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Temporal Soliton Solution of the Generalized Davey-Stewartson System of Partial Differential Equations

Nadeem Salamat
Department of Mathematics & Statistics,
Karakoram International University, Pakistan.
E-mail: nadeem.salamat@kiu.edu.pk

Muhammad Mustahsan
Department of Mathematics,
The Islamia University,
Bahawlpur, Pakistan.
E-mail: Muhammad.mustahsan@iub.edu.pk

Abstract

In this paper, the generalized Davey-Stewartson system of partial differential equations are studied and a solitary wave solution is developed using the modified simple equation method. This method provides a new temporal soliton solution of the system.

AMS subject classification:
Keywords: Davey-Stewartson system of partial differential equation, soliton solution, modified simple equation method.

1. Introduction

The Non Linear Partial Differential Equations (NLPDEs) are extensively used in mathematical models of biology and physics. Many methods for solving the nonlinear evaluation equations, such as Modified Simple Equation Method (MSEM) [10, 15], Differential Transformation Method (DTM) [1], has been introduced in the last decade. The other methods include the sine-cosine method [18, 2], exponential function method [9], sn-ns method [12]. There are some other methods also discussed in literature. In these methods, a partial differential equation is transformed into a ordinary differential equation through the traveling wave approximation. Finally, a reverse approximation is used to determined the solution to the original problem.

The Devay-Stewartson system of partial differential equations is classified into integrable and non integrable partial differential equations [8, 7]. At the same time, these equations are also categorized into the four types of partial differential equations. The solution of particular DS systems (DS-II) are developed by $\frac{G'}{G}$ expansion method [19], power law non linearity [6] methods etc. The existence of the generalized solution of the system is granted in [5], and estimation of the rough waves produced by the system is taken into account in [11], and special solutions are developed in [4]. The phenomenon of the production of excitation and solitary solutions are considered into [13, 14, 16, 17].

In this paper, the generalized davey-stewartson system of partial differential equation [3] is considered and the exact solutions are explored with the method of modified simple differential equation method. This system of partial differential equation is written as

$$iu_t + \delta u_{xx} + u_{yy} + \lambda \mid u \mid^2 u - v_x u = 0 \tag{1.1}$$

$$\alpha v_{xx} + v_{yy} + \gamma(\mid u \mid^2)_x = 0 \tag{1.2}$$

where $\delta, \lambda = \pm 1$ and α may be positive or negative according to the system and $\gamma > 0$. The complex valued function $u(x, y, t)$ describe the amplitude of the wave and $v(x, y, t)$ describe the mean flow of the wave phenomena. This system of equations is classified into four classes depending on the nature of and range of constants δ and α.

This paper is arranged as, the Section 2 describes the method of modified simple equation method in brief. The solution of the DS system is developed in section 3 and Section 4 consist of the interpretation of the soliton solutions of the DS-system and Section 5 concludes the paper.

2. Description of the Modified Simple Equation Method

Let a Non-Linear Partial Differential Equation (NLPDE) is written as

$$F(u, u_t, u_x, u_y, \ldots, u) = 0 \tag{2.1}$$

This method is discussed in [10], the method consist of the following four steps.

Step-1 First of all use the transformation $Z = x + y \pm t$. This transformation converts the above equation into a simple non linear ODE, described as

$$P(u, u_t, u_x, u_y, \ldots, u) = 0 \tag{2.2}$$

Step-2 Let us consider the

$$u = \sum_{k=0}^{N} A_k \left(\frac{G'}{G}\right)^k \tag{2.3}$$

be the solution, where N is any positive integer, and $A_0, A_1, \ldots, A_N$ be the constants and G is a function of variable z. All these variables and constants have to be determine latter.

Step-3 Find N and determine the required derivatives of u. The method of determining the quantity N is based on homogenous balance principal and linearly ordered differential equation theory.

Step-4 Put all the above values in equation 2.2. It is assumed that the traveling wave solution is expressed by a polynomial in G. Equate all the coefficients of G and its powers to zero. Solve the resultant system of algebraic equations and determine the values of $A_0, A_1, \ldots, A_N$ and G. Put these values in the solution.

3. The solution of Davey-Stewartson System

Let us consider the DS-system of partial differential equation

$$iu_t + \delta u_{xx} + u_{yy} + \lambda \mid u \mid^2 u - v_x u = 0 \tag{3.1}$$

$$\alpha v_{xx} + v_{yy} + \gamma(\mid u \mid^2)_x = 0 \tag{3.2}$$

Let us use the substitution $z = x + y - t$ this converts the Eq(3.1) and Eq(3.2) into the equations of ODEs as

$$-iu' + \delta u'' + u'' + \lambda \mid u \mid^2 u - v' u = 0 \tag{3.3}$$

$$\alpha v'' + v'' + \gamma(\mid u \mid^2)' = 0 \tag{3.4}$$

By simplifying and integrating the Eq. (3.4)

$$v'' = \frac{-\gamma}{\alpha+1}(\mid u \mid^2)' \quad \Rightarrow \quad v' = \frac{-\gamma}{\alpha+1}(\mid u \mid^2) \tag{3.5}$$

Put the value of v' into the Eq. (3.3), we obtain

$$-iu' + (\delta+1)u'' + \lambda \mid u \mid^2 u + \frac{\gamma}{\alpha+1}u^3 = 0 \tag{3.6}$$

$$(\delta+1)u'' - iu' + (\lambda + \frac{\gamma}{\alpha+1})u^3 = 0 \tag{3.7}$$

Let Eq. (2.3) be the solution of the system, first of all, we have to determine the positive integer N. By balancing the highest order derivative and the nonlinear terms, we obtained $N = 1$, then the solution becomes

$$u = \sum_{k=0}^{1} A_k \left(\frac{G'}{G}\right)^k \qquad \text{or} \qquad u = A_0 + A_1 \left(\frac{G'}{G}\right). \tag{3.8}$$

By differentiating this equation for the required derivatives of u, simplifying the Eq. (3.7, 3.8) and put the coefficients of G^0, G^{-1}, G^{-2} and G^{-3} equal to zero,we get the following algebraic system of equations.

$$A_0^3 = 0 \tag{3.9}$$

$$(\delta+1)A_1G^{'''} - iA_1G^{''} + 3A_0^2A_1\left(\lambda + \frac{\gamma}{\alpha+1}\right)G^{'} = 0 \tag{3.10}$$

$$3A_0A_1^2\left(\lambda + \frac{\gamma}{\alpha+1}\right)(G^{'})^2 + iA_1(G^{'})^2 - 3A_1(\delta+1)G^{'}G^{''} = 0 \tag{3.11}$$

$$A_1(G^{'})^3\left[2(\delta+1) + \left(\lambda + \frac{\gamma}{\alpha+1}\right)A_1^2\right] = 0 \tag{3.12}$$

Now $A_1 \neq 0$, and from the equation (3.12), we get

$$A_1 = \pm\sqrt{\frac{-2(\alpha+1)(\delta+1)}{\lambda(\alpha+1)+\gamma}} \tag{3.13}$$

The quantity A_1 becomes zero for $\alpha = -1$ or $\delta = -1$, Simplifying the equations (3.9,3.10,3.11), we get

$$G^{'''} + \frac{1}{3(\delta+1)^2}G^{'} = 0 \tag{3.14}$$

Solution of this equation can be written as

$$G^{'}(z) = C_1\cos(k_1z) + C_2\sin(k_1z) \tag{3.15}$$

where C_1 and C_2 are constants of integration and constant term $k1$ is represented as

$$k_1 = \sqrt{\frac{1}{3(\delta+1)^2}} \tag{3.16}$$

and by integrating equation (3.15), we get

$$G(z) = \frac{C_1}{k_1}\sin(k_1z) - \frac{C_2}{k_1}\cos(k_1z) \tag{3.17}$$

From equation (3.8, 3.15) and (3.17), we get

$$u(x,y,t) = A_1k_1\left[\frac{C_1\cos(k_1(x+y-t)) + C_2\sin(k_1(x+y-t))}{C_1\sin(k_1(x+y-t)) - C_2\cos(k_1(x+y-t))}\right] \tag{3.18}$$

After solving the equation (3.4) and equation (3.18)

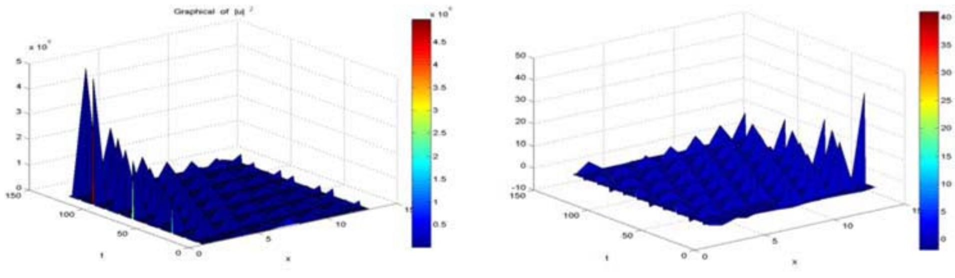
$$V(x,y,t) = -2A_1^2k_1^2\frac{\arctan(k2)[C_2^2k_2^2 - C_2^2 + 2C_1k_2] + k_2(C_1^2 + C_2^2)}{(2k_2 + k_2^2 - C_2)k_1C_2} \tag{3.19}$$

where k_2 is defined as $k_2(x,y,t) = \tan\left(\frac{k_1}{2}(x+y-t)\right)$. This complex integral is solved with the help of soft wear Maple.

4. Interpretation of the Davey-Stewartson System of Equations

In this section, we describe the DS-systems for the multiple parameters. The solution diverges for $\delta = \alpha = -1$, due to the definition of A_1andk_1 in equations Eq. (3.13) and Eq. (3.16). We propose to use the value of the parameter other than -1. According to the value of parameters, these equations can be classified into the following four classes. Numerical computations are computed for a fixed parameters $\gamma = 0.005$

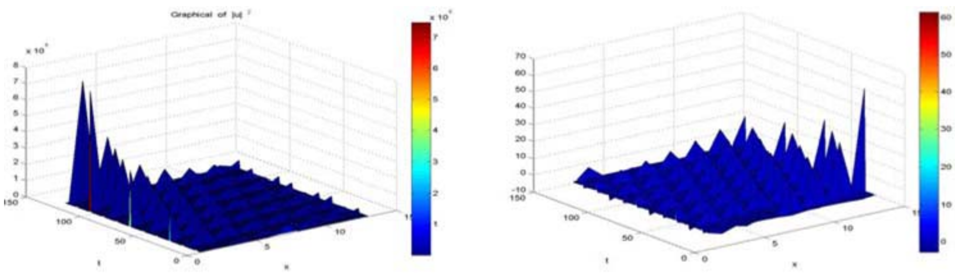
Hyperbolic - Hyperbolic Case: The Davey-Stewartson system of equation represents Hyperbolic - Hyperbolic type of the system of differential equations when the parameters are adjusted in such a way that $(\delta, \alpha) = (-, -)$. The graphical representation of the soliton for the $\delta = \alpha = 0.99$ is represented in figure (1).



(a) $|u(x, y, t)|^2$ (b) v(x,y,t)

Figure 1: Family of temporal soliton in equation(3.18 and 3.19) for a fixed $y = 0.005$.

Hyperbolic - Elliptic Case: The Davey-Stewartson system of equation represents Hyperbolic - Hyperbolic type of the system of differential equations when the parameters are adjusted in such a way that $(\delta, \alpha) = (-, +)$. The $U(x, y, t)$ is a complex valued function. The graphical representation of the soliton for the $\delta = \alpha = 0.99$

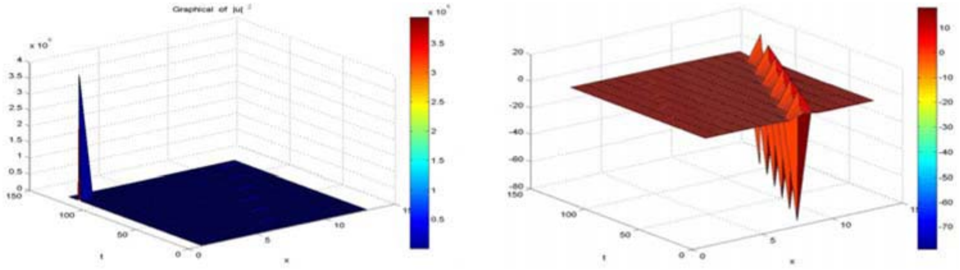


(a) $|u(x, y, t)|^2$ (b) v(x,y,t)

Figure 2: Family of temporal soliton in equation(3.18 and 3.19) for a fixed $y = 0.005$.

Elliptic - Hyperbolic Case: The Davey-Stewartson system of equation represents Hyperbolic - Hyperbolic type of the system of differential equations when the parameters are adjusted in such a way that $(\delta, \alpha) = (+, -)$. The function $U(x, y, t)$

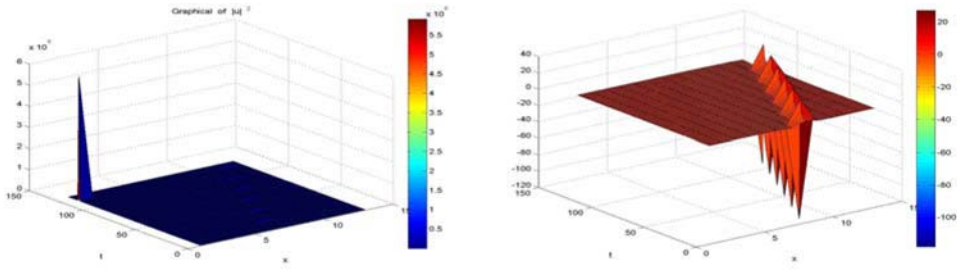
is a complex valued function. The graphical representation of the soliton for the $\delta = \alpha = 0.99$



(a) $|u(x, y, t)|^2$ (b) v(x,y,t)

Figure 3: Family of temporal soliton in equation(3.18 and 3.19) for a fixed $y = 0.005$.

Elliptic - Elliptic Case: The Davey-Stewartson system of equation represents Elliptic - Elliptic type of the system of differential equations when the parameters are adjusted in such a way that $(\delta, \alpha) = (+, +)$. The $U(x, y, t)$ represents the complex valued function. The graphical representation of the soliton for the $\delta = \alpha = 0.99$



(a) $|u(x, y, t)|^2$ (b) v(x,y,t)

Figure 4: Family of temporal soliton in equation(3.18 and 3.19) for a fixed $y = 0.005$.

5. Conclusion

In this paper, the temporal soliton solution of the generalized DS- system of partial differential equations is obtained by the modified simple equation method. These soliton solutions are important to explain the physical phenomenon in physics and evolutionary mathematics, where the energy moves in packets.

References

[1] Marwan Taiseer Alquran, *Applying Differential Transform Method to Nonlinear Partial Differential Equations: A Modified Approach*, Applications and Applied

Mathematics: An International Journal (AAM) **7** (2012), no. 1, 155–163.

[2] Anwar Jáafar Mohamad Jawad, *THE SINE-COSINE FUNCTION METHOD FOR THE EXACT SOLUTIONS OF NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS*, International Journal of Research and Reviews in Applied Sciences (IJRRAS) **13** (2012), no. 1, 186—191.

[3] A. Davey and K. Stewartson, *On three-dimensional packets of surface waves*, Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences, vol. 338, Jun. 4, 1974, no. 1613, Royal Society of London, 1974, pp. 101–110.

[4] Ghodrat Ebadi and Anjan Biswas, *The G'-G Method and 1-soliton Solution of the Davey-Stewartson Equation*, Math. Comput. Model. **53** (2011), no. 5-6, 694—698.

[5] A. Eden and T.B. Gürel, *On the existence of special solutions of the generalized davey-stewartson equations*, Applied Mathematics Letters **22** (2009), no. 2009, 1174–1177.

[6] Y. Gurefe, E. Misirli, Y. Pandir, A. Sonmezoglu, and M. Ekici, *New Exact Solutions of the Davey-Stewartson Equation with Power-Law Nonlinearity*, Bulletin of the Malaysian Mathematical Sciences Society **2013** (2013).

[7] Benoit Huard and Vladimir Novikov, *On classification of integrable davey–stewartson type equations*, Journal of Physics A: Mathematical and Theoretical **46** (2013), no. 27, 275202.

[8] Ponce Gustavo Linares, Felipe, *On the davey-stewartson systems*, Annales de l'institut Henri Poincaré (C) Analyse non linéaire **10** (1993), no. 5, 523—548 (eng).

[9] Emine Misirli and Yusuf Gurefe, *EXP-FUNCTION METHOD FOR SOLVING NONLINEAR EVOLUTION EQUATIONS*, Mathematical and Computational Applications **16** (2011), no. 1, 258—266.

[10] Abdulkadir ERTAS Mustfa MIZRAK, *The modified simple equation method for solving some nonlinear evaluation equations*, DUFED **1** (2012), no. 1, 39–45.

[11] Yasuhiro Ohta and JiankeYang, *Dynamics of rogue waves in the davey–stewartson ii equation*, Journal of Physics A: Mathematical and Theoretical **46** (2013), no. 10, 105202.

[12] Alvaro H. Salas, *Solving nonlinear partial differential equations by the sn-ns method*, Abstract and Applied Analysis **2012** (2012), 25 pages, Article ID= 340824.

[13] X Y Tang, KWChow, and S Y Lou, *Some novel nonlinear coherent excitations of the davey–stewartson system*, Journal of Physics A: Mathematical and General **38** (2005), no. 48, 10361.

[14] R.-J. Wang and Y.-C. Huang, *Exact solutions and excitations for the Davey-Stewartson equations with nonlinear and gain terms*, European Physical Journal D **57** (2010), 395–401.

[15] Elsayed M. E. Zayed and Hoda Ibrahim S. A., *Modified simple equation method and its applications for some nonlinear evolution equations in mathematical physics*, International Journal of Computer Applications **67** (2013), no. 6, 39–44, Published by Foundation of Computer Science, New York, USA.

[16] E. M. E. Zayed, Ali Filiz, Mehmet Ekici, Abdullah Sonmezoglu, and Durgun Duran, *Exact solutions of the ZK-MEW equation and the Davey-Stewartson equation*, International Journal of Applied Mathematical Research **3** (2014), no. 2, 122–132, Science Publishing Corporation.

[17] Hassan A. Zedan and Ahmed Al Saedi, *Periodic and solitary wave solutions of the davey-stewartson equation*, An International Journal of Applied Mathematics & Information Sciences **4** (2010), no. 2, 253–260.

[18] Monaquel Shatha Jameel Zedan, Hassan A., *The sine-cosine method for the daveystewartson equations*, Applied Mathematics E-Notes [electronic only] **10** (2010), 103–111 (eng).

[19] MingXing Zhu, Li Wu, ChuChu Yin, and MingYuan Zhang, *Solving (2 + 1) - Dimensional Davey - Stewartson Equation by (G'- G) Expansion Method*, International Journal of Nonlinear Science **12** (2011), no. 2, 147–150.