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# Quadratic Loss Bayesian Approximation Approach for Scale $\delta$ of Generalized Compound Rayleigh Distribution

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**Abstract**

In this article, the Bayesian and non-Bayesian approaches have been used to obtain the estimators of the parameter Generalized Compound Rayleigh Distribution. Bayes estimators have been developed under symmetric loss function. These estimators are derived using Natural Conjugate prior distribution for Scale parameter $\gamma$. We compare the performance of the presented Bayes estimators with known, non-Bayesian, estimators such as the maximum likelihood (ML).

**Keywords:** Bayesian and non-Bayesian approaches, Generalized Compound Rayleigh Distribution, Natural Conjugate prior distribution, Quadratic Loss function.

## 1. Introduction

The three-parameter Generalized Compound Rayleigh Distribution is derived from the three-parameter Burr type XII distribution(Burr(1942)). Mostert Roux, and Bekker (1999) used this distribution as a Gamma mixture of Rayleigh distribution and obtained the Compound Rayleigh model with unimodal hazard function. Bain and Engelhardt (1991) studied this distribution (also known as the Compound Weibull distribution (Dubey 1968) from a Poisson perspective. The pdf of Generalized Compound Rayleigh Distribution is given by

$$f(x;\theta,\varphi,\delta)=\frac{\theta}{\delta}\varphi^{\frac{1}{\delta}}x^{(\theta-1)}\left(\varphi+x^{\delta}\right)^{-(\delta+1)}\ ;\ x,\theta,\varphi,\delta>0 \tag{1.1}$$

The Quadratic loss function is commonly used loss function in estimation problems, given as $L(\hat{\Delta},\Delta)=k(\hat{\Delta}-\Delta)^2$ where $\hat{\Delta}$ is the estimate of $\Delta$, the loss function is called quadratic weighed loss function if k=1, we have

$$L(\hat{\Delta}, \Delta) = (\hat{\Delta} - \Delta)^2 \tag{1.2}$$

A loss function that satisfies a symmetric condition because it associates the equal importance to the losses due to overestimation and under estimation with equal magnitudes. [Ferguson (1985), Canfield (1970), Basu and Ebrahimi (1991), Zellner (1986) and Varion(1975)]. Soliman (2001) derived and discussed the properties of symmetric and Asymmetric Loss Function and demonstrated its usefulness in weakly-supervised learning, e.g., one can use asymmetric loss to simplify a risk estimator in learning from positive-unlabeled data.

We have studied the sensitivity of the Approximate Bayes estimators of model and presented a numerical study to illustrate the above technique on generated observations and the comparison is done by R-programming.

## 2. The Estimators

Let $x_1 \leq x_2 \leq \ldots\ldots\ldots \leq x_n$ be the n failures in complete sample case. The likelihood function is given by

$$L(\underline{x} \mid \theta, \varphi, \delta) = \left(\frac{\theta}{\delta}\right)^n U e^{-T/\delta} \tag{2.1}$$

where

$$T = \sum_{j=1}^{n} log\left[1 + \frac{{x_j}^{\theta}}{\varphi}\right] \quad \text{and} \quad U = \prod_{j=1}^{n} \frac{{x_j}^{\theta-1}}{\varphi + {x_j}^{\theta}}$$

from equation(2.1), the log likelihood function is

$$Log\ L = n \log\theta + \frac{n}{\delta}\log\varphi - n\log\delta + (\theta - 1)\sum_{j=1}^{n} log x_j - \left(\frac{1}{\delta} + 1\right)\sum_{j=1}^{n} log\left(\varphi + {x_j}^{\theta}\right) \tag{2.2}$$

differentiating the equation(2.2) with respect to $\theta, \varphi$ and $\delta$ yields respectively we obtain the Maximum Likelihood Estimator of $\theta, \varphi\ and\ \delta$.

Applying the Newton-Raphson method $\hat{\theta}_{MLE}$ and $\hat{\varphi}_{MLE}$ can be derived and then from them $\hat{\delta}_{MLE}$ can be obtained.

## 3. Bayes estimators for $\delta$ with known parameter $\theta$ *and* $\varphi$

If $\hat{\theta}$ and $\hat{\varphi}$ is known we assume $\delta(a, b)$ as conjugate prior for $\delta$ as

$$g(\delta|\underline{x}) = \frac{b^a}{\Gamma a}\left(\frac{1}{\delta}\right)^{a+1} e^{-\frac{b}{\delta}}; \qquad (a, b, \delta) > 0 \tag{3.1}$$

combining the likelihood function equation(2.1) and prior density equation(3.1), we obtain the posterior density of $\delta$ in the form

$$h(\delta|\underline{x}) = \frac{\delta^{-(n+\theta-1)} e^{-\frac{(b+T)}{\delta}}}{\int_0^\infty \delta^{-(n+\theta-1)} e^{-\frac{(b+T)}{\delta}} d\delta} \tag{3.2}$$

assuming

$T = \sum_{j=1}^{n} \log\left(1 + \frac{x_j{}^{\theta}}{\varphi}\right)$ and $U = \prod_{j=1}^{n} \frac{x_j{}^{\theta-1}}{(\varphi + x_j{}^{\theta})}$

$$h(\delta|\underline{x}) = \frac{\gamma^{-(n+\theta-1)} e^{-\frac{(b+T)}{\delta}} (b+T)^{(n+a)}}{\Gamma(n+a)} \tag{3.3}$$

**Bayes Estimator of $\delta$ under Quadratic loss function (QLF)**

Under Quadratic loss function Bayes Estimator is

$$\hat{\delta}_{BSQ} = \int_0^\infty \delta \frac{(b+T)^{(n+a)}}{\Gamma(n+a)} \left(\frac{1}{\delta}\right)^{(n+a+1)} e^{-(b+T)/\delta} d\delta \tag{3.4}$$

$$\hat{\delta}_{BSQ} = \frac{(b+T)^{(n+a)}}{\Gamma(n+a)} \int_0^\infty \left(\frac{1}{\delta}\right)^{(n+a)} exp^{-(b+T)/\delta}$$

$$substituting \quad y = \frac{b+T}{\delta}$$

On solving which gives

$$\hat{\delta}_{BSQ} = \frac{(b+T)}{(n+a-1)} \tag{3.5}$$

## 4. Approximate Bayes Estimator of the unknown Location parameter $\varphi$.

The Joint prior density of the parameters $\theta, \varphi, \delta$ is given by

$$G(\theta, \varphi, \delta) = \mathrm{g}_1(\theta)\mathrm{g}_2(\varphi)\mathrm{g}_3(\delta|\varphi)$$

$$= \frac{c}{\delta\Gamma\xi} \varphi^{-\xi} \delta^{\xi+1} exp\left[-\left(\frac{\delta}{\varphi} + \frac{\varphi}{\mu}\right)\right] \tag{4.1}$$

where

$$g_1(\theta) = c \tag{4.2}$$

$$g_2(\varphi) = \frac{1}{\mu} e^{-\frac{\delta}{\mu}} \tag{4.3}$$

$$g_3(\delta) = \frac{1}{\Gamma\xi} \varphi^{-\xi} \delta^{\xi+1} e^{-\frac{\delta}{\varphi}} \tag{4.4}$$

The Joint posterior combing the likelihood equation(2.2) and joint prior equation(4.1) is

$$h^*(\theta, \varphi, \delta|\underline{x}) = \frac{\varphi^{-\xi}\delta^{\xi+1} exp\left[-\left(\frac{\delta}{\varphi} + \frac{\varphi}{\mu}\right)\right] . L(\underline{x}|\theta, \varphi, \delta)}{\int_\theta \int_\varphi \int_\delta \beta^{-\xi}\gamma^{\xi+1} exp\left[-\left(\frac{\gamma}{\beta} + \frac{\beta}{\delta}\right)\right] . L(\underline{x}|\theta, \varphi, \delta) d\theta d\varphi d\delta} \tag{4.5}$$

**The Approximate Bayes Estimator is given by**

$$Y(\Theta) = Y(\theta, \varphi, \delta) \tag{4.6}$$

$$\hat{Y}_{ABSQ} = E(Y|\underline{x}) = \frac{\int_\theta \int_\varphi \int_\delta Y(\alpha,\beta,\gamma) G^*(\theta,\varphi,\delta) d\theta d\varphi d\delta}{\int_\theta \int_\varphi \int_\delta G^*(\theta,\varphi,\delta) d\theta d\varphi d\delta} \tag{4.7}$$

**Lindley Approximation Procedure**

The Bayes estimators of a function $\nu = \nu(\vartheta, \rho)$ of the unknown parameter $\vartheta$ and $\rho$ under quadratic loss is the posterior mean

$$\hat{\nu}_{ABSQ} = E(\nu|\underline{x}) = \frac{\iint \nu(\vartheta,\rho) h^*(\theta, p|\underline{x}) d\vartheta\, d\rho}{\iint h^*(\theta, p|\underline{x}) d\vartheta\, d\rho} \tag{4.7a}$$

The ratio of integrals in equation (4.7a) does not seem to take a closed form so we must consider the Lindley approximation procedure as

$$E(\nu(\vartheta,\rho)|\underline{x}) = \frac{\int \nu(\vartheta).e^{(l(\vartheta)+\rho(\vartheta))} d\vartheta}{\int e^{(l(\vartheta)+\rho(\vartheta))}.d\vartheta} \tag{4.7b}$$

Lindley developed approximate procedure for evaluation of posterior expectation of $\nu(\vartheta)$. Several other authors have used this technique to obtain Bayes estimators (see Berger(1980), Sinha(1986), Sinha and sloan(1988),Soliman(2001)).The posterior expectation of Lindley approximation procedure to evaluate of $\nu(\vartheta)$in equation (4.7a and 4.7b) under SELF, where where $\rho(\vartheta) = \log g(\vartheta)$ , and $g(\vartheta)$ is an arbitrary function of $\vartheta$ and $l(\vartheta)$ is the logarithm likelihood function (Lindley (1980)).

The modified form of equation (4.7) is given by

$$E(Y(\alpha,\beta,\gamma|\underline{x}) = Y(\Theta) + \frac{1}{2}[A(Y_1\sigma_{11} + Y_2\sigma_{12} + Y_3\sigma_{13}) + B(Y_1\sigma_{21} + Y_2\sigma_{22} + Y_3\sigma_{23}) + P\ (Y_1\sigma_{31} + Y_2\sigma_{32} + Y_3\sigma_{33})] + (Y_1a_1 + Y_2a_2 + Y_3a_3 + a_4 + a_5) + 0\left(\frac{1}{n^2}\right) \tag{4.8}$$

Above equation is evaluated at MLE = $(\hat{\theta}, \hat{\varphi}, \hat{\delta})$

where

$$a_1 = \rho_1\sigma_{11} + \rho_2\sigma_{12} + \rho_3\sigma_{13} \tag{4.9}$$

$$a_2 = \rho_1\sigma_{21} + \rho_2\sigma_{22} + \rho_3\sigma_{23} \tag{4.10}$$

$$a_3 = \rho_1\sigma_{31} + \rho_2\sigma_{32} + \rho_3\sigma_{33} \tag{4.11}$$

$$a_4 = Y_{12}\sigma_{12} + Y_{13}\sigma_{13} + Y_{23}\sigma_{23} \tag{4.12}$$

$$a_5 = \frac{1}{2}(Y_{11}\sigma_{11} + Y_{22}\sigma_{22} + Y_{33}\sigma_{33})\,; \tag{4.13}$$

And

$$A = [\sigma_{11}l_{111} + 2\sigma_{12}l_{121} + 2\sigma_{13}l_{131} + 2\sigma_{23}l_{231} + \sigma_{22}l_{221} + \sigma_{33}l_{331}] \tag{4.14}$$

$$B = [\sigma_{11}l_{112} + 2\sigma_{12}l_{122} + 2\sigma_{13}l_{132} + 2\sigma_{23}l_{232} + \sigma_{22}l_{222} + \sigma_{33}l_{332}] \tag{4.15}$$

$$P = [\sigma_{11}l_{113} + 2\sigma_{13}l_{133} + 2\sigma_{12}l_{123} + 2\sigma_{23}l_{233} + \sigma_{22}l_{223} + \sigma_{33}l_{333}] \tag{4.16}$$

To apply Lindley approximation on equation (4.8) , we first obtain

$\sigma_{ij} = [-l_{ijk}]^{-1} i, j, k = 1,2,3$

Likelihood function from equation (2.2) is

$$Log\ L = n \log \theta + \frac{n}{\delta} \log \varphi - n \log \delta + (\theta - 1) \sum_{j=1}^{n} log x_j - \left(\frac{1}{\delta} + 1\right) \sum_{j=1}^{n} log\left(\varphi + x_j{}^{\theta}\right) \quad (4.17)$$

Now differentiating log likelihood function with respect to $\theta, \varphi\ and\ \delta$ ,we get

$$l_{111} = \frac{2n}{\theta^3} - \left(\frac{1}{\gamma} + 1\right) \omega_{133} \quad \text{where} \quad \omega_{133} = \sum \frac{x_j{}^{\theta}(\varphi - x_j{}^{\theta})(\log x_j)^3}{(\varphi + x_j{}^{\theta})^3} \quad (4.18)$$

$$l_{222} = \frac{2n}{\delta\varphi^3} - 2\left(\frac{1}{\gamma} + 1\right) \delta_{13} \quad \text{where} \quad \delta_{13} = \sum_{j=1}^{n} \frac{1}{(\varphi + x_j)^3} \quad (4.19)$$

$$l_{333} = -\frac{2n}{\delta^3} - \frac{6n\ log\varphi}{\delta^4} + \frac{6}{\delta^4} \delta_{10} \quad \text{where} \quad \delta_{10} = \sum_{j=1}^{n} log\left(\varphi + x_j{}^{\theta}\right) \quad (4.20)$$

$$l_{112} = \left(\frac{1}{\delta} + 1\right) \omega_{123} \quad \text{where} \quad \omega_{123} = \sum_{j=1}^{n} \frac{x_j{}^{\theta}(\varphi - x_j{}^{\theta})(\log x_j)^2}{(\varphi + x_j{}^{\theta})^3} \quad (4.21)$$

and $l_{112} = l_{121}$

$$l_{113} = \frac{\varphi}{\delta^2} \omega_{122} \quad \text{where} \quad \omega_{122} = \sum_{j=1}^{n} \frac{x_j{}^{\theta}(log x_j)^2}{(\varphi + x_j{}^{\theta})^2} \quad (4.22)$$

$$l_{221} == -2\left(\frac{1}{\delta} + 1\right) \omega_{113} \quad \text{where} \quad \omega_{113} = \sum_{j=1}^{n} \frac{x_j{}^{\theta} log x_j}{(\varphi + x_j{}^{\theta})^3} \quad (4.23)$$

$l_{221} = l_{212}$

$$l_{223} = \frac{n}{(\delta\varphi)^2} - \frac{1}{(\delta)^2} \delta_{12} \quad \text{where} \quad \delta_{12} = \sum_{j=1}^{n} \frac{1}{(\varphi + x_j{}^{\theta})^2} \quad (4.24)$$

$$l_{331} = -\frac{2}{\delta^3} \omega_{11} \quad \text{where} \quad \omega_{11} = \sum_{j=1}^{n} \frac{x_j{}^{\delta} log x_j}{(\varphi + x_j{}^{\theta})} \quad (4.25)$$

$l_{331} = l_{313}$

$$l_{332} = \frac{\partial}{\partial\delta}\left(\frac{\partial^2 L}{\partial\delta\partial\varphi}\right) = \frac{2}{\delta^3}\left(\frac{n}{\varphi} - \delta_{11}\right) \quad (4.26)$$

$l_{332} = l_{323}$

$$l_{231} = -\frac{\omega_{14}}{\delta^2} \quad (4.27)$$

$l_{231} = l_{213}$

$$l_{123} = -\frac{\omega_{14}}{\delta^2} \quad (4.28)$$

$l_{123} = l_{132}$

$$l_{133} = \frac{-2}{\delta^2} \sum_{j=1}^{n} \frac{x_j{}^{\theta} \log x_j}{(\varphi + x_j{}^{\theta})} = -\frac{2}{\delta^2} \omega_{11} \quad (4.29)$$

$l_{122} = -2\left(\frac{1}{\delta}+1\right)\omega_{113}$ where $\omega_{113} = \sum_{j=1}^{n} \frac{x_j{}^{\theta}\log x_j}{(\varphi+x_j{}^{\theta})^3}$ (4.30)

$l_{233} = \frac{2n}{\varphi\delta^3} - \frac{2}{\delta^3}\sum_{j=1}^{n}\frac{1}{\varphi+x_j{}^{\theta}} = \frac{2}{\delta^3}\left(\frac{n}{\varphi} - \delta_{11}\right)$ (4.31)

The matrix of derivatives is given as

$$[-l_{ijk}] = -\begin{bmatrix} l_{111} & l_{112} & l_{113} \\ l_{221} & l_{222} & l_{223} \\ l_{331} & l_{332} & l_{333} \end{bmatrix} \quad (4.32)$$

$$= \begin{bmatrix} \frac{2n}{\theta^3} - \left(\frac{1}{\delta}+1\right)\omega_{133} & ,\left(\frac{1}{\delta}+1\right)\omega_{123} & -\frac{\varphi}{\delta^2}\omega_{122} \\ -2\left(\frac{1}{\delta}+1\right)\omega_{113} & ,\frac{2n\delta}{\delta\varphi^3} - 2\left(\frac{1}{\delta}+1\right)\delta_{13} & \frac{n}{(\delta\beta)^2} - \frac{1}{\delta^2}\delta_{12} \\ \frac{-2}{\delta^3}\omega_{11} & ,\frac{-2}{\delta^3}\left(\frac{n}{\delta} - \delta_{11}\right) & ,-\frac{2n}{\delta^3} - \frac{6n\, log\varphi}{\delta^4} + \frac{6}{\delta^4}\delta_{10} \end{bmatrix}$$

$$[-l_{ijk}] = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} \\ Q_{21} & Q_{22} & Q_{23} \\ Q_{31} & Q_{32} & Q_{33} \end{bmatrix}$$

$Determinant\ of \quad [-l_{ijk}]$

$D = \{Q_{11}[Q_{22}Q_{33} - Q_{23}Q_{32}] - Q_{12}[Q_{21}Q_{33} - Q_{31}Q_{23}] + Q_{13}[Q_{21}Q_{32} - Q_{22}Q_{33}]\}$ (4.45)

$$[-l_{ijk}]^{-1} = \frac{(Adjoint\ of\ [-l_{ijk}])'}{D}$$

$$[-l_{ijk}]^{-1} = \begin{bmatrix} \frac{U_{11}}{D} & \frac{U_{12}}{D} & \frac{U_{13}}{D} \\ \frac{U_{21}}{D} & \frac{U_{22}}{D} & \frac{U_{23}}{D} \\ \frac{U_{31}}{D} & \frac{U_{32}}{D} & \frac{U_{33}}{D} \end{bmatrix}$$

$$[-l_{ijk}]^{-1} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}; \quad (4.33)$$

## Approximate Bayes Estimator

$Y(\alpha,\beta,\gamma) = Y$

$\hat{Y}_{ABSQ} = E(Y|\underline{x})$

evaluated from equation number and from joint prior density , we have

$$G(\theta,\varphi,\delta) = g(\theta)g_2(\varphi)g_3(\delta|\varphi)$$

$$= \frac{c}{\varepsilon\Gamma\xi}\beta^{-\xi}\gamma^{\xi-1}exp\left[-\left(\frac{\delta}{\varphi}+\frac{\varphi}{\varepsilon}\right)\right];$$

$$\rho = \log G = \log C - log\varepsilon - log[\xi + (\xi - 1)log\delta - \xi log\varphi - \left(\frac{\delta}{\varphi} + \frac{\varphi}{\varepsilon}\right) \quad (4.34)$$

Log G= $constant - \xi log\beta + (\xi - 1)log\gamma - \frac{\delta}{\varphi} - \frac{\varphi}{\varepsilon}$

$$\rho_1 = \frac{\delta\rho}{\delta\theta} = 0 \quad (4.35)$$

$$\rho_2 = \frac{\delta\rho}{\delta\varphi} = \frac{-\xi}{\varphi} + \frac{\delta}{\varphi^2} - \frac{1}{\varepsilon} \quad (4.36)$$

$$\rho_3 = \frac{\delta\rho}{\delta\delta} = \frac{\xi-1}{\delta} - \frac{1}{\varphi} \quad (4.37)$$

Using equation (4.14) to equation (4.37), we have

$$A = \frac{1}{D}\Big[U_{11}\left(\frac{2n}{\theta^3} - \left(\frac{1}{\delta} + 1\right)\omega_{133}\right) + 2U_{12}\left(\frac{1}{\delta} + 1\right)\omega_{123} + 2U_{13}\frac{\varphi}{\delta^3}\omega_{122} - 2U_{23}\frac{\omega_{14}}{\delta^2} - 2U_{22}\left(\frac{1}{\delta} + 1\right)\omega_{113} - \frac{2}{\delta^3}U_{33}\omega_{11}\Big] \quad (4.38)$$

$$B = \frac{1}{D}\left[\left(\frac{1}{\delta} + 1\right)\omega_{123}U_{11} - 4U_{12}\left(\frac{1}{\delta} + 1\right)\omega_{113} - 2U_{13}\left(-\frac{\omega_{14}}{\delta^2}\right) + (U_{22}+2U_{23})\left(\frac{n}{(\delta\varphi)^2} - \frac{1}{\delta^2}\delta_{12}\right) + U_{33}\left(-\frac{2}{\delta^3}\left(\frac{n}{\varphi} - \delta_{11}\right)\right)\right] \quad (4.39)$$

$$P = \frac{1}{D}\Big[\frac{U_{11}\varphi}{\delta^2}\omega_{122} - \frac{2U_{12}\omega_{14}}{\delta^4} - \frac{4U_{13}\omega_{11}}{\delta^3} + \frac{4U_{23}}{\delta^3}\left(\frac{n}{\delta} - \delta_{11}\right) + U_{22}\left(\frac{n}{\delta^2\varphi^2} - \frac{1}{\delta^2}\delta_{12}\right) + U_{33}\left(-\frac{2n}{\delta^3} - \frac{6n\,log\varphi}{\delta^4} + \frac{6}{\delta^4}\delta_{10}\right)\Big] \quad (4.40)$$

Now

$$\hat{Y}_{ABSQ} = E(Y|\underline{x})$$

$$E(Y|\underline{x}) = u + (u_1a_1 + u_2a_2 + u_3a_3 + a_{4+}a_5) + \frac{1}{2}[A(u_1\sigma_{11} + u_2\sigma_{12} + u_3\sigma_{13}) + B(u_1\sigma_{21} + u_2\sigma_{22} + u_3\sigma_{23}) + P(u_1\sigma_{31} + u_2\sigma_{32} + u_3\sigma_{33})] + 0\left(\frac{1}{n^2}\right) \quad (4.41)$$

$$E(Y|\underline{x}) = U + \psi_1 + \psi_2$$

where

$$\psi_1 = u_1a_1 + u_2a_2 + u_3a_3 + a_4 + a_5 \quad (4.42)$$

$$\psi_2 = \frac{1}{2}[(A\sigma_{11} + B\sigma_{21} + P\sigma_{31}).Y_1 + (A\sigma_{12} + B\sigma_{22} + P\sigma_{32}).Y_2 + (A\sigma_{13} + B\sigma_{23} + P\sigma_{33})Y_3] \quad (4.43)$$

evaluated at the MLE $\hat{Y} = (\hat{\theta}, \hat{\varphi}, \hat{\delta})$ where

$$a_1 = \sigma_{11} + \left(\frac{-\varepsilon}{\varphi} + \frac{\delta}{\varphi^2} - \frac{1}{\varepsilon}\right)\frac{Y_{12}}{D} + \left(\frac{\varepsilon-1}{\delta} - \frac{1}{\varphi}\right)\frac{Y_{13}}{D} \quad (4.44)$$

$$a_2 = 0.\sigma_{21} + \left(\frac{-\xi}{\beta} + \frac{\gamma}{\beta} - \frac{1}{\delta}\right)\frac{Y_{22}}{D} + \left(\frac{\xi-1}{\gamma} - \frac{1}{\beta}\right)\frac{Y_{23}}{D} \quad (4.45)$$

$$a_3 = \sigma_{31} + \left(\frac{-\xi}{\varphi} + \frac{\delta}{\varphi^2} - \frac{1}{\varepsilon}\right)\frac{Y_{32}}{D} + \left(\frac{\varepsilon-1}{\delta} - \frac{1}{\varphi}\right)\frac{Y_{33}}{D} \tag{4.46}$$

$$a_4 = \frac{Y_{12}}{D}U_{12} + \frac{Y_{13}}{D}U_{13} + \frac{Y_{23}}{D}U_{23} \tag{4.47}$$

$$a_5 = \frac{1}{2D}(U_{11}Y_{11} + U_{22}Y_{22} + U_{33}Y_{33}) \tag{4.48}$$

**Approximate Bayes Estimate Under Quadratic Loss Function**

$\hat{Y}_{ABSQ} = E(\vartheta) = \vartheta$

where

$$E_Y(\vartheta|\underline{x}) = \frac{\int_\theta \int_\varphi \int_\delta \theta G^*(\theta,\psi,\delta)\partial\theta\partial\psi\partial\delta}{\int_\theta \int_\varphi \int_\delta G^*(\theta,\psi,\delta)\partial\theta\partial\varphi\partial\delta} \tag{4.49}$$

The above equation (4.49) is evaluated by method of Lindley approximation by replacing $\vartheta$ by $Y(\theta,\psi,\delta)$ in equation (4.49)

**Special cases:–**

$Y(\theta,\varphi,\delta) = Y$

$\hat{Y}_{ABSQ} = E(Y|\underline{x})$

**3. Approximate Bayes Estimate of $\boldsymbol{\delta}$**

$\hat{Y}_{ABSQ} = E(Y|\underline{x})$

$= Y + \psi_1 + \psi_2$

$Y = \delta$ ; $\quad Y_3 = \dfrac{\partial Y}{\partial \delta} = 1, \quad Y_{31} = Y_{32} = Y_{33} = 0$

$Y_2 = Y_{22} = Y_{21} = Y_{23} = 0$

$Y_1 = Y_{11} = Y_{12} = Y_{13} = 0$

$$\hat{\delta}_{ABSQ} = \delta + \psi_1 + \psi_2 \tag{4.50}$$

*where* $\psi_1 = Y_1a_1 + Y_2a_2 + Y_3a_3 + a_4 + a_5$

$$\psi_1 = \left(\frac{-\xi}{\delta} + \frac{\delta}{\beta^2} - \frac{1}{\varepsilon}\right)\frac{U_{32}}{D} + \left(\frac{\xi-1}{\delta} - \frac{1}{\varphi}\right)\frac{Y_{33}}{D}$$

$$\text{and} \quad \psi_2 = \frac{1}{2}[(A\sigma_{11} + B\sigma_{21} + P\sigma_{31}) \cup_1 + (A\sigma_{12} + B\sigma_{22} + P\sigma_{32}) \cup_2 + (A\sigma_{13} + B\sigma_{23} + P\sigma_{33}) \cup_3]$$

$$\psi_2 = \frac{1}{2}(A\sigma_{13} + B\sigma_{23} + P\sigma_{33})$$

$$\hat{\delta}_{ABSQ} = \delta + \left(\frac{-\xi}{\delta} + \frac{\delta}{\varphi^2} - \frac{1}{\varepsilon}\right)\frac{Y_{32}}{D} + \left(\frac{\xi - 1}{\delta} - \frac{1}{\varphi}\right)\frac{Y_{33}}{D} + \frac{1}{2}(A\sigma_{13} + B\sigma_{23} + P\sigma_{33})$$

$$\hat{\delta}_{ABSQ} = \delta + \Delta; \text{ at } \left(\hat{\theta}_{ML}, \hat{\varphi}_{ML,}\hat{\delta}_{ML}\right) \tag{4.51}$$

Where

$$\Delta = \left(\frac{-\xi}{\varphi} - \frac{\delta}{\varphi^2} - \frac{1}{\varepsilon}\right)\frac{U_{32}}{D} + \left(\frac{\xi-1}{\delta} - \frac{1}{\varphi}\right)\frac{U_{33}}{D} + \frac{1}{2}\frac{U_{13}}{D}\left[\frac{U_{11}}{D}\left(\left(\frac{2n}{\theta^3} - \left(\frac{1}{\delta}+1\right)\omega_{133}\right) + 2U_{12}\left(\frac{1}{\delta}+1\right)\omega_{123} + 2U_{13}\frac{\varphi}{\delta^2}\omega_{122} - 2U_{23}\frac{\omega_{14}}{D\delta^2} - 2\frac{U_{22}}{D}\left(\frac{1}{\delta}+1\right)\omega_{113} - \frac{2}{\delta^2}\frac{U_{33}}{D}\omega_{11}\right)\right] + \frac{1}{2}\frac{U_{23}}{D}\left[\left(\frac{1}{\delta}+1\right)\omega_{123}\frac{U_{11}}{D} - \frac{4U_{12}}{D}\left(\frac{1}{\delta}+1\right)\omega_{113} - \frac{2U_{13}}{D}\frac{\omega_{14}}{\delta^2} + \left(\frac{U_{22}+2U_{23}}{D}\right).\left(\frac{n}{\delta^2\varphi^2} - \frac{\delta_{12}}{\delta^2}\right) + \frac{U_{33}}{D}\left(\frac{2}{\delta^3}\left(\frac{n}{\varphi} - \delta_{10}\right)\right)\right] + \frac{1}{2}\frac{U_{33}}{D}\left[\frac{Y_{11}\varphi}{D\delta^2}\omega_{122} - \frac{2Y_{12}}{D}\frac{\omega_{14}}{\varphi^4} - 4\frac{U_{13}}{D}\frac{\omega_{14}}{\delta^3} + 4\frac{U_{23}}{D}\frac{1}{\delta^3}\left(\frac{n}{\varphi} - \delta_{11}\right) + \frac{U_{22}}{D}\left(\frac{n}{\delta^2\varphi^2} - \frac{\delta_{12}}{\delta^2}\right) + \frac{U_{33}}{D}\left(\frac{-2n}{\delta^3} - \frac{6n\, log\varphi}{\delta^4} + \frac{6}{\delta^4}\delta_{10}\right)\right] \tag{4.52}$$

**Numerical Comparison**

The numerical calculations are presented in table below.

1. The Random variable of Generalized Compound Rayleigh Distribution is generated by R-Language programming by taking the values of the parameters $\theta, \varphi, \delta$ , taken as $\theta = 1$ , $\varphi = 0.5$ and $\delta = 0.8$ in the equations[(4.2)-(4.4)] and equation(1.1).

2. Taking the different sizes of samples n=10(10)80 with complete sample, MLE's, the Approximate Bayes estimator, and their respective MSE's (in parenthesis) are obtained by repeating the steps 500 times, are presented in the tables(1) with parameters of prior distribution $a$ =2 and $b$ =3.

3. Table (1) present the MLE of parameter of $\delta$ (for known $\theta$ *and* $\varphi$ ), Bayes Estimator of $\delta$ under QLF and Approximate Bayes estimator under QLF (for $\theta$, $\varphi$ *and* $\delta$ unknown) and their respective MSE's. It also presents the mean and MSE's of $\delta$ and Approximate Bayes estimator of $\delta$ under QLF. All the estimators have minimum MSE's for large sample sizes, as the sample sizes decrease, the MSE's increased. The MSE's in all above cases are presented in parenthesis.

**Table (2.1) Mean and MSE'S of $\delta$**

| **n** | **10** | **20** | **30** | **40** | **50** | **60** | **70** | **80** |
|---|---|---|---|---|---|---|---|---|
| $\hat{\delta}_{ML}$ | 0.5900124 | 0.6922601 | 0.6865478 | 0.8697568 | 0.849001 | 0.949011 | 0.945454 | 0.965732 |
| **MSE** | **[0.02564]** | **[0.02657]** | **[0.09658]** | **[0.00365]** | **[0.00465]** | **[0.00426]** | **[0.00367]** | **[0.00145]** |
| $\hat{\delta}_{BS}$ | 0.5501482 | 0.5555857 | 0.6521463 | 0.8170933 | 0.875761 | 0.975763 | 0.924443 | 1.052443 |
| **MSE** | **[1.69e-05]** | **[1.59e-05]** | **[1.69e-05]** | **[0.00184]** | **[0.00162]** | **[0.00164]** | **[0.00318]** | **[0.00412]** |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $\hat{\delta}_{ABS}$ | 0.54632 58 | 0.56913 77 | 0.87458 96 | 0.8801 81 | 0.8592 358 | 0.9792 358 | 0.9873 98 | 1.0073 98 |
| **MS E** | **[0.0042 51]** | **[0.0008 22]** | **[0.0009 56]** | **[0.0157 8]** | **[0.0154 7]** | **[0.0154 7]** | **[0.013 74]** | **[0.015 44]** |

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