Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 7, Number 1 (2011), pp. 89–99

http://www.ripublication.com/gjpam.htm

I_{s^*g} -closed Sets in Ideal Topological Spaces

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Abstract

In this paper we introduce and study the notion of I_{s^*g} -closed sets and I_{s^*g} -open sets in ideal topological spaces. A characterization of s^* -normal spaces is given in terms of I_{s^*g} -open sets. It will be seen that I_g -closed sets and I_{s^*g} -closed sets coincide in *-topology as well as in T_1-spaces.

2000 AMS Classification: 54A05.

Keywords: local-function, semi-open set, I_{s^*g} -closed set, I_{s^*g} -open set, s^*g -closed set, s^*g -open set, $*$-closed set, s^* -normal space.

Introduction

Ideals in topological spaces have been considered since 1930. In 1990, Jankovic and Hamlett [9], once again initiated the application of topological ideals and generalized the most fundamental properties in topological spaces. In this regard, compactness [6,16,18], connectedness, resolvability [5], submaximal spaces, extremally disconnected spaces and separation axioms [1] have been generalized via topological ideals in the recent years.

I_g -closed sets were first introduced by Dontchev et al. [4] in 1999. Recently, Navaneethakrishnan and Joseph [15] further investigated and characterized I_g -closed sets and I_g -open sets and obtained some of their properties. In this paper, we define and characterize I_{s^*g} -closed sets and I_{s^*g} -open sets in ideal topological spaces and investigate some of their properties. It will be seen that I_g -closed sets and I_{s^*g} -

closed sets coincide in *-topology as well as in T_1-spaces. A characterization of s^*-normal spaces in terms of I_{s^*g}-open sets is given. It is seen that every s^*g-closed set is I_{s^*g}-closed and every I_{s^*g}-closed set is I_g-closed. Counter examples are given to show that reverse implications are not true in general.

Preliminaries

An ideal *I* on a topological space (X,τ) is a collection of subsets of *X* which satisfies the following properties:

(i) $A\in I$ and $B\subseteq A$ implies $B\in I$.

(ii) $A\in I$ and $B\in I$ implies $A\cup B\in I$.

(X,τ,I) represents the topological space with an ideal *I*. Let *P(X)* be the set of all subsets of *X*, a set operator $()^*:P(X)\to P(X)$, called the local function [11] of *A* with respect to τ and *I*, is defined as follows: for $A\subseteq X$, $A^*(I,\tau)=\{x\in X / U\cap A\notin I$ for every open set *U* containing *x*}. We simply write A^* instead of $A^*(I,\tau)$ in case there is no confusion. X^* is often a proper subset of *X*. For every ideal topological space (X,τ,I), there exists a topology $\tau^*(I)$, finer than τ, generated by $\beta(I,\tau)=\{U\setminus J : U\in\tau \text{ and } J\in I\}$. It is known in [9] that $\beta(I,\tau)$ is not always a topology on *X*. A subset *A* of an ideal space (X,τ,I) is called τ^*-closed [9] or simply *-closed (resp. *-dense in it self [7]) if $A^*\subseteq A$ (resp. $A\subseteq A^*$). A Kuratowski closure operator $cl^*()$ for a topology $\tau^*(I,\tau)$, called the *-topology, is defined by $cl^*(A)=A\cup A^*(\tau,I)$ [20]. By a space, we always mean a topological space (X,τ) with no separation properties assumed. For a subset *A* of *X*, *cl(A)* (resp. *scl(A)*) and *Int(A)* (resp. *sInt(A)*) denotes the closure (resp. semi-closure) of *A* and the interior (resp. semi-interior) of *A* in (X,τ). Similarly $cl^*(A)$ and $\text{int}^*(A)$ will represent the closure and the interior of *A* respectively in (X,τ^*).

Definition 2.1 A subset A of an ideal space (X,τ) is said to be semi-open [12] if there exists an open set *U* in *X* such that $U\subseteq A\subseteq cl(U)$.

α-open [17] if $A\subset Int(cl(Int(A)))$.

I_g-closed [4] if $A^*\subseteq U$ whenever $A\subseteq U$ and U is open in *X*.

g-closed [13] if $cl(A)\subseteq U$ whenever $A\subseteq U$ and *U* is open in *X*.

s^*g-closed [10] if $cl(A)\subseteq U$ whenever $A\subseteq U$ and U is semi-open in *X*.

$g\alpha$-closed [14] if $\alpha cl(A)\subseteq$ U whenever $A\subseteq U$ and *U* is $\alpha-open$ in *X*.

gs-closed [2] if $scl(A)\subseteq U$ whenever $A\subseteq U$ and *U* is open in *X*.

The complement of a semi-open (resp. α-open, I_g-closed) set is semi-closed (resp. α-closed, I_g-open). $SO(X)$ (resp. $SC(X,x)$) represents the collection of all semi-open sets (resp. semi-closed sets containing *x*) in *X*.

I_{s^*g} -closed sets

Definition 3.1 A subset A of an ideal space (X,τ,I) is said to be I_{s*g}-closed if $A^* \subseteq U$ whenever $A \subseteq U$ and U is semi-open in *X*. The complement of an I_{s*g}-closed set is said to be I_{s*g}-open.

Remark 3.1 Every I_{s*g}-closed set is I_g-closed but the converse is not true in general. To see this, let $X = \{a,b,c,d\}$ with $\tau = \{\phi, X, \{a,b\}\}$ and $I = \{\phi, \{a\}\}$. Then $A = \{d\}$ is I_g-closed set but it is not I_{s*g}-closed, since $A^* = \{c,d\}$ and $\{a,b,d\}$ is a semi-open set containing A but it is not containing A^*.

Remark 3.2 (1) Every $*$-closed set is I_{s*g}-closed but not conversely. To see this, let $X = \{a,b,c\}$ with $\tau = \{\phi, X, \{a,b\}, \{c\}\}$ and $I = \{\phi, \{a\}, \{c\}, \{a,c\}\}$. Then $A = \{b,c\}$ is I_{s*g}-closed but it is not $*$-closed.

(2) Every $*$-closed set is I_g-closed. Converse is true if X is a T_1-space.

(3) In T_1-space, I_{s*g}-closed sets and I_g-closed sets coincide.

Remark 3.3

(1) I is I_{s*g}-closed in an ideal space (X,τ,I).

(2) A^* is I_{s*g}-closed for every subset A of (X,τ,I).

Remark 3.4 The following diagram shows the interrelation between the resulting notion of I_{s*g}-closed sets and related concepts. Reverse implications do not hold.

$$
\begin{array}{ccccc}
 & & *-closed & & \\
 & & \swarrow \qquad \searrow & & \\
 & & I_{s^*g}-closed & \rightarrow & I_g-closed \\
 & & \uparrow & & \uparrow \\
closed & \rightarrow & s^*g-closed & \rightarrow & g-closed \\
\downarrow & & \downarrow & & \downarrow \\
\alpha-closed & \rightarrow & g\alpha-closed & \rightarrow & gs-closed
\end{array}
$$

Remark 3.5 In an ideal space (X,τ,I), I_{s*g}-closed sets are generalization of s^*g-closed sets which is itself a generalization of the closed set. An I_{s*g}-closed set may not be s^*g-closed. To see this, let $X=\{a,b,c,d\}$ with $\tau=\{\phi,X,\{a,b\},\{a,b,c\}\}$ and $I=\{\phi,\{a\},\{c\},\{a,c\}\}$. Then $A=\{a,d\}$ is I_{s*g}-closed set but it is not s^*g-closed. Since $\{a,b,d\}$ is a semi-open set containing A but it is not containing $cl(A)$. An I_{s*g}-closed set is s^*g-closed if $I=\{\phi\}$.

Theorem 3.1 Let (X,τ,I) be an ideal space and A a non-empty subset of X. Then the following statements are equivalent:

(1) A is I_{s*g}-closed.

(2) $cl^*(A)\subseteq U$ for every semi-open set U containing A.

(3) For all $x\in cl^*(A)$, $scl(\{x\})\cap A\neq\phi$.

(4) $cl^*(A)-A$ contains no non empty semi-closed set.

(5) A^*-A contains no non empty semi-closed set.

Proof. $(1)\Rightarrow(2)$ Let A be an I_{s*g}-closed set. Then clearly $cl^*(A)\subseteq U$ whenever $A\subseteq U$ and U is semi-open in X.

$(2)\Rightarrow(3)$. Suppose $x\in cl^*(A)$. If $scl(\{x\})\cap A=\phi$, then $A\subseteq X-scl(\{x\})$ where $X-scl(\{x\})$ is a semi-open set. By (2), $cl^*(A)\subseteq X-scl(\{x\})$. This contradicts the fact that $x\in cl^*(A)$. Hence $scl(\{x\})\cap A\neq\phi$. This proves (3).

$(3)\Rightarrow(4)$. Suppose $F\subseteq cl^*(A)-A$ where $F\in SC(X,x)$. Since $F\subset X-A$ and $\{x\}\subseteq F$. This implies $scl\{x\}\subseteq F$ and $scl(\{x\})\cap A=\phi$. Since $x\in cl^*(A)$, by (3) $scl(\{x\})\cap A\neq\phi$, a contradiction. This proves (4).

$(4)\Rightarrow(5)$. Assume that $F\subseteq A^*-A$ where $F\in SC(X)$ and $F\neq\phi$. This gives $F\subseteq cl^*(A)-A$. This contradicts (4).

$(5)\Rightarrow(1)$ Let $A\subseteq U$ where $U\in SO(X)$ such that $A^*\not\subset U$. This gives $A^*\cap(X-U)\neq\phi$ or $A^*-[X-(X-U)]\neq\phi$. This gives $A^*-U\neq\phi$. Moreover, $A^*-U=A^*\cap(X-U)$ is semi-closed in X since $A^*=cl(A^*)$ is closed in X by [9, Theorem 2.3(c)] and $(X-U)\in SC(X)$. Also $A^*-U\subseteq A^*-A$. This gives that A^*-A contains a non empty semi-closed set. This contradicts (5). This completes the proof.

Theorem 3.2 Let (X,τ,I) be an ideal space and A be a I_{s*g}-closed set. Then following statements are equivalent:

(1) A is $*$-closed set.

(2) $cl^*(A)-A$ is a semi-closed set.

(3) $A^* - A$ is a semi-closed set.

Proof. $(1) \Rightarrow (2)$ Let A be $*$-closed set. Then $A^* - A = \phi$. Now $A^* - A = cl^*(A) - A$ gives $cl^*(A) - A = \phi$. This proves that $cl^*(A) - A$ is semi-closed set.
$(2) \Rightarrow (3)$. This is trivial.
$(3) \Rightarrow (1)$. Let $A^* - A$ be a semi-closed set. Now A is I_{s^*g}-closed and by Theorem 2.1(5), $A^* - A$ contains no non-empty semi-closed set, therefore $A^* - A = \phi$. This proves $A^* \subset A$ and hence A is $*$-closed.

Theorem 3.3 In an ideal space (X, τ, I), an I_{s^*g}-closed and $*$-dense set in-itself is s^*g-closed.

Proof. Suppose A is $*$-dense in itself and I_{s^*g}-closed in X. Let U be any semi-open set containing A, then by Theorem 2.1 (2) $cl^*(A) \subset U$. Since A is $*$-dense in itself, $A \subset A^*$. By [19, Theorem 5] $A^* = cl(A^*) = cl(A) = cl^*(A)$. We get $cl(A) \subset U$ whenever $A \subset U$. This proves that A is s^*g-closed.

Corollary 3.1 Let A be a semi-open and I_{s^*g}-closed subset of an ideal space (X, τ, I) where I is codense in X. Then A is s^*g-closed.

Proof. By [19, Theorem 3], A is $*$-dense in itself and hence by Theorem 2.3, A is s^*g-closed.

Theorem 3.4 Let (X, τ, I) be an ideal space. If A and B are subsets of X such that $A \subset B \subset cl^*(A)$ and A is I_{s^*g}-closed then B is I_{s^*g}-closed.

Proof. Since A is I_{s^*g}-closed set, by the Theorem 2.1(5), $cl^*(A) - A$ contains no non-empty semi-closed set. Since, $A \subset B \subset cl^*(A)$ implies, $cl^*(B) - B \subset cl^*(A) - A$. So $cl^*(B) - B$ contains no non-empty semi-closed set. By Theorem 2.1(4), B is I_{s^*g}-closed.

Theorem 3.5 Let (X, τ, I) be an ideal space and $A \subseteq X$. Then A is I_{s^*g}-closed if and only if $A = F - N$, where F is $*$-closed and N contains no non-empty semi-closed set.

Proof. If A is I_{s^*g}-closed set then by the Theorem 2.1(5), $N = A^* - A$ contains no non-empty semi-closed set. Let $F = cl^*(A)$, then F is $*$-closed set and $F - N = (A \cup A^*) - (A^* - A) = A$.

Conversely, let U be any semi-open set in X containing A, then $F - N \subseteq U$ implies $F \cap (X - U) \subseteq F \cap [X - (F \cap N')] = F \cap [(X - F) \cup N] = F \cap N \subseteq N$. By hypothesis $A \subseteq F$ and $F^* \subset F$ as F is *-closed gives $A^* \cap (X - U) \subset F^* \cap (X - U) \subset F \cap (X - U) \subset N$, where $A^* \cap (X - U)$ is a semi-closed set. By hypothesis $A^* \cap (X - U) = \phi$ or $A^* \subseteq U$ implies A is I_{s*g}-closed set.

Lemma 3.1 [4, Lemma 2.6] If A *and* B are subsets of an ideal space (X, τ, I), then $(A \cap B)^* \subseteq A^* \cap B^*$.

Theorem 3.6 Let (X, τ, I) be an ideal space. If A is I_{s*g}-closed and B is $*$-closed in X, then $A \cap B$ is I_{s*g}-closed.

Proof. Let U be a semi-open set in X containing $A \cap B$. Then $A \subseteq U \cup (X - B)$. Since A is I_{s*g}-closed, therefore $A^* \subseteq U \cup (X - B)$ or $B \cap A^* \subseteq U$. Using Lemma 2.1, $(A \cap B)^* \subseteq A^* \cap B^* \subseteq A^* \cap B \subseteq U$ because B is *-closed. This proves that $A \cap B$ is I_{s*g}-closed.

Theorem 3.7 Let (X, τ, I) be an ideal space and A a non empty subset of X. A is I_{s*g}-open if and only if $F \subseteq \text{int}^*(A)$ whenever $F \subseteq A$ and $F \in SC(X)$.

Proof. Suppose A is I_{s*g}-open set and $F \subseteq A$, where $F \in SC(X)$. Then $X - A \subseteq X - F$.

By Theorem 3.1(2), $cl^*(X - A) \subseteq X - F$. This proves $F \subseteq \text{int}^*(A)$. Conversely, Let U be any semi-open set containing $X - A$. Then $X - U \subseteq A$. By hypothesis, $X - U \subseteq \text{int}^*(A)$. This implies $cl^*(X - A) \subseteq U$. By Theorem 3.1(1) $X - A$ is I_{s*g}-closed or A is I_{s*g}-open.

Theorem 3.8 Let A be an I_{s*g}-open set in an ideal space (X, τ, I) and $\text{int}^*(A) \subset B \subset A$. Then B is I_{s*g}-open.

Proof. Let F be any semi-closed set in X contained in B. Then $F \subseteq A$. Since A is I_{s*g}-open. Therefore, by Theorem 3.7, $F \subseteq \text{int}^*(A)$. But $\text{int}^*(A) \subseteq \text{int}^*(B)$, implies $F \subseteq \text{int}^*(B)$. By Theorem 3.7, B is I_{s*g}-open.

Theorem 3.9 Let (X, τ, I) be an ideal space and A a non empty subset of X. Then A is I_{s*g} -closed if and only if $A \cup (X - A^*)$ is I_{s*g} -closed.

Proof. Suppose A is I_{s*g}-closed. Let U be a semi-open set such that $A \cup (X - A^*) \subset U$. Then $X - U \subset X - (A \cup (X - A^*)) = A^* - A$.

Since A is I_{s*g}-closed, by Theorem 2.1(5) , $X - U = \phi$ and hence $X = U$. Thus X is the only set containing $A \cup (X - A^*)$. This gives $[A \cup (X - A^*)]^* \subset X$. This proves $A \cup (X - A^*)$ is I_{s*g} -closed.

Conversely, let F be any semi-closed set such that $F \subset A^* - A$. Since $A^* - A = X - (A \cup (X - A^*))$. This gives $A \cup (X - A^*) \subset X - F$ and $X - F$ is semi-open. By hypothesis, $(A \cup (X - A^*))^* = X - F$ and hence $F \subset X - A^*$. Since $F \subset A^* - A$ it proves that $F = \phi$ and hence $A^* \subset X - F \in SO(X)$. This proves that A is I_{s*g} -closed.

Theorem 3.10 Let (X, τ, I) be an ideal space and $A \subseteq X$. Then $A \cup (X - A^*)$ is I_{s*g} -closed if and only if $A^* - A$ is I_{s*g} -open.

Proof. Let $A \cup (X - A^*)$ be I_{s*g} -closed. We show that $X - (A^* - A)$ is I_{s*g} -closed. Let U a be semi-open set containing $X - (A^* - A)$. Then $X - U \subseteq A^* - A$. By Theorem 2.1(5), $X - U = \phi$. Therefore X is the only semi-open set which contains $X - (A^* - A)$ and hence $(X - (A^* - A))^* \subseteq X$. This proves $X - (A^* - A)$ is I_{s*g} -closed or $A^* - A$ is I_{s*g} -open.

Conversely, let $A^* - A$ be I_{s*g} -open. Then $X - (A^* - A) = A \cup (X - A^*)$ is I_{s*g} -closed.

Corollary 3.2 Let (X, τ, I) be an ideal space and $A \subseteq X$. Then A is I_{s*g} -closed if and only if $A^* - A$ is I_{s*g} -open.

Theorem 3.11 Let (X, τ, I) be an ideal space. Then every subset of X is I_{s*g} -closed if and only if every semi-open set is $*$-closed.

Proof. Suppose every subset of X is I_{s*g} -closed. Let U be a semi-open set then U is I_{s*g} -closed and $U^* \subset U$. Hence U is $*$-closed.

Conversely, suppose that every semi open set is $*$-closed. Let A be non empty subset of X contained in a semi-open set U. Then $A^* \subset U^*$ implies $A^* \subset U$. This proves that A is I_{s*g} -closed.

Example 3.1 Consider $\Re$ the set of all real numbers with the usual topology. If $I = P(\Re)$ *then* $A^* = \phi$ for every subset A of X or $A^* \subset A$. This proves that A is *-closed.

Definition 3.2 [10] The intersection of all semi-open subsets of a space X containing set A is known as semi kernel of A and is denoted by $s\ker(A)$.

Lemma 3.2 A *-dense in itself subset A of a space X is I_{s*g} -closed if and only if $A^* \subseteq s\ker(A)$.

Proof. Assume that an I_{s*g} -closed set A is a *-dense in itself. Then by [19,Theorem 5], $A^* = cl(A)$. But $A^* \subseteq \bigcap\{G : A \subseteq G \text{ and } G \in SO(X)\} = s\ker(A)$. The converse is trivial.

Lemma 3.3 [8, Lemma 2]Every singleton $\{x\}$ in a space X is either no-where dense or preopen.

Theorem 3.12 Arbitrary intersection of *-dense in itself, I_{s*g} -closed sets in an ideal space (X, τ, I) is I_{s*g} -closed.

Proof. Let $\{A_\alpha : \alpha \in \Omega\}$ be an arbitrary collection of *-dense, I_{s*g} -closed sets in an ideal space (X, τ, I) and let $A = \bigcap A_\alpha$. Let $x \in A^*$. In view of Lemma 2.3, we consider the following two cases.

Case 1: $\{x\}$ is no-where-dense. If $x \notin A$, then for some $j \in \Omega$, we have $x \notin A_j$. Since no-where-dense subsets are semi-closed [3, Theorem 1.3], therefore $x \notin s\ker(A_j)$. Again by Lemma 2.2, $A_j^* \subseteq s\ker(A_j)$. Since A_j is *-dense in itself, I_{s*g} -closed implies $x \in A^* = cl(A) \subseteq cl(A_j) \subseteq s\ker(A_j)$. By contradiction $x \in A$ and hence $x \in s\ker(A)$. This proves that $A^* \subseteq s\ker(A)$ and hence by Lemma 2.2, A is I_{s*g} -closed.

Case2: $\{x\}$ is pre-open. Let $F = Int(Cl(\{x\}))$. Assume that $x \notin s\ker(A)$. Then, there exist a semi-closed set C containing x such that $C \cap A = \phi$. Now by [3, Theorem 1.2] $x \in F = Int(Cl(\{x\})) \subseteq Int(Cl(C)) \subseteq C$. Since F is an open set containing x and $x \in cl(A) = A^*$, therefore, $F \cap A \neq \phi$. Since $F \subseteq C$ therefore $C \cap A \neq \phi$. A contradiction. Hence $x \in s\ker(A)$. By Lemma 2.2, A is I_{s*g} -closed.

Lemma 3.4 [4]Let $\{A_i : i \in \Omega\}$ be a locally finite family of sets in an ideal space (X, τ, I). Then $\bigcup_{i\in\Omega} A_i^*(I) = (\bigcup_{i\in\Omega} A_i)^*(I)$.

Theorem 3.13 Let (X, τ, I) be an ideal space. If $\{A_i : i \in \Omega\}$ is a locally finite family of sets and each A_i is I_{s*g} -closed in (X, τ, I). Then $\bigcup_{i\in\Omega} A_i$ is I_{s*g} -closed.

Proof. Let $\bigcup_{i\in\Omega} A_i \subseteq U$ where U is semi-open set in X. Since for each i, A_i is I_{s*g} -closed, $A_i^* \subseteq U$ for each $i \in \Omega$. Hence $\bigcup_{i\in\Omega} A_i^* \subseteq U$. Using Lemma 2.4, $(\bigcup_{i\in\Omega} A_i)^* \subseteq U$. Hence $\bigcup_{i\in\Omega} A_i$ is I_{s*g} -closed.

Theorem 3.14 Union of two I_{s*g} -closed sets is I_{s*g} -closed.

Proof. Let A, B be I_{s*g} -closed sets and W be a semi-closed set such that $A \cup B \subseteq W$. This implies $A^* \subseteq W$ *and* $B^* \subseteq W$. This implies $A^* \cup B^* = (A \cup B)^* \subseteq W$. This proves that $A \cup B$ is I_{s*g} -closed set.

Example 3.2 Let X = N and $\mathcal{T}$ be the cofinite topology. Let $\{A_n : A_n = \{2,3,...,n+1\}, n \in N\}$ be a collection of I_{s*g} -closed sets in *X*. Then $\bigcup_{n\in N} A_n = N \setminus \{1\}$ = A (say) having a finite complement is open and hence semi-open but not closed. As $A^* = cl(A) = N \not\subset A$ for $I = \phi$, gives that A is not I_{s*g} -closed but $A^* = \phi \subseteq A$ for $I = P(X)$. In this case arbitrary union of I_{s*g} -closed sets is I_{s*g} -closed.

Theorem 3.15 Every open set is I_{s*g} -open.

Proof. Let U be an open set. We need to show U is I_{s*g} -open. For this we show that $X - U$ is I_{s*g} -closed. Let $X - U \subset G$ where $G \in SO(X)$. Since $X - U$ is closed. So by [9, Theorem 2.3] $(X - U)^* \subseteq cl(X - U) = X - U$ or $(X - U)^* \subseteq X - U \subset G$. This proves that $X - U$ is I_{s*g} -closed or U is I_{s*g} -open.

Definition 3.2.A space X is s*-normal [10], if for each pair of disjoint semi-closed sets A and B, there exist disjoint open sets U *and* V such that A $\subset U$ and B $\subset V$.

Theorem 3.16 Let (X, τ, I) be an ideal space where I is completely co dense. Then

the following statements are equivalent:
(1) X is s^*-normal.
(2) For any disjoint semi-closed sets A and B, there exit disjoint I_{s*g}-open sets U and V containing A and B respectively.
(3) For any semi-closed set A and semi-open set V containing A there exists an I_{s*g}-open set U such that $A \subset U \subset cl^*(U) \subset V$.

Proof. (1) $\Rightarrow$ (2) This proof follows from the fact that every open set is I_{s*g}-open set.
(2) $\Rightarrow$ (3) Suppose A is semi-closed and V is a semi-open set containing A. Since A and $X - V$ are disjoint semi-closed sets, there exist disjoint I_{s*g}-open sets U and W such that $A \subset U$ and $X - V \subset W$. Since $X - V$ is semi closed and W is I_{s*g}-open By Theorem 2.7, $X - V \subset \text{int}^*(W)$ and hence $X - \text{int}^*(W) \subset V$. Again $U \cap W = \phi$ implies $U \cap \text{int}^*(W) = \phi$ and hence $cl^*(U) \subset X - \text{int}\ (W) \subset V$. Thus U is the required I_{s*g}-open set. This implies $A \subset U \subset cl^*(U) \subset V$.
(3) $\Rightarrow$ (1) Let A and B be two disjoint semi-closed subsets of X. By hypothesis there exists an I_{s*g}-open set U such that $A \subset U \subset cl^*(U) \subset X - B$. Since U is I_{s*g}-open set and $A \subset U$, by Theorem 2.7, $A \subset \text{int}^*(U)$. Since I is completely co- dense, By [19, Theorem 6], $\tau^* \subset \tau^\alpha$ and so $\text{int}^*(U)$ *and* $X - cl\ (U) \in \tau^\alpha$.

Hence $A \subset \text{int}^*(U) \subset \text{int}(cl(\text{int}(\text{int}^*(U)))) = G$ and $B \subset X - cl^*(U) \subset \text{int}(cl(\text{int}(X - cl^*(U)))) = H$. Hence, G and H are required disjoint open sets containing A and B respectively. This proves (1). This completes the proof.

References

[1] F. G. Arenas, J. Dontchev and M. L. Puertas, Idealization of some weak separation axioms, (preprint).
[2] S.P. Arya and T. Nour, Characterizations of s-normal spaces, Indian J. Pure. Appl. Math., 21(8) (1990), 717-719.
[3] S.G. Crossley and S.K. Hilderbrand, semi-closure, Texas J. Sci., 22 (1971), 99-112.
[4] J. Dontchev, M. Ganster and T. Noiri, Unified approach of generalized closed sets via topological ideals, Math. Japon., 49 (1999), 395-401.
[5] J. Dontchev, M. Ganster and D. Rose, Ideal resolvability, Topology Appl., 93 (1) (1999), 1-16.
[6] M. K. Gupta and T. Noiri, C-compactness modulo an ideal, Internat. J. Math. Math. Sci., vol. 2006, Article ID 78135, pages 1-120 DOI 10.1155/IJMMS/2006/78135.

[7] E. Hayashi, Topologies defined by local properties, Math. Ann., 156 (1964), 205-215.

[8] D. Jankovic and I. L. Reilly, On Semi-Separation Properties, Indian J. Pure Appl. Math., 16(9) (1985), 957-964.

[9] D. Jankovic and T. R. Hamlett, New topologies from old via ideals, Amer. Math. Monthly, 97 (4) (1990), 295-310.

[10] M. Khan , T. Noiri and M. Hussain, On s*g-closed sets and s*-normal spaces, J. Natur. Sci. Math., (to appear in vol 48).

[11] K. Kuratowski, Topology I, Warszawa, 1933.

[12] N. Levine, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, 70(1) (1963), 36-41.

[13] N. Levine, Generalized closed sets in topology, Rend. Circ. Mat. Palermo (2), 19 (1970), 89-96.

[14] H. Maki, R. Devi and K. Balachandran, Generalized $\alpha - closed$ sets in topology, Bull. Fukuoka Univ. Ed. Part III, 42(1993), 13-21.

[15] M. Navaneethakrishnan and J. Paularaj Joseph, g-closed sets in ideal topological spaces, Acta Math. Hungar., 2007, DOI: 10.1007/s10474-007-7050-1.

[16] R.L. Newcomb, Topologies which are compact modulo an Ideal, Ph.D. Dissertation, Univ. of Cal. at Santa Barbara, 1967.

[17] O. Nja˚stad, On some classes of nearly open sets, Pacific J. Math., 15, (1965), 961-970.

[18] D.V. Rancin, Compactness modulo an Ideal, Soviet Math. Dokl., 13 (1)(1972), 193-197.

[19] V. Renuka Devi, D. Sivaraj and T. Tamizh Chelvam, Codense and completely codense ideals, Acta Math. Hungarica 108 (3) (2005), 197-205.

[20] R. Vaidyanathaswamy, Set Topology, Chelsea Publishing Company, 1946.