0x_0)^5, \tag{7}$$

where $D_n = \partial/\partial T_n$. Solving the first equation in (7) for $x_0(T_0,T_1)$, we have

$$x_0(T_0,T_1)=A(T_1)e^{iT_0}+\overline{A}(T_1)e^{-iT_0}, \tag{8}$$

Substituting Equation (8) into the second Equation of (7), then to eliminate the secular terms yields

$$-2i\omega_0A'-2i\mu\omega_0A+2k_{11}A+2i\omega_0k_{12}A-3\alpha A^2\overline{A}-10\delta A^3\overline{A}^2+\frac{1}{2}fe^{i\sigma T_1}+3k_{31}A^2\overline{A}$$
$$+i\omega_0k_{32}A^2\overline{A}+\omega_0^2k_{33}A^2\overline{A}+3i\omega_0^3k_{34}A^2\overline{A}+10k_{51}A^3\overline{A}^2+2i\omega_0k_{52}A^3\overline{A}^2+2\omega_0^2k_{53}A^3\overline{A}^2 \tag{9}$$
$$+2i\omega_0^3k_{54}A^3\overline{A}^2+2\omega_0^4k_{55}A^3\overline{A}^2+10i\omega_0^5k_{56}A^3\overline{A}^2=0,$$

Now, suppose

$$A(T_1)\equiv\frac{1}{2}a(T_1)e^{i\beta(T_1)} \tag{10}$$

By substituting Equation (10) into (9) and separating the real part and the imaginary part, we obtain a set of autonomous differential equations that govern the amplitude $a(T_1)$ and the phase $\gamma(T_1)$

$$\begin{cases} a'=-(\mu-k_{12})a+(k_{32}+3\omega_0^2k_{34})\frac{1}{8}a^3+(k_{52}+\omega_0^2k_{54}+5\omega_0^4k_{56})\frac{a^5}{16}+\frac{f}{2\omega_0}\sin(\gamma), \\ a\gamma'=(\sigma+\frac{k_{11}}{\omega_0})a-(3\alpha-3k_{31}-\omega_0^2k_{33})\frac{a^3}{8\omega_0}-(5\delta-5k_{51}-\omega_0^2k_{53}-\omega_0^4k_{55})\frac{a^5}{16\omega_0} \\ \qquad +\frac{f}{2\omega_0}\cos(\gamma). \end{cases} \tag{11}$$

where $\gamma=\sigma T_1-\beta$.

From Equation (11), we have a set of algebraic equations for amplitude a and phase γ of the steady- state primary resonance

$$\begin{cases} -(\mu-k_{12})a+(k_{32}+3\omega_0^2k_{34})\frac{1}{8}a^3+(k_{52}+\omega_0^2k_{54}+5\omega_0^4k_{56})\frac{a^5}{16}+\frac{f}{2\omega_0}\sin(\gamma)=0, \\ (\sigma+\frac{k_{11}}{\omega_0})a-(3\alpha-3k_{31}-\omega_0^2k_{33})\frac{a^3}{8\omega_0}-(5\delta-5k_{51}-\omega_0^2k_{53}-\omega_0^4k_{55})\frac{a^5}{16\omega_0} \\ \qquad +\frac{f}{2\omega_0}\cos(\gamma)=0. \end{cases} \tag{12}$$

whereby we derive the frequency- response relation between a and σ in the form

$$\left[-\mu_1 a + p_1 a^3 + p_1^* a^5\right]^2 + \left[\sigma_1 a - p a^3 - p^* a^5\right]^2 = \frac{f^2}{4\omega_0^2} \tag{13}$$

where,

$$\mu_1 = \mu - k_{12}, \quad \sigma_1 = \sigma + \frac{k_{11}}{\omega_0}, \quad p_1 = \frac{(k_{32} + 3\omega_0^2 k_{34})}{8}, \quad p = \frac{(3\alpha - 3k_{31} - \omega_0^2 k_{33})}{8\omega}$$

$$p_1^* = \frac{(k_{52} + \omega_0^2 k_{54} + 5\omega_0^4 k_{56})}{16}, \qquad p^* = \frac{(5\delta - 5k_{51} - \omega_0^2 k_{53} - \omega_0^4 k_{55})}{16\omega_0}$$

A first-order approximation for the solution of equation (3) can be derived as

$$x = a\cos(\Omega t - \gamma) + O(\varepsilon), \tag{14}$$

Equation (11) is an autonomous dynamical system. The fixed points of this system correspond to the periodic solutions of the original system (3). If all the feedback gains $k_{ij} = 0$, equation (11) corresponds to the modulation equations for the uncontrolled system. It is clearly seen that the addition of feedback control modifies the modulation equations. Consequently, the feedback control is possible to change the nonlinear dynamic characteristics associated with the system. The stability of the steady-state response of the controlled system (3) is associated with that of the fixed points of system (11), which is determined by the eigenvalues of the corresponding Jacobian matrix of equation (11). The eigenvalues are the roots of

$$\begin{aligned} &\lambda^2 + 2(\mu_1 - 2p_1 a^2 - 3p_1^* a^4)\lambda + (\mu_1 - p_1 a^2 - p_1^* a^4)(\mu_1 - 3p_1 a^2 - 5p_1^* a^4) \\ &+ (\sigma_1 - 3pa^2 - 5p^* a^4)(\sigma_1 - pa^2 - p^* a^4) = 0, \end{aligned} \tag{15}$$

From the Routh-Hurwitz criterion, the steady-state vibration is asymptotically stable if and only if the following two inequalities hold simultaneously

$$\mu_1 - 2p_1 a^2 - 3p_1^* a^4 \succ 0,$$

$$(\mu_1 - p_1 a^2 - p_1^* a^4)(\mu_1 - 3p_1 a^2 - 5p_1^* a^4) + (\sigma_1 - 3pa^2 - 5p^* a^4)(\sigma_1 - pa^2 - p^* a^4) \succ 0$$

and are otherwise the system is unstable.

Now, it is necessary to know what is happening in an uncontrolled system due to the existence of bifurcation . The eigenvalues of the Jacobian matrix associated with the steady-state solutions for the corresponding uncontrolled system are obtained from equation (15) by letting all $k_{ij} = 0$, and become the roots of

$$\lambda^2 + 2\mu\lambda + \mu^2 + (\sigma - \frac{3}{8\omega_0}\alpha a^2 - \frac{5}{16\omega_0}\delta a^4)(\sigma - \frac{9}{8\omega_0}\alpha a^2 - \frac{25}{16\omega_0}\delta a^4) = 0. \tag{16}$$

From equation (16), it is easy to see that the sum of the eigenvalues is -2μ, which is negative for $\mu > 0$. Then, this fact rejects the occurrence of a Hopf bifurcation (a pair of purely imaginary eigenvalues). It is found that one of the eigenvalues is zero when the last two terms of equation (11) are equal to zero, where a saddle-node bifurcation occurs. In this case the steady-state response of the system appears a jump. We use this fact to controll the system and a void the appearance of a saddle- node bifurcation by controlling and modifying the appearance of a zero eigenvalue of the associated Jacobian matrix.

From studying for the uncontrolled system if the inequalities $\sigma\alpha > 0$ and $\sigma\delta \succ 0$ are satisfied, there always exists at least a certain value of σ such that an eigenvalue becomes zero, where a saddle-node bifurcation occurs. For simplicity, the proposed feedback formula can then be reduced by setting $k_{12} = 0, k_{32} = k_{34} = 0$ and $k_{52} = k_{54} = k_{56} = 0$.

Based on the foregoing discussion, there are two cases to be considered in the design of the feedback control law.

Case I: to transform the inequalities $\sigma\alpha > 0$ and $\sigma\delta \succ 0$ for the uncontrolled system into the inequalities $\sigma_1 p < 0$ and $\sigma_1 p^* \prec 0$ for the controlled system, hence eliminating the existence of a zero eigenvalue.

Case II: to delay the occurrence of a zero eigenvalue for $\sigma\alpha > 0$ and $\sigma_1 p^* \prec 0$ to larger values of the excitation parameters.

It is clear from equation (15) that the parameter σ_1 for the controlled system can be modified by the linear feedback component k_{11}, whereas the parameters p and p^* can be changed by the nonlinear feedback components $k_{31}, k_{33}, k_{51}, k_{53}$ and k_{55} . This suggests that the bifurcation control can be achieved by three types of feedback.

Controlling the system by using a pure linear feedback

Using the pure linear feedback control $u = 2\varepsilon k_{11} x$, the eigenvalues of the corresponding controlled system are given by

$$\lambda^2 + 2\mu\lambda + \mu^2 + (\sigma + \frac{k_{11}}{\omega_0} - \frac{3}{8\omega_0}\alpha a^2 - \frac{5}{16\omega_0}\delta a^4)(\sigma + \frac{k_{11}}{\omega_0} - \frac{9}{8\omega_0}\alpha a^2 - \frac{25}{16\omega_0}\delta a^4) = 0. \qquad (17)$$

for a suitable choice of linear feedback gain k_{11} renders $(\sigma + \frac{k_{11}}{\omega_0})\alpha < 0$ and $(\sigma + \frac{k_{11}}{\omega_0})\delta < 0$, the two eigenvalues possess negative real parts. The saddle-node bifurcations do not occur in the controlled system and hence this system will not exhibit jump and hysteresis phenomena. Under such a linear feedback, the saddle-node bifurcations are removed from the interval of σ.

Controlling the system by using a pure nonlinear feedback

In this case, the pure nonlinear feedback cannot eliminate the saddle node bifurcations. However, the saddle-node bifurcations can be delayed to reappear at a larger value of the excitation amplitude. In terms of the foregoing discussion, the pure nonlinear feedback is assumed to be of the form $u = \varepsilon(k_{31}x^3 + k_{33}x\dot{x}^2 + k_{51}x^5 + k_{53}x^3\dot{x}^2 + k_{55}x\dot{x}^4)$. Under this pure nonlinear feedback control, the so-called frequency-response equation is of the form

$$a^2[\mu^2 + (\sigma - pa^2 - p^* a^4)^2] = (\frac{f}{2\omega_0})^2 \tag{18}$$

Since the saddle-node bifurcation points exist at the locations of vertical tangency[19,20]. Differentiation of equation (18) implicitly with respect to a^2 and setting $d\sigma/da^2 = 0$, leads to the condition

$$\sigma^2 - (4pa^2 + 6p^* a^4)\sigma + \mu^2 + 3p^2 a^4 + 8pp^* a^6 + 5p^* a^8 = 0, \tag{19}$$

with solutions

$$\sigma_{\pm} = (2pa^2 + 3p^* a^4) \pm \sqrt{(pa^2 + 2p^* a^4)^2 - \mu^2}. \tag{20}$$

For $\mu^2 < (pa^2 + 2p^* a^4)^2$, there exists an interval $\sigma_- < \sigma < \sigma_+$ in which three real solutions of equation (18) exist. In the limit $\mu^2 = (pa^2 + 2p^* a^4)^2$, this interval leads to the point $\sigma = 2pa^2 + 3p^* a^4$. The critical excitation amplitude obtained from equation (18) is

$$f_{crit} = 2\mu\omega_0 \left| \frac{-p + \sqrt{p^2 + 8p^* \mu}}{2p^*} \right|^{\frac{1}{2}}. \tag{21}$$

For $f < f_{crit}$, there is only one solution, while for $f > f_{crit}$, there are three. It can be concluded that, if the nonlinear feedback gains k_{31}, k_{33}, k_{51}, k_{53} and k_{55} are chosen with the same sign of parameters α and δ, the critical excitation amplitude for the controlled system will be greater than that for the uncontrolled system, for instance $f_{crit}(\omega_0 = 3.0, \mu = 0.05, \alpha = 1.0, \delta = 0.5, \Omega = 3.0) = 0.23869$ for the uncontrolled system and $f_{crit}(\omega_0 = 3.0,\ \mu = 0.05,\ \alpha = 1.0,\ \delta = 0.5,\ \Omega = 3.0,\ k_{31} = 0.1,\ k_{33} = 0.1,\ k_{51} = 0.3,$ $k_{53} = 0.1,\ k_{55} = 0.05) = 0.31498$ for the controlled system. As a result, the saddle-node bifurcation is delayed to occur at a higher value of forcing amplitude.

Conrolling the system by using linear and nonlinear feedback

In this case, the candidate feedback control formula is chosen as $u = \varepsilon(2k_{11}x + k_{31}x^3 + k_{33}x\dot{x}^2 + k_{51}x^5 + k_{53}x^3\dot{x}^2 + k_{55}\dot{x}^5)$, hence equation (15) now becomes

$$\lambda^2 + 2\mu\lambda + \mu^2 + (\sigma_1 - 3pa^2 - 5p^* a^4)(\sigma_1 - pa^2 - p^* a^4) = 0. \tag{22}$$

The parameters σ_1, p and p^* can be assigned any desired values by adjusting the feedback gains k_{11}, k_{31}, k_{33}, k_{51}, k_{53} and k_{55}. Thus, the saddle-node bifurcations can be eliminated or delayed to desirable values by an appropriate selection of feedback gains. It should be noted that the peak amplitude a_p of the forced response for the controlled system is given by

$$a_p = \frac{f}{2\mu\omega_0},$$

which is the same as that of the uncontrolled system. This indicates that the control gains modify the bifurcation behavior and do not change the maximum of the forced response amplitude.

Illustration

This section illustrates the applicability of the feedback control developed in section 2. Whenever numerical simulations are performed, the values for the system parameters are chosen as follows: $\omega_0 = 3.0,\ \mu = 0.05,\ \alpha = 1.0,\ \delta = 0.5,\ \Omega = 3.0$ for the primary resonance. For all the frequency – response curves presented here, the solid and broken lines correspond to the stable and unstable solutions, respectively.

For the primary resonance response, the critical excitation amplitude for the uncontrolled system with the given values of the system parameters is $f_{crit} = 0.23869$. If the excitation amplitude is less than this critical value, the uncontrolled system does not exhibit saddle-node bifurcation. On the other hand, if the excitation amplitude is exceeds this critical value, the primary resonance response of the uncontrolled system (1) with $\alpha, \delta \succ 0$ (resp. $\alpha, \delta \prec 0$) will exhibit jump and hystresis phenomena in a certain interval of region $\sigma \succ 0$ (resp. $\sigma \prec 0$). Figure 1 shows typical frequency- response curves of the uncontrolled system (1) under the excitation amplitude $f = 0.26$. The saddle-node bifurcations in the frequency response for the uncontrolled system occur at the external detuning values c1 and c2 respectively. If an experiment was conducted to construct the frequency response curves, jump phenomenon occurring from one stable branch to another stable one would be observed at the saddle-node bifurcations. For the controlled system under introducing a pure linear feedback to the uncontrolled system., no saddle- node bifurcations appear in the frequency- response curves , thereby rejecting jump and hystresis phenomena. Figure 2 shows the frequency- response curves of the controlled system for several linear feedback gains. The linear feedback gains are chosen according to the inequality $(\sigma + \frac{k_{11}}{\omega_0})\alpha \prec 0$ and $(\sigma + \frac{k_{11}}{\omega_0})\delta < 0$. We can see that an appropriate choice of the linear feedback gains removed the saddle-node bifurcations occurring in the uncontrolled system in the same region of detuning σ for the controlled system. Also, this choice can reduce the amplitude of steady- state response. Moreover , the controlled system does not exhibit jump and hysteresis phenomena in the same interval of detuning σ for the uncontrolled system. Figure 3 shows the effect of nonlinear feedback gains on the critical value of the excitation amplitude for appearance of saddle-node bifurcations where the critical excitation amplitude is plotted as a function of a nonlinear feedback gain. We show that the nonlinear feedback gains k_{31} and k_{33} are more effective than the nonlinear feedback gains k_{51}, k_{53} and k_{55} on the critical excitation amplitude also we notice that the feedback gain k_{55} has the weakest effect on f_{crit}. Then the saddle- node bifurcation can be delayed to occur by using an appropriate choice of the nonlinear feedback gains only. The saddle-node bifurcations can also be controlled by a combination of linear and nonlinear feedback as we show in Figure 4 or by using anyone of nonlinear feedback control gains alone as we show in Figure 5 . It can be seen that the saddle-node bifurcations are delayed to occur compared with Figure 1. Moreover, it is easy to note that the amplitude of the steady- state response is greatly reduced (Figure 6). Now, it is shown by illustrative examples that the proposed linear plus nonlinear feedback are effective for controlling the primary resonance response.

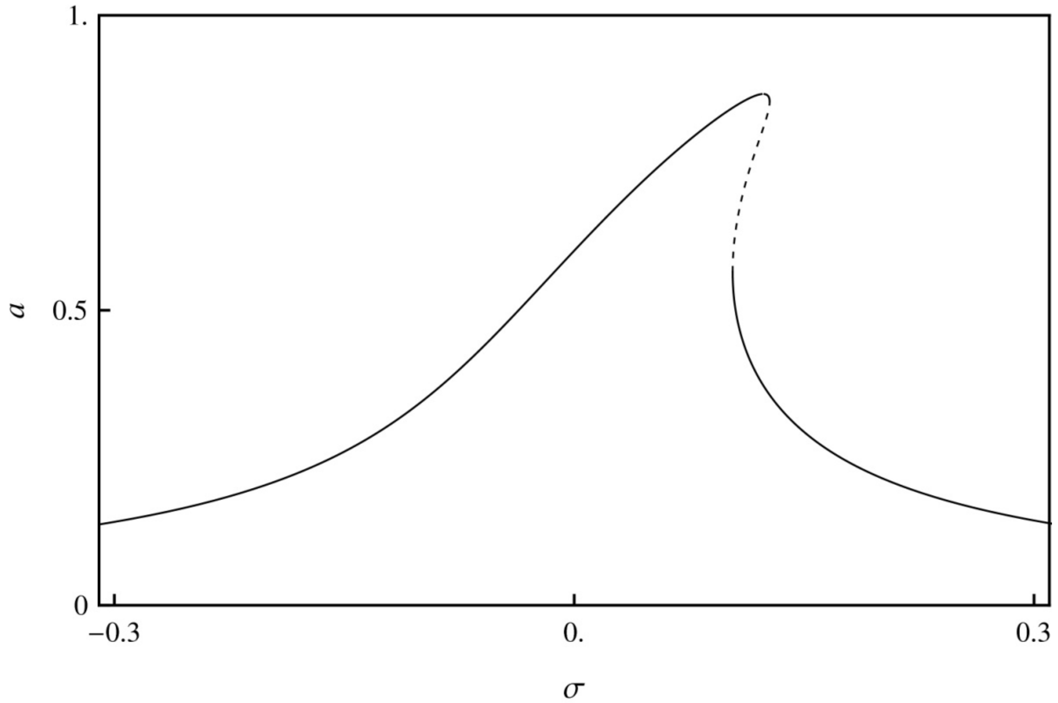


Figure 1: Frequency-response curves for the primary resonance of the uncontrolled system under forcing amplitude $f = 0.26$. Saddle-node bifurcations occur at points C_1 and C_2.

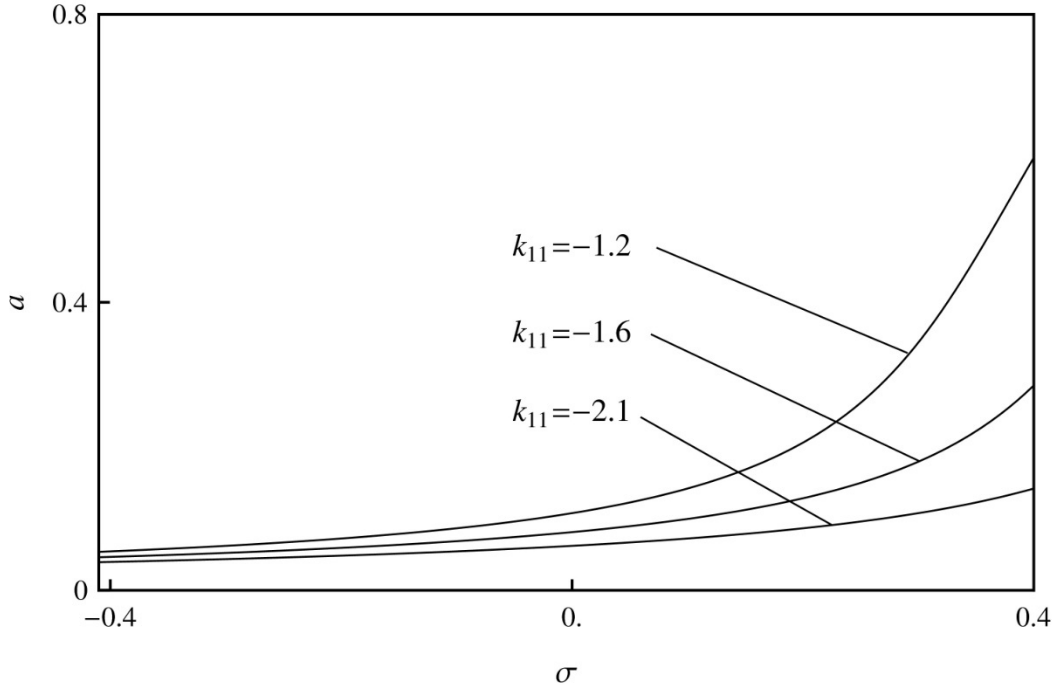
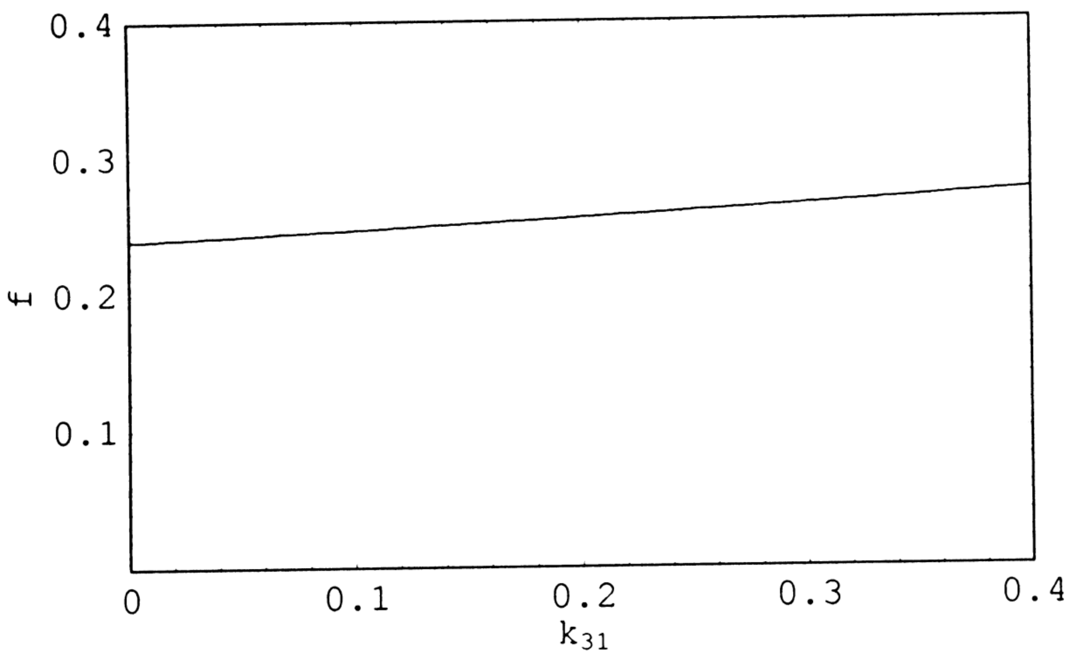


Figure 2: The effect of linear feedback gains on the frequency-response curves for the primary resonance.



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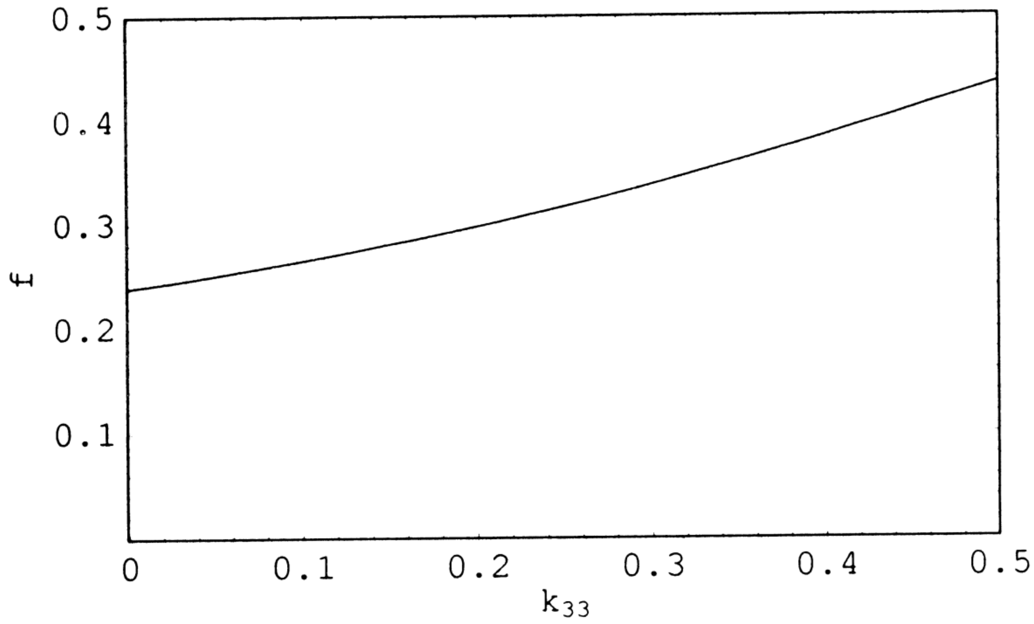

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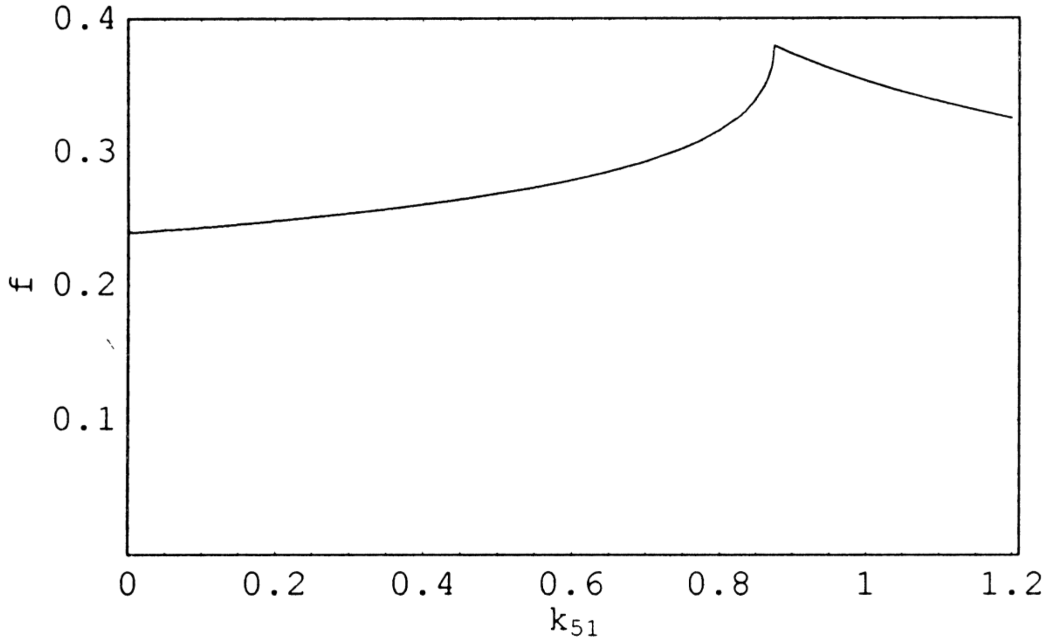

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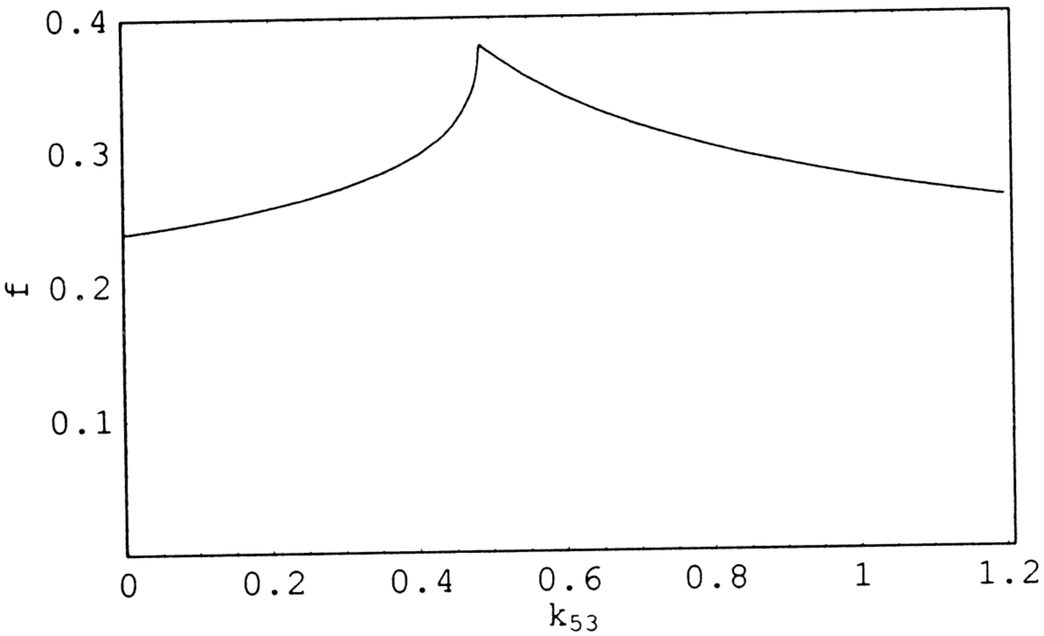

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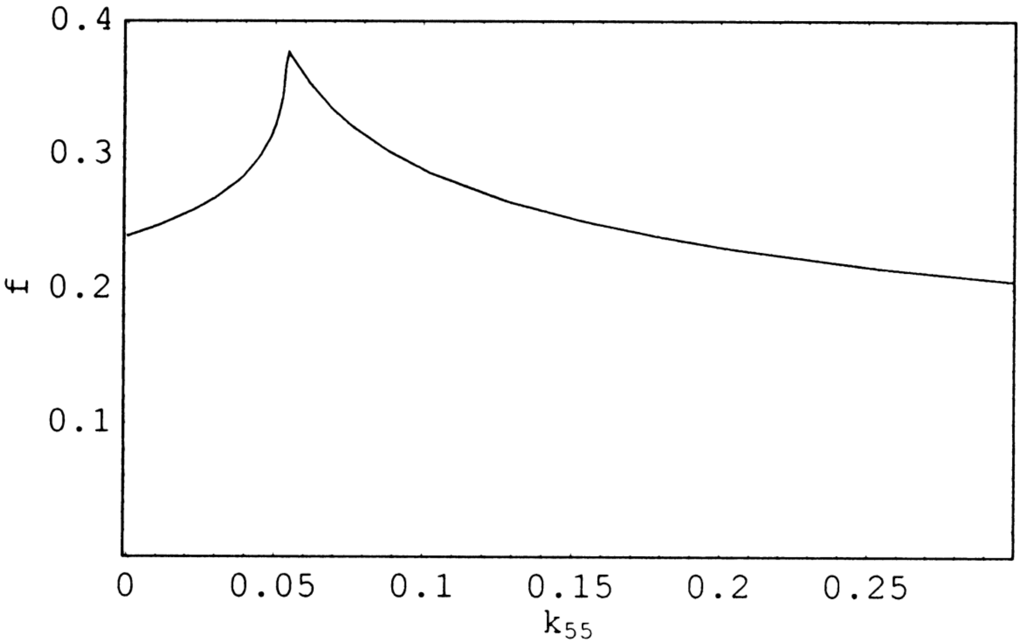

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Figure 3: The effect of nonlinear feedback gains on the critical value of the excitation amplitude for appearance of saddle-node bifurcations.

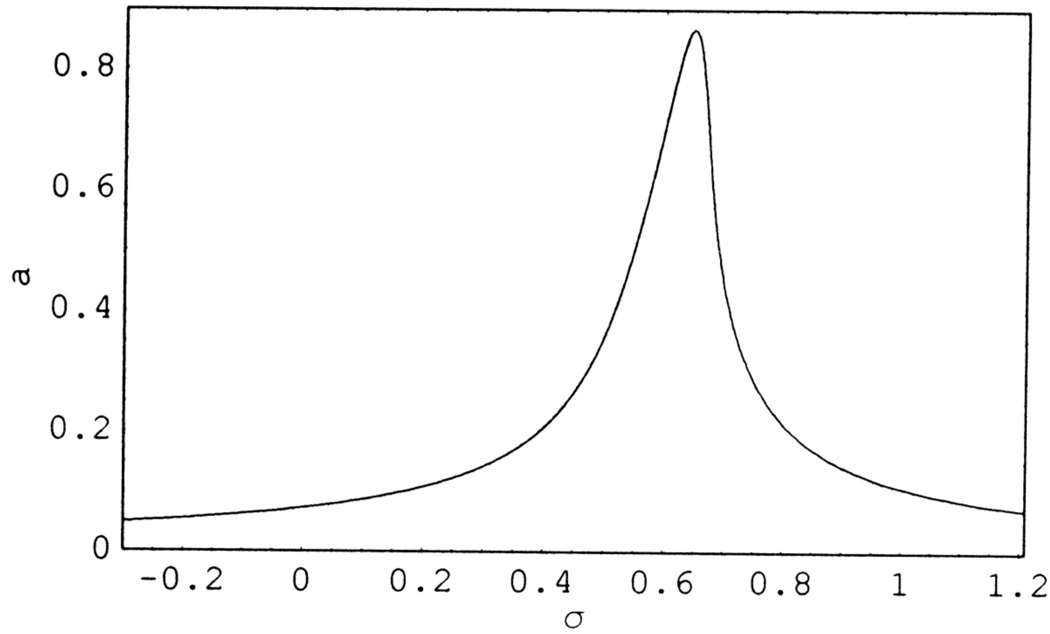
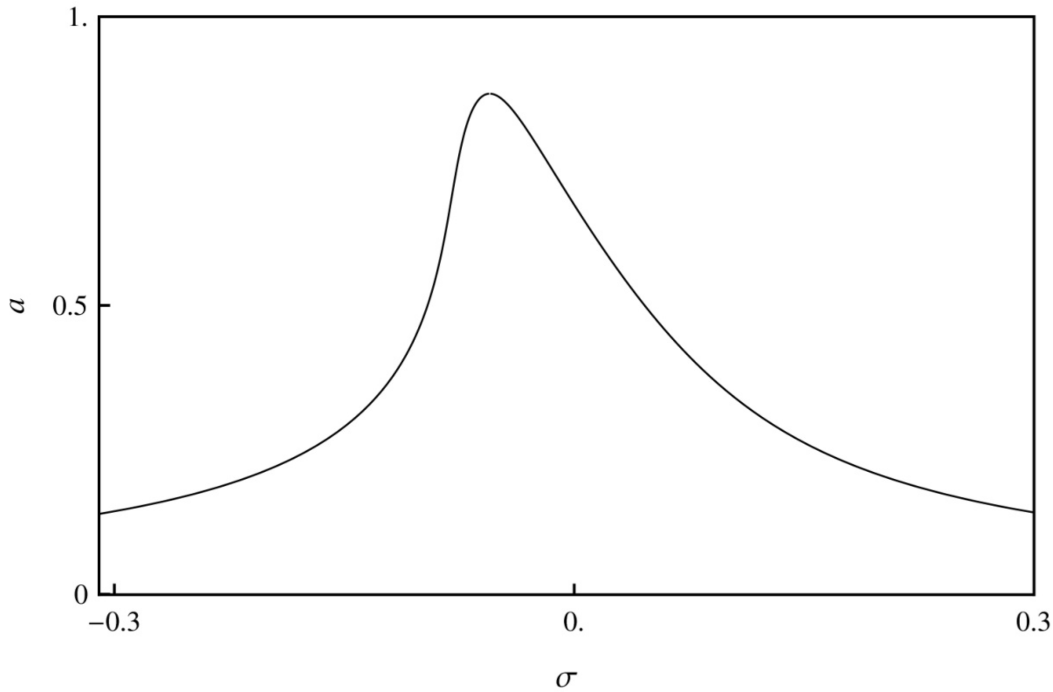


Figure 4: The frequency- response curves for the primary resonance under a combination of linear and nonlinear feedback with control gain $k_{11} = -1.8,\ k_{31} = 0.04,\ k_{33} = 0.07,\ k_{51} = 0.05,\ k_{53} = 0.01,$ and $k_{55} = 0.05.$



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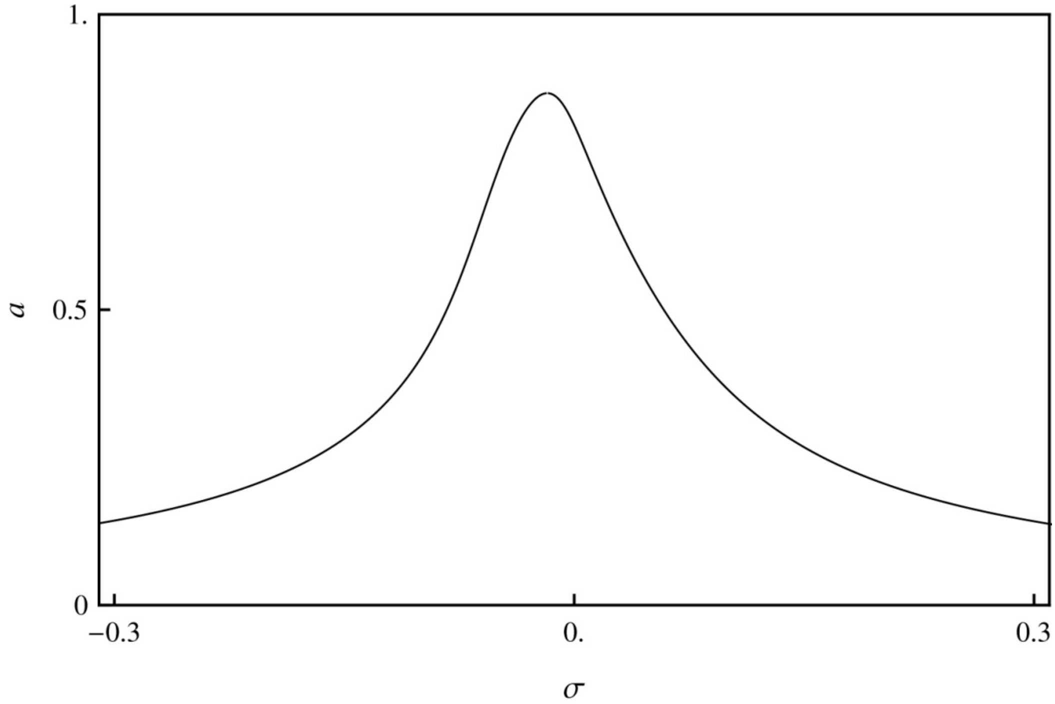

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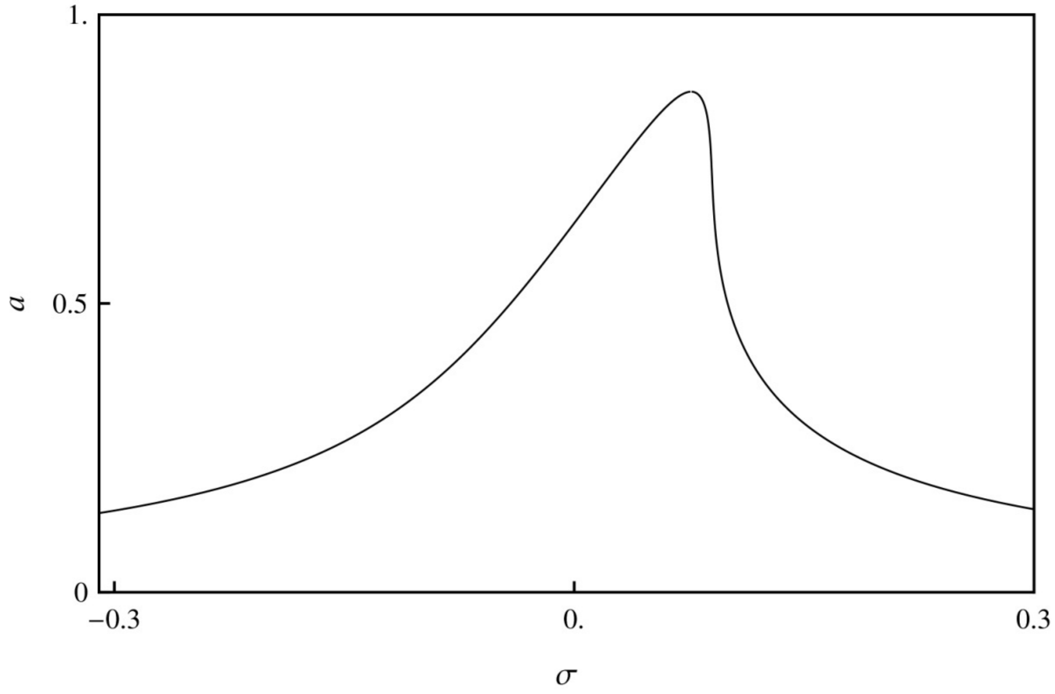

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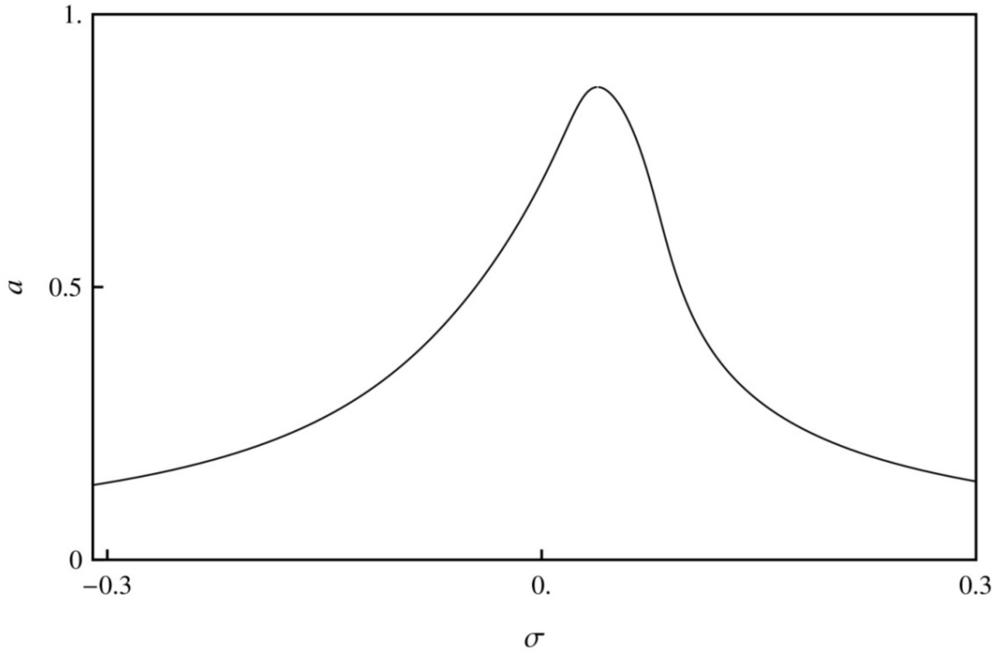
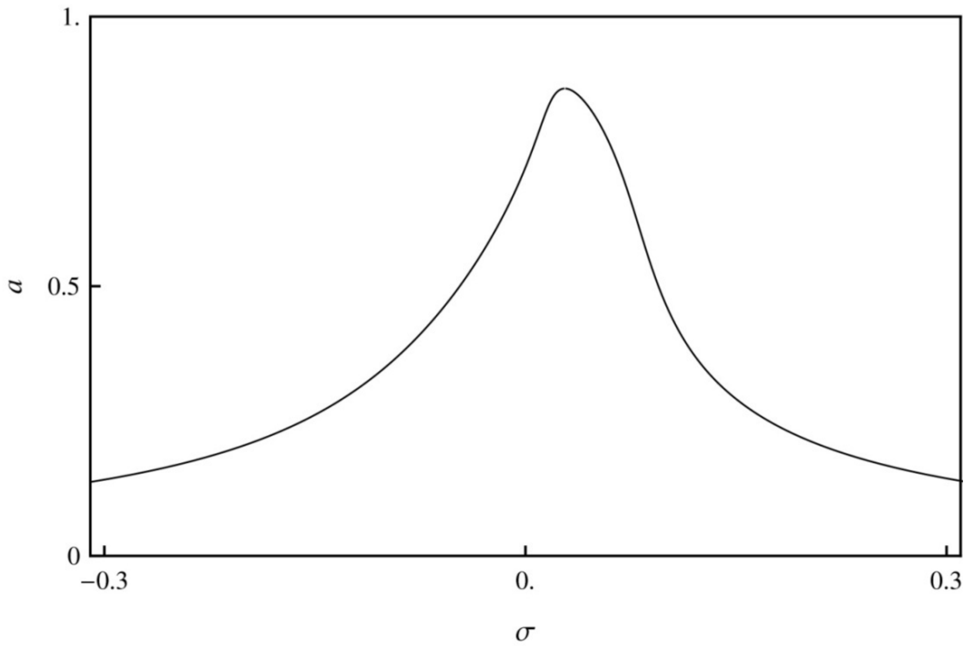


(d)



(e)

Figure 5: The frequency- response curves for the primary resonance by using one nonlinear feedback control gain only. $(a)\,k_{31} = 1.9$, $(b)\,k_{33} = 0.5$, $(c)\,k_{51} = 0.8$, $(d)\,k_{53} = 0.8$, $(e)\,k_{55} = 0.1$.

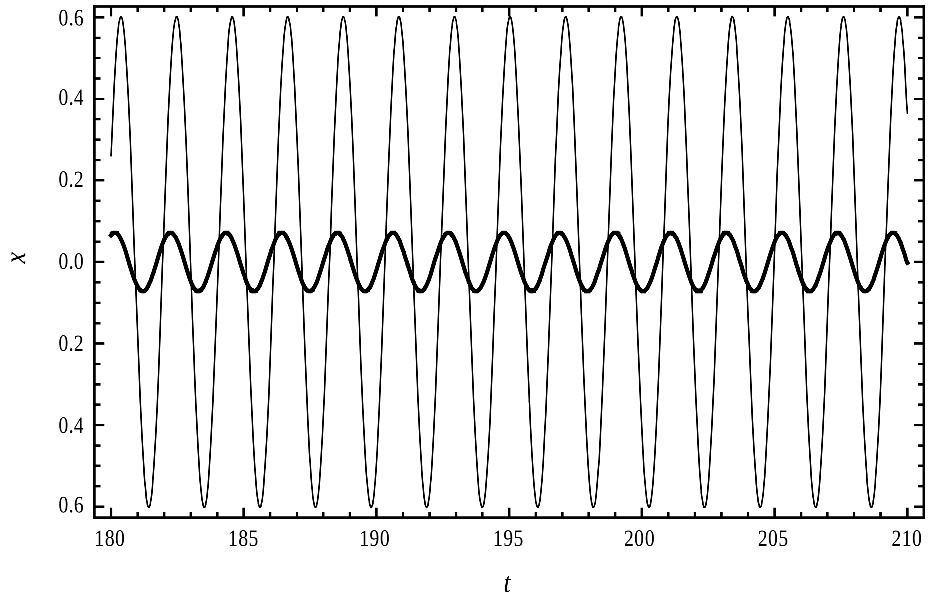


Figure 6: The time histories; thick lines for the controlled system at ($x(0)=0.5, \dot{x}(0)=0.0, k_{11}=-1.8,\ k_{31}=0.04, k_{33}=0.07, k_{51}=0.05, k_{53}=0.01, k_{55}=0.05$) while thin lines are for the uncontrolled system.

Conclusions

The steady-state response of a forced SDOF nonlinear system can exhibit jump and hysteresis phenomena in the case of primary resonance. The saddle-node bifurcations of the fixed points of the averaged equations are responsible for the nonlinear behavior. A general feedback formula is designed to control the saddle-node bifurcations in the steady-state response. It is found that the linear feedback component can eliminate the saddle-node bifurcations which occur in the uncontrolled system, while the nonlinear feedback components can delay the occurrence of saddle-node bifurcations. Also, an appropriate choice of any one of the nonlinear feedback gains can delay the saddle-node bifurcations alone.

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