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Unsteady Convective Nanofluid Flow past a Flat Porous Plate Moving Through a Binary Mixture

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Abstract

In this paper, the problem of unsteady convective nanofluid flow with chemical reaction past a flat porous plate moving through a binary mixture in an optically thin environment is considered. The governing boundary layer equations are converted to non-linear ordinary differential equations by similarity transformation and then solved numerically by MATLAB “bvp4c” function. The velocity, temperature and concentration profiles are presented graphically for various values of the material parameters.

Key-Words: - Porous medium, Diffusion, Binary mixture, Arrhenius kinetics, Nanofluids and heat transfer.

1 Introduction

Combined heat and mass transfer problems with chemical reaction are of importance in many processes and have, therefore, received a considerable amount of attention in research. In processes such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler, heat and the mass transfer occur simultaneously. Study towards boundary layer flow of a binary mixture of fluids is very important in view of its application in various branches of engineering and technology. A familiar example is an emulsion which is the dispersion of one fluid within another fluid. Typical emulsions are oil dispersed within water or water within oil. Another example where the mixture of fluids plays an important role is in multigrade oils. Some polymeric type fluids are added to the base oil so as to enhance the lubrication properties of mineral oil [1]. Moreover through chemical reaction, all industrial chemical processes are designed to transform cheaper raw materials to high value products. Naturally these transformations occur in reactors. Fluid dynamics plays a pivotal role in establishing relationship between the reactor hardware and reactor performance. Unsteady free convection boundary layer flows with heat and mass transfer encounter an important criterion of species chemical reaction with finite Arrhenius activation energy defined by Makinde[2]. Khanafer et al. [3] developed a stream function vorticity based numerical algorithm based on the thermal dispersion model. They found that the nanofluid heat transfer rate increases with increase in nanofluid volume fraction. Buongiorno [4] proposed a mathematical model to capture the nano particle/base fluid slip by treating nanofluid as a two component mixture with Brownian diffusion and thermophoresis as the important factors in the convective transport process in a nanofluid. He introduced these terms in the conservation equations for mass and energy. Recently Sastry et.al [5] has studied the case of unsteady MHD convective flow past a flat porous plate moving through a binary mixture with suction and injection. Truesdell [6] has first obtained the thermo-mechanical balance equations for a mixture of general materials. Chamkha [7] studied the unsteady convective heat and mass transfer flow on a semi-infinite porous moving plate with heat absorption. Vajravelu and Hadjinicolaou [8] observed the heat transfer characteristics in the laminar boundary layer of a viscous fluid over a stretching sheet with viscous dissipation or frictional heating and internal heat generation. The aim of this paper is to study the effect of suction/injection, Grashof number, chemical reaction and Damköhler number on nanofluid flow. Nano particle Cu is taken into consideration with water as base fluid.

2 Problem Formulation

Consider an unsteady one-dimensional convective flow with chemical reaction and past a vertical porous plate moving through a binary mixture in water based nano fluid containing nano particles Cu (Copper). Assume that the boundary wall to be of infinite extended so that all quantities are homogeneous in *x* and hence all derivatives with respect to *x* are omitted. Let the *x*-axis be directed in up- word direction along the plate and the *y*-axis is normal to the plate. Let *u* and *v* be the velocity components along the *x*- and *y*- axes, respectively. The momentum, energy and chemical species concentration balance equations which govern the flow may be written as follows.

$$\frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\rho_{nf}\left(\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y}\right) = \mu_{nf}\frac{\partial^2 u}{\partial y^2} + g(\rho\beta)_{nf}(T - T_\infty) + g(\rho\beta^*)_{nf}(\varphi - \varphi_\infty) \tag{2}$$

$$(\rho C_p)_{nf}\left(\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y}\right) = k_{nf}\frac{\partial^2 T}{\partial y^2} + Q \tag{3}$$

$$\frac{\partial \varphi}{\partial t} + v\frac{\partial \varphi}{\partial y} = D\frac{\partial^2 \varphi}{\partial y^2} - R_A \tag{4}$$

where $Q = (-\Delta H)R_A$ is the heat of chemical reaction and is called the activation enthalpy and

$$R_A = k_r e^{-E_A/R_G T}\varphi^n \tag{5}$$

is the Arrhenius type of the n^{th} order irreversible reaction, k_r is the chemical reaction rate, R_G is the universal gas constant and E_A is the activation energy parameter. The boundary conditions of the above problem are assumed to be

$$u(y,0) = 0, T(y,0) = T_w, \varphi(y,0) = \varphi_w$$
$$u(0,t) = U_0, T(0,t) = T_w, \varphi(0,t) = \varphi_w, t > 0$$
$$u \to 0, T \to T_\infty \; and \; \varphi \to \varphi_\infty \text{ as } y \to \infty, t > 0$$

where U_0 is the plate characteristic velocity. From the equation of continuity (1), it can be noted that v is either constant or a function of time. Following Makinde [2], we take

$$v = -c\left(\frac{\upsilon}{t}\right)^{1/2} \tag{6}$$

where $c > 0$ is the suction parameter $c < 0$ is the injection parameter. Introducing the dimensionless quantities and parameters

$$u = U_0 f(\eta), (\theta, \theta_w) = \frac{(T, T_w)}{T_\infty},$$
$$(h, h_w) = \frac{(\varphi, \varphi_w)}{\varphi_\infty} G_r = \frac{4tg\beta_f T_\infty}{U_0}, G_c = \frac{4tg\beta_f{}^* \varphi_\infty}{U_0}, P_r = \frac{\upsilon_f}{\lambda_f}$$
$$\lambda_f = \frac{k_f}{(\rho C_p)_f}, S_C = \frac{\upsilon}{D}, \gamma = \frac{E_A}{R_G T_\infty}, \eta = \frac{y}{2\sqrt{\upsilon_f t}}, b = \frac{(-\Delta H)\varphi_\infty}{(\rho C_p)_f T_\infty}$$
$$D_a = 4tk_0 \varphi_\infty{}^{n-1}, k_0 = k_r e^{-E_A/R_G T_\infty}$$

Also

$$\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}$$
$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s$$
$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}$$
$$(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s$$
$$\frac{k_{nf}}{k_f} = \frac{(k_s + 2k_f) - 2\phi(k_f - k_s)}{(k_s + 2k_f) + \phi(k_f - k_s)}$$
$$(\rho\beta)_{nf} = (1-\phi)(\rho\beta)_f + \phi(\rho\beta)_s$$

With the above equations one can obtain the following differential equations

$$\frac{f''}{(1-\emptyset)^{2.5}} + 2(\eta + c)f'\emptyset_1 = -G_r\emptyset_2(\theta - 1) - G_c\emptyset_3(h - 1) \tag{9}$$
$$\frac{\emptyset_4}{P_r}\theta'' + 2(\eta + c)\theta'\emptyset_5 = -bD_a h^n \exp\left\{\gamma\left(1 - \frac{1}{\theta}\right)\right\} \tag{10}$$
$$\frac{1}{S_c}h'' + 2(\eta + c)h' = D_a h^n \exp\left\{\gamma\left(1 - \frac{1}{\theta}\right)\right\} \tag{11}$$

with boundary conditions

$$f(0) = 1, \theta(0) = \theta_w, h(0) = h_w, f(\infty) = 0, \theta(\infty) = 1, h(\infty) = 1 \qquad (12)$$

where *Da* is the Damköhler number, γ is the activation energy parameter, *Gr* is the thermal Grashof number and *Gc* is the solutal Grashof number, k_r is the chemical reaction rate, *Pr* is the Prandtl number.

Also one can obtain Nusselt number *Nu* $\propto -\theta'(0)$ and Sherwood number *Sh* $\propto -h'(0)$

2.1 Calculation of skin friction

By the definition of shearing stress,

$$\tau_w = \mu_{nf} \left(\frac{\partial u}{\partial y}\right)_{y=0}$$

The coefficient of skin friction is obtained as

$$C_f = -\frac{1}{(1-\phi)^{2.5}} \frac{f'(0)}{R_e}$$

where $R_e = \sqrt{\frac{\rho_f t}{\mu_f}}\, U_0$ is Reynolds number.

The heat flux at the surface of the plate is given by

$$q_w = -k_{nf} \left[\frac{\partial T}{\partial y}\right]_{y=0} = \frac{-k_f T_\infty \theta'}{2(\upsilon_f t)^{1/2}}$$

Nusselt number

$$N_u = \frac{q_w (\upsilon_f t)^{1/2}}{k_f T_\infty} = \frac{-1}{2} \theta'(0)$$

Sherwood number

$$Sh = \frac{j_w (\upsilon_f t)^{1/2}}{D_f \varphi_\infty} = \frac{-1}{2} h'(0)$$

where $j_w = D_f \left(\frac{\partial \varphi}{\partial y}\right)_{y=0}$ is mass flux at the wall surface

3 Problem Solution

The governing boundary layer equations are converted to non-linear ordinary differential equations by similarity transformation and then solved numerically by MATLAB “bvp4c” function. The velocity, temperature and concentration profiles are presented graphically for various values of the material parameters.

4 Results and Discussion

To get a clear insight of the physical problem, we have assigned various numerical values to the parameters that are incorporated in the problem with which one can discuss the profiles of velocity, temperature and concentration. We have taken the values of Schmidt number (Sc) for Water vapour 0.62 at temperature 25ºC and one atmospheric pressure. The value of Prandtl number is chosen to be Pr = 0.71 which represents air at temperature 25ºC and one atmospheric pressure. More over the focus is made towards the positive values of the buoyancy parameters i.e. Grashof number $Gr > 0$, corresponds to the cooling problem, and solutal Grashof number $Gc > 0$, indicates the concentration in the free stream region is less than the concentration at the boundary surface. Figures 1 and 2 illustrate the effect of buoyancy forces on the horizontal velocity component in the momentum boundary layer. Fluid velocity is highest at the moving plate surface and decreases to free stream zero velocity far away from the plate satisfying the boundary conditions. Also it is observed that in presence of uniform suction at the plate surface, increase of buoyancy forces lead to retardation in the flow and thereby giving rise to a decrease in the velocity profiles i.e., momentum boundary layer thickness decreases with an increase of buoyancy forces. It is observed that a reverse flow occurs within the boundary layer as the intensity of buoyancy forces increases. Also it is observed that velocity decreases with the increase in the solid volume fraction of the nanofluid particles. Figure 3 depicts the effect of wall suction and injection on the horizontal velocity in momentum boundary layer. It is observed that the momentum boundary layer thickness decreases with increasing the wall suction ($c>0$) and increases with increasing wall injection ($c<0$). It is seen from figure 4, the variation of temperature profile against similarity variable η for varying values of wall temperature parameter under uniform suction. The temperature increases towards the free stream temperature whenever the surface temperature is lower than the free stream temperature. The effect of chemical reaction parameter Da is showed in figure 5. It is very interesting that chemical reaction increases the rate of interfacial mass transfer. The reaction reduces the local concentration, thus increasing its concentration gradient and its flux. From the graph, it can be seen that increase in Da causes a decrease in the concentration of the chemical species in the boundary layer. Figure 6 depicts the effect of Prandtl number on heat transfer rate under various values of volume fraction. It is seen that increase in the volume fraction results in decrease in the heat transfer rate at the same time increase in the Prandtl number enhances the rate of heat transfer. This change is more in free stream fluid. Figure 7 illustrates the effect of Prandtl number on Sherwood number. Mass transfer rate decreases with increase in the Prandtl number. Also it is noticed that increase in the volume fraction enhances the mass transfer rate in the nanofluid flow.

5 Conclusions:

The problem of unsteady convective nanofluid flow with chemical reaction past a flat porous plate moving through a binary mixture in an optically thin environment is considered. The governing boundary layer equations are converted to non-linear ordinary differential equations by similarity transformation and then solved numerically by MATLAB "bvp4c" function. The velocity, temperature and concentration profiles are presented graphically for various values of the material parameters. It is observed that velocity decreases with the increase in the solid volume fraction of the nanofluid particles. Increase in the volume fraction results in decrease in the heat transfer rate at the same time increase in the Prandtl number enhances the rate of heat transfer. Mass transfer rate decreases with increase in the Prandtl number

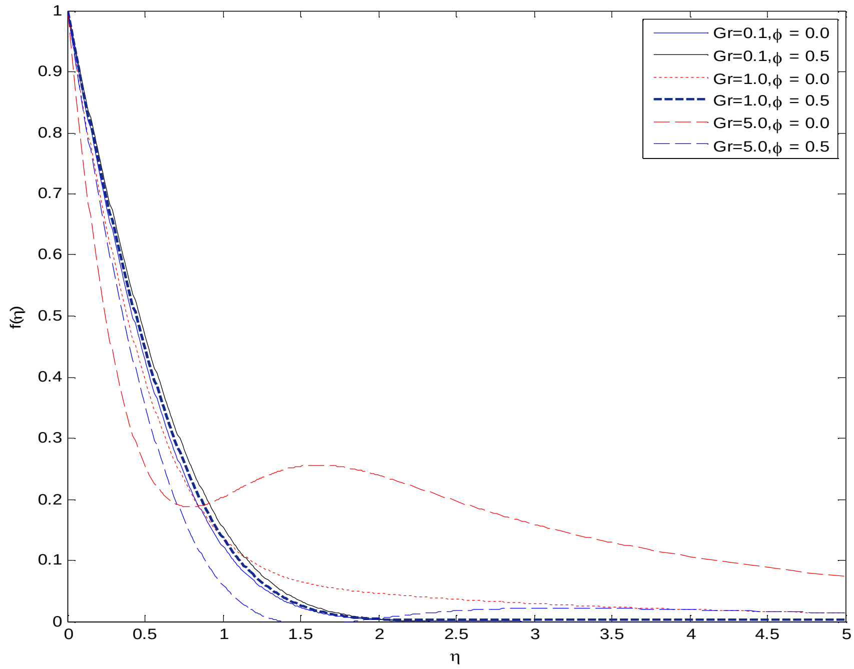

Fig1. Effect of thermal Grashof number on velocity profiles when $(\boldsymbol{Da} = \boldsymbol{c} = \boldsymbol{Ra} = \boldsymbol{Gc} = \boldsymbol{\gamma} = \boldsymbol{\theta_w} = \boldsymbol{h_w} = \mathbf{0.1}, \boldsymbol{Sc} = \mathbf{0.62}, \boldsymbol{b} = \boldsymbol{n} = \mathbf{1})$

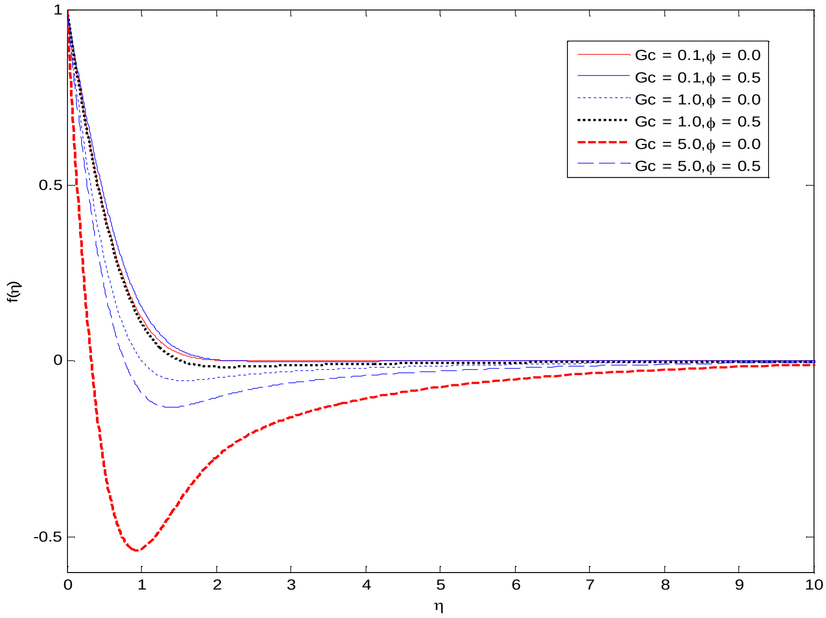

Fig 2. Effect of solutal Grashof number on velocity profiles when $(Da = c = Ra = Gr = \gamma = \theta_w = h_w = 0.1, Sc = 0.62, b = n = 1)$

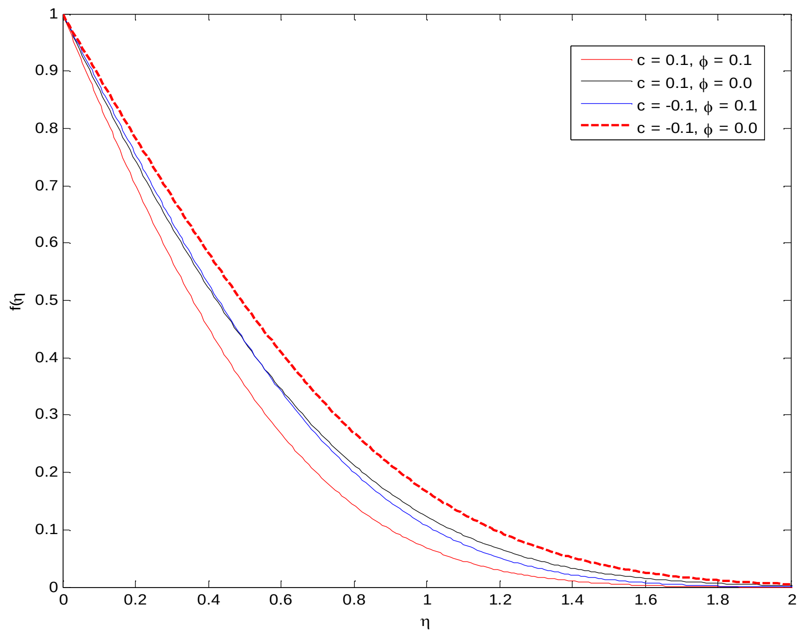

Fig 3. Effect of suction/injection parameter on velocity profiles $(Da = Ra = Gr = Gc = \gamma = \theta_w = h_w = 0.1, Sc = 0.62, b = n = 1)$

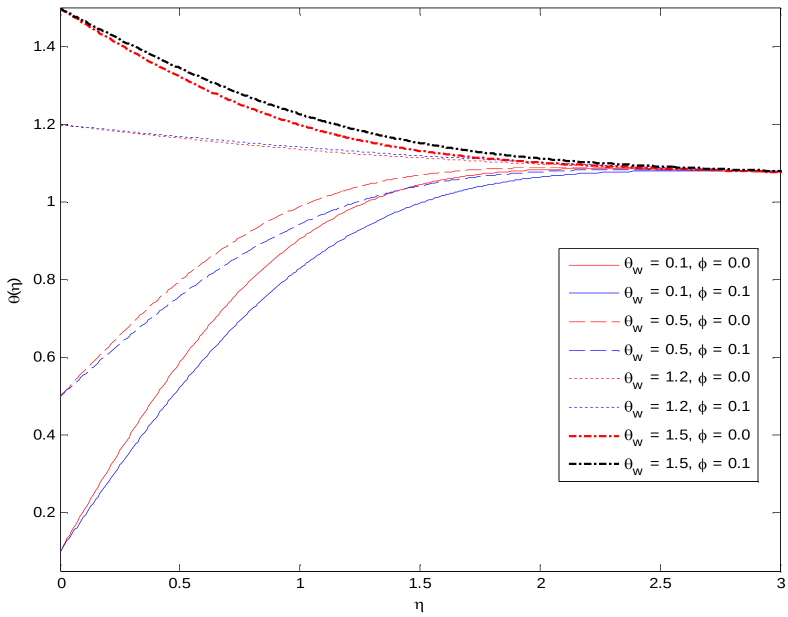

Fig 4. Effect of wall temperature on temperature profile $Da = c = Ra = Gr = Gc = \gamma = h_w = 0.1, Sc = 0.62, b = n = 1)$

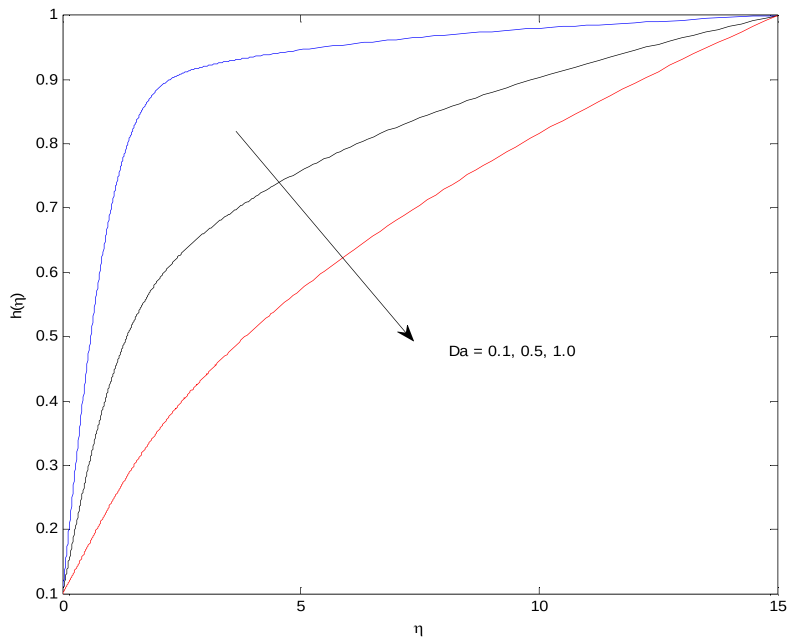

Fig.5: Effect of chemical reaction on concentration profiles when $(c = Ra = Gr = Gc = \gamma = \theta_w = h_w = 0.1, Sc = 0.62, b = n = 1)$

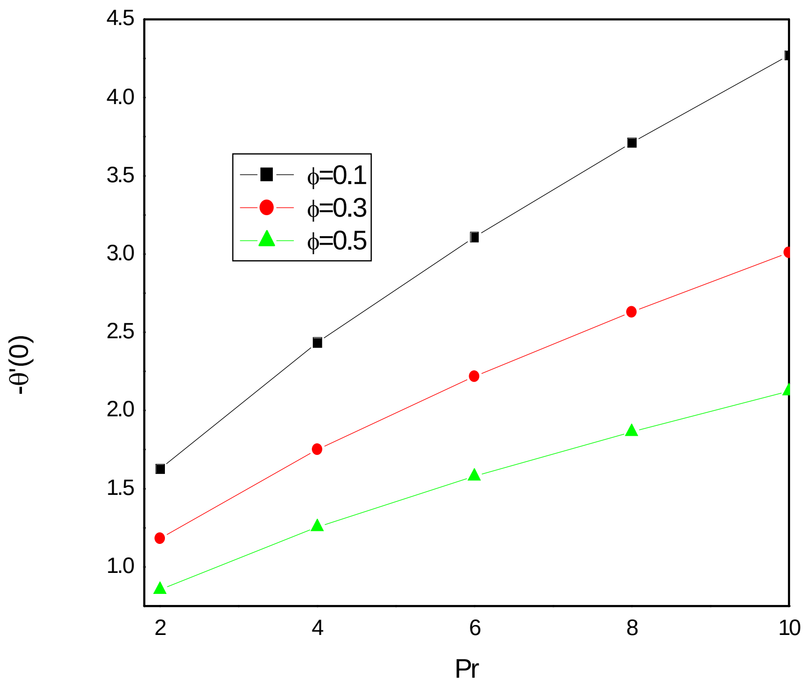

Fig 6. Effect of Prandtl number on Heat transfer coefficient

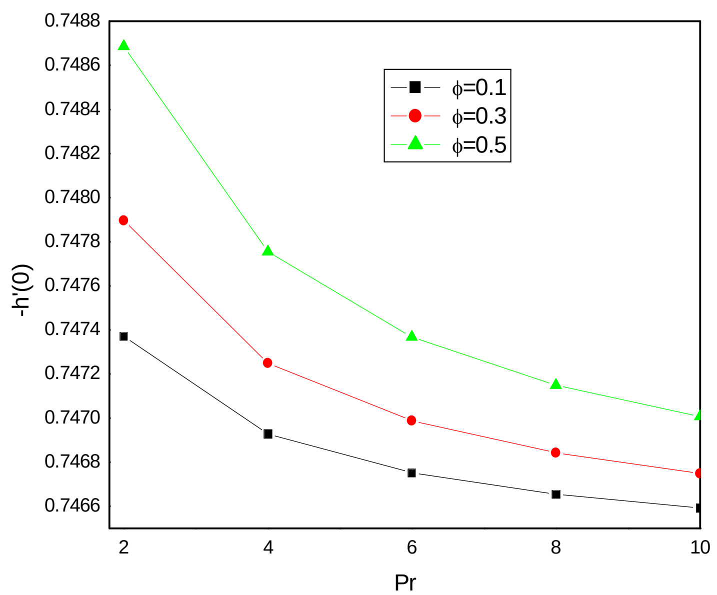

Fig 7. Effect of Prandtl number on Mass transfer coefficient

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