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Some of Properties of Intuitionistic Fuzzy Normal Subrings

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Abstract

In this paper, we introduce the definition of intuitionistic fuzzy normal subrings. We also made an attempt to study the algebraic nature of intuitionistic fuzzy normal subrings of a ring under homomorphism and anti-homomorphism.

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Key words: Intuitionistic fuzzy subsets, intuitionistic fuzzy subrings**,** intuitionistic fuzzy normal subrings

Introduction

After an introduction of fuzzy sets by L.A. Zadeh several researchers explored on the generalization of the notion of fuzzy set. The concept of intuitionistic fuzzy set was introduced K.T. Atanassov[2] as a generalization of the notion of a fuzzy set. In this paper, we discuss algebraic nature of intuitionistic fuzzy normal subrings and prove some results on these.

1. Preliminaries

1.1 Definition

An **intuitionistic fuzzy subset**(IFS) A in X is defined as an object of the form A = { < x, $\mu_A(x)$, $\nu_A(x)$ > / x $\in$ X }, where $\mu_A : X \to [0, 1]$ and $\nu_A : X \to [0, 1]$ define the degree of membership and the degree of non-membership of the element x $\in$ X respectively and for every x $\in$ X satisfying $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

1.2 Definition

Let (R, +, .) be a ring. An intuitionistic fuzzy subset A of R is said to be an intuitionistic fuzzy subring of R (IFSR) if it satisfies the following axioms:

(i) $\mu_A(x - y) \geq \min\{ \mu_A(x), \mu_A(y) \}$

(ii) $\mu_A(xy) \geq \min\{ \mu_A(x), \mu_A(y) \}$

(iii) $\nu_A(x - y) \leq \max\{ \nu_A(x), \nu_A(y) \}$

(iv) $\nu_A(xy) \leq \max\{ \nu_A(x), \nu_A(y) \}$, for all x, y $\in$ R.

1.3 Definition

Let R be a ring. An intuitionistic fuzzy subring A of R is said to be an intuitionistic fuzzy normal subring(IFNSR) of R if it satisfies the following axioms:

(i) $\mu_A(x + y) = \mu_A(y + x)$

(ii) $\mu_A(xy) = \mu_A(yx)$

(iii) $\nu_A(x + y) = \nu_A(y + x)$

(iv) $\nu_A(xy) = \nu_A(yx)$, for all x, y $\in$ R.

1.1 Theorem

Let A and B be intuitionistic fuzzy subsets of the rings with an identity R_1 and R_2 respectively and AxB is an intuitionistic fuzzy subring of R_1xR_2. Then the following are true :

(i) if $\mu_A(x) \leq \mu_B(e^l)$ and $\nu_A(x) \geq \nu_B(e^l)$, then A is an intuitionistic fuzzy subring of R_1.

(ii) if $\mu_B(x) \leq \mu_A(e)$ and $\nu_B(x) \geq \nu_A(e)$, then B is an intuitionistic fuzzy subring of R_2.

(iii) either A is an intuitionistic fuzzy subring of R_1 or B is an intuitionistic fuzzy subring of R_2.

1.4 Definition

Let A and B be two intuitionistic fuzzy subrings of rings R_1 and R_2, respectively. The product of A and B, denoted by AxB, is defined as AxB = { $\langle$ (x, y), $\mu_{AxB}(x, y)$, $\nu_{AxB}(x, y)$ $\rangle$ / for all x $\in$ R_1 and y $\in$ R_2 }, where $\mu_{AxB}(x, y) = \min\{ \mu_A(x), \mu_B(y) \}$ and $\nu_{AxB}(x, y) = \max\{ \nu_A(x), \nu_B(y) \}$.

2. Some properties of intuitionistic fuzzy normal subring of a ring

2.1 Theorem

Let (R, +, .) be a ring. If A and B are two intuitionistic fuzzy normal subrings of R, then their intersection $(A \cap B)$ is an intuitionistic fuzzy normal subring of R.

Proof:

Let $x, y \in R$.

Let $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in R \}$and

$B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle / x \in R \}$ be intuitionistic fuzzy normal subrings of a ring R.

Let $C = A \cap B$ and $C = \{ \langle x, \mu_C(x), \nu_C(x) \rangle / x \in R \}$,

where $\min\{ \mu_A(x), \mu_B(x) \} = \mu_C(x)$ and $\max\{ \nu_A(x), \nu_B(x) \} = \nu_C(x)$.

Clearly C is an intuitionistic fuzzy subring of a ring R, since A and B are two intuitionistic fuzzy subrings of a ring R.

Now,

$\mu_C(x + y) = \min \{ \mu_A(x + y), \mu_B(x + y) \}$

$= \min \{\mu_A(y + x) , \mu_B(y + x) \}$, by 1.3

$= \mu_C(y + x)$.

Therefore,

$\mu_C(x + y) = \mu_C(y + x)$, for all $x, y \in R$.

Also,

$\mu_C(xy) = \min \{ \mu_A(xy) , \mu_B(xy) \}$

$= \min \{\mu_A(yx) , \mu_B(yx) \}$, by 1.3

$= \mu_C(yx)$.

Therefore,

$\mu_C(xy) = \mu_C(yx)$, for all $x, y \in R$.

Again,

$\nu_C(x + y) = \max \{ \nu_A(x + y) , \nu_B(x + y) \}$

$= \max \{ \nu_A(y + x) , \nu_B(y + x)\}$, by 1.3

$= \nu_C(y + x)$.

Therefore,

$\nu_C(x + y) = \nu_C(y + x)$, for all $x, y \in R$.

Also,

$\nu_C(xy) = \max \{ \nu_A(xy), \nu_B(xy) \}$

$= \max \{\nu_A(yx), \nu_B(yx)\}$, by 1.3

$= \nu_C(yx)$.

Therefore,

$\nu_C(xy) = \nu_C(yx)$, for all $x, y \in R$.

Hence intersection of any intuitionistic fuzzy normal subrings is an intuitionistic fuzzy normal subring of a ring R.

2.2 Theorem:

Let A and B be intuitionistic fuzzy subsets of the rings R_1 and R_2 respectively and AxB is an intuitionistic fuzzy normal subring of R_1xR_2. Then the following are true :

(i) if $\mu_A(x) \leq \mu_B(e^l)$ and $\nu_A(x) \geq \nu_B(e^l)$, then A is an intuitionistic fuzzy normal subring of R_1.
(ii) if $\mu_B(x) \leq \mu_A(e)$ and $\nu_B(x) \geq \nu_A(e)$, then B is an intuitionistic fuzzy normal subring of R_2.
(iii) either A is an intuitionistic fuzzy normal subring of R_1 or B is an intuitionistic fuzzy normal subring of R_2.

Proof

Let AxB be an intuitionistic fuzzy normal subring of R_1xR_2 and x, y in $R_1$and $e^l \in R_2$.

Then (x, e^l) and (y, e^l) are in R_1xR_2.

Clearly AxB is an intuitionistic fuzzy subring of R_1xR_2.

Now, using the property that $\mu_A(x) \leq \mu_B(e^l)$ and $\nu_A(x) \geq \nu_B(e^l)$, for all x in R_1,

Clearly A is an intuitionistic fuzzy subring of R_1, by Theorem 1.1.

Now,

$$\begin{aligned}\mu_A(x + y) &= \min\{ \mu_A(x + y), \mu_B(e^l + e^l) \} \\ &= \mu_{AxB}((x + y), (e^l + e^l)) \\ &= \mu_{AxB}[(x, e^l) + (y, e^l)] \\ &= \mu_{AxB}[(y, e^l) + (x, e^l)], \text{ by 1.3} \\ &= \mu_{AxB}[(y + x), (e^l + e^l)] \\ &= \min\{ \mu_A(y + x), \mu_B(e^l + e^l) \} \\ &= \mu_A(y + x).\end{aligned}$$

Therefore,

$\mu_A(x + y) = \mu_A(y + x)$, for all $x, y \in R$.

Also,

$$\begin{aligned}\mu_A(xy) &= \min\{ \mu_A(xy), \mu_B(e^le^l) \} \\ &= \mu_{AxB}((xy), (e^le^l)) \\ &= \mu_{AxB}[(x, e^l)(y, e^l)] \\ &= \mu_{AxB}[(y, e^l)(x, e^l)], \text{ by 1.3} \\ &= \mu_{AxB}[(yx), (e^le^l)] \\ &= \min\{ \mu_A(yx), \mu_B(e^le^l) \} \\ &= \mu_A(yx).\end{aligned}$$

Therefore,

$\mu_A(xy) = \mu_A(yx)$, for all $x, y \in R$.

And,

$$\begin{aligned}\nu_A(x + y) &= \max\{ \nu_A(x + y), \nu_B(e^l + e^l) \} \\ &= \nu_{AxB}((x + y), (e^l + e^l)) \\ &= \nu_{AxB}[(x, e^l) + (y, e^l)] \\ &= \nu_{AxB}[(y, e^l) + (x, e^l)], \text{ by 1.3} \\ &= \nu_{AxB}[(y + x), (e^l + e^l)] \\ &= \max\{ \nu_A(y + x), \nu_B(e^l + e^l) \} \\ &= \nu_A(y + x).\end{aligned}$$

Therefore,

$\nu_A(x + y) = \nu_A(y + x)$, for all $x, y \in R$.

Also,

$\nu_A(xy) = \max \{ \nu_A(xy) , \nu_B(e^I e^I) \}$
$= \nu_{AxB} ((xy), (e^I e^I))$
$= \nu_{AxB} [(x, e^I) (y, e^I)]$
$= \nu_{AxB} [(y, e^I) (x, e^I)]$, by 1.3
$= \nu_{AxB} [(yx), (e^I e^I)]$
$= \max\{ \nu_A(yx) , \nu_B(e^I e^I) \}$
$= \nu_A(yx)$.

Therefore,

$\nu_A(xy) = \nu_A(yx)$, for all $x, y \in R$.

Hence A is an intuitionistic fuzzy normal subring of R_1.

Thus (i) is proved.

Now, using the property that $\mu_B(x) \leq \mu_A(e)$ and $\nu_B(x) \geq \nu_A(e)$, for all x in R_2, let $x, y \in R_2$ and $e \in R_1$.

Then (e, x) and (e, y) are in $R_1 x R_2$.

Clearly B is an intuitionistic fuzzy subring of R_2, by Theorem 1.1.

Now,

$\mu_B(x + y) = \min\{ \mu_B(x + y) , \mu_A(e + e) \}$
$= \min\{ \mu_A(e + e) , \mu_B(x + y) \}$
$= \mu_{AxB} ((e + e), (x + y))$
$= \mu_{AxB} [(e, x) + (e, y)]$
$= \mu_{AxB} [(e, y) + (e, x)]$, by 1.3
$= \mu_{AxB} [(e + e), (y + x)]$
$= \min \{ \mu_A(e + e) , \mu_B(y + x) \}$
$= \mu_B(y + x)$.

Therefore,

$\mu_B(x + y) = \mu_B(y + x)$, for all $x, y \in R$.

Also,

$\mu_B(xy) = \min\{ \mu_B(xy) , \mu_A(ee) \}$
$= \min\{ \mu_A(ee) , \mu_B(xy) \}$
$= \mu_{AxB} ((ee), (xy))$
$= \mu_{AxB} [(e, x)(e, y)]$
$= \mu_{AxB} [(e, y) (e, x)]$, by 1.3
$= \mu_{AxB} [(ee), (yx)]$
$= \min\{ \mu_A(ee) , \mu_B(yx) \}$
$= \mu_B(yx)$.

Therefore,

$\mu_B(xy) = \mu_B(yx)$, for all $x, y \in R$.

And,

$\nu_B(x + y) = \max\{ \nu_B(x + y) , \nu_A(e + e) \}$
$= \max\{ \nu_A(e + e) , \nu_B(x + y) \}$
$= \nu_{AxB} ((e + e), (x + y))$
$= \nu_{AxB} [(e, x) + (e, y)]$

$= \nu_{AxB}$ [(e, y) + (e, x)], by 1.3
$= \nu_{AxB}$ [(e + e), (y + x)]
$= \max\{ \nu_A(e + e) , \nu_B(y + x) \}$
$= \nu_B(y + x)$.

Therefore,
$\nu_B(x + y) = \nu_B(y + x)$, for all x, y $\in$ R.
Also,
$\nu_B(xy) = \max\{ \nu_B(xy) , \nu_A(ee) \}$
$= \max\{ \nu_A(ee) , \nu_B(xy) \}$
$= \nu_{AxB}$ ((ee), (xy))
$= \nu_{AxB}$ [(e, x) (e, y)]
$= \nu_{AxB}$ [(e, y) (e, x)], by 1.3
$= \nu_{AxB}$ [(ee), (yx)]
$= \max\{ \nu_A(ee), \nu_B(yx) \}$
$= \nu_B(yx)$.

Therefore,
$\nu_B(xy) = \nu_B(yx)$, for all x, y $\in$ R.
Hence B is an intuitionistic fuzzy normal subring of R_2.
Thus (ii) is proved.
(iii) is clear.

2.3 Theorem

If A is an intuitionistic fuzzy normal subring of a ring R, then   A is an intuitionistic fuzzy normal subring of a ring R.

Proof

Let   A = B = { (x, μ_B(x), ν_B(x) }.

Let   A is an intuitionistic fuzzy subring of a ring R, since A is an intuitionistic fuzzy subring of a ring R

Let x, y $\in$ R
Then, clearly
$\mu_B(x + y) = \mu_B(y + x)$ and $\mu_B(xy) = \mu_B(yx)$
And also, $\mu_A(x + y) = \mu_A(y + x)$
which implies that
$1 – \nu_B(x + y) = 1 – \nu_B(y + x)$.
That is, $\nu_B(x + y) = \nu_B(y + x)$ and $\mu_A(xy) = \mu_A(yx)$
which implies that $1 – \nu_B(xy) = 1 – \nu_B(yx)$
That is, $\nu_B(xy) = \nu_B(yx)$
Hence B =   A is an intuitionistic fuzzy normal subring of a ring R.

2.4 Theorem

If A is an intuitionistic fuzzy normal subring of a ring R, then ◊A is an intuitionistic fuzzy normal subring of a ring R.

Proof

Let ◊A = B ={ (x, $\mu_B(x)$, $\nu_B(x)$) }.

Clearly B is an intuitionistic fuzzy subring of a ring R, since A is an intuitionistic fuzzy subring of a ring R.

Then, clearly,

$\nu_B(x + y) = \nu_B(y + x)$ and $\nu_B(xy) = \nu_B(yx)$

Now, $\nu_A(x + y) = \nu_A(y + x)$

which implies that $1 - \mu_B(x + y) = 1 - \mu_B(y + x)$.

That is, $\mu_B(x + y) = \mu_B(y + x)$

$\nu_A(xy) = \nu_A(yx)$

which implies that $1 - \mu_B(xy) = 1 - \mu_B(yx)$

That is, $\mu_B(xy) = \mu_B(yx)$

Hence B = ◊A is an intuitionistic fuzzy normal subring of a ring R.

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