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Threshold Results For A Host-Mortal Commensal Ecosystem With Limited Resources

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Abstract

This paper focus on phase plane diagrams for two species ecological model comprising a mortal commensal and the host species. This model is characterized by a couple of first order non-linear ordinary differential equations. All existing three equilibrium points of the system are identified and the nature of the ecological interaction between the species around the equilibrium points are investigated and the threshold diagram are also illustrated where ever necessary with some considered values of the parameters.

Keywords: Commensalism Interaction, Commensal species, Host species, Equilibrium points, Non-linear system, Phase - Plane diagrams.

2010 Mathematics Subject Classification: 92D25, 92D40.

Introduction

Phase plane is a plane whose points represent possible pairs of values for the two $N_1(t)$ and $N_2(t)$ functions being studied. Those techniques will be almost entirely qualitative concerning the general shape and long term behavior of solutions rather than their precise values. The basic strategy in analyzing the phase plane is to first identify the points where solution trajectories must be exactly horizontal or exactly vertical this will partition the phase plane into a number of regions and the general direction of trajectories in each region can be identified. From here it is often possible to see the general behavior of trajectories even without explicitly solving for them.

The main aim of this paper is to know the general behavior of the ecological commensalism interaction between the two species mortal commensal species and the host species without finding the solution of the system. This model is characteristized by the couple of first order non-linear ordinary differential equations. The possible three equilibrium points of the model are identified and the threshold results are illustrated with some selected values.

To construct this ecological model here we use the following notations:

$N_1(t)$	:	The population of the commensal (S_1) at time t.
$N_2(t)$	:	The population of the host (S_2) at time t.
d_1	:	The mortal rate of the commensal (S_1).
a_2	:	The rate of natural growth of the host (S_2).
a_{11}	:	The rate of decrease of the commensal (S_1) due to the limitations of its natural resources.
a_{22}	:	The rate of decrease of the host (S_2) due to the limitations of its Natural resources.
a_{12}	:	The rate of increase of the commensal (S_1) due to the support given by the host (S_2).
$k_2(= a_2 / a_{22})$	:	The carrying capacity of S_2.
$c(= a_{12} / a_{11})$	:	The coefficient of the commensal.
$e_1(= d_1 / a_{11})$	:	The mortality coefficient of S_1.

Basic Equations

Employing the notation given above the model equations for a two species commensal-host eco-system with mortality rate for the commensal species are given by the following system of non-linear coupled ordinary differential equations.

(i). Growth rate equation for the Mortal commensal species (S_1):

$$\frac{dN_1}{dt} = a_{11}N_1\left[-e_1 - N_1 + cN_2\right] \tag{1}$$

(ii). Growth rate equation for the Host species (S_2):

$$\frac{dN_2}{dt} = a_{22}N_2\left[k_2 - N_2\right] \tag{2}$$

Equilibrium Points

The system under investigation has three equilibrium states $(E_1)-(E_3)$ resulting from

$$\frac{dN_1}{dt} = 0 \;;\; \frac{dN_2}{dt} = 0 \,.$$

E_1 : The fully washed out state (or **extinct** equilibrium state)

$$\overline{N_1} = 0 \quad ; \quad \overline{N_2} = 0 \tag{3}$$

E_2 : The state in which the commensal *species* is washed out while the host *species* only survives

$$\overline{N_1} = 0 \quad ; \quad \overline{N_2} = k_2 \tag{4}$$

E_3 : The state in which both the commensal and the host co-exist

$$\overline{N_1} = ck_2 - e_1 \quad ; \quad \overline{N_2} = k_2 \tag{5}$$

The state E_3, also called the **"normal steady state"**, would exists only when $e_1 < ck_2$. When $e_1 = ck_2$ this equilibrium point merges with E_2.

These equilibrium points are the turning points in the variation of N_1 and N_2 with respect to time t. The lines (straight lines and/or curves) given by $\frac{dN_1}{dt} = 0$ and $\frac{dN_2}{dt} = 0$ in the $N_1 - N_2$ plane may be referred as the **threshold lines** or **Null clines** these are $N_1 = 0$, $N_1 - cN_2 + e_1 = 0$, $N_2 = 0$, $N_2 = k_2$. These lines divide the first quadrant of the $N_1 - N_2$ plane into four regions I, II, III and IV are called the **threshold regions**. The diagram showing the threshold lines and regions is called the **threshold/phase-plane diagram**. This diagram shows the direction of variations of the species around the stable/unstable equilibrium points. The threshold regions and the direction of the field lines in the threshold regions for $a_{11} = 1$, $e_1 = 1$, $c = 2$, $a_{22} = 1$, $k_2 = 2$ of the model are shown in the following Fig.1 & Fig.2.

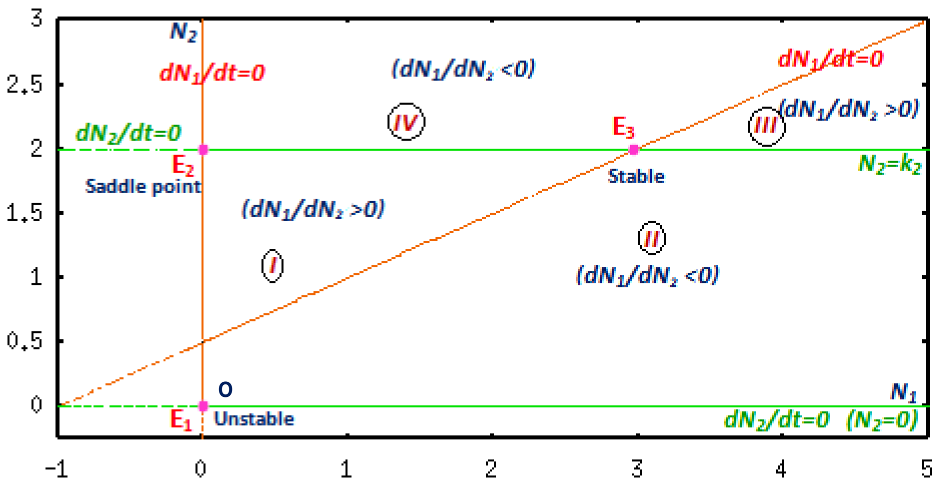

Figure 1: Threshold Regions

Region I: In this region $\frac{dN_1}{dt} > 0$ and $\frac{dN_2}{dt} > 0$ $\Rightarrow \frac{dN_1}{dN_2} > 0$ then $N_1(t)$ is an increasing function of $N_2(t)$ and the trajectories move up and right (The perturbations in both the commensal and host species flourish each other with time t).

Region II: Here $\frac{dN_1}{dt} < 0$ and $\frac{dN_2}{dt} > 0$ $\Rightarrow \frac{dN_1}{dN_2} < 0$ then $N_1(t)$ is a decreasing function of $N_2(t)$ and the trajectories move up and left (The perturbations in the commensal declines where as in the host species flourishes with time t).

Region III: Here $\frac{dN_1}{dt} < 0$ and $\frac{dN_2}{dt} < 0$ $\Rightarrow \frac{dN_1}{dN_2} > 0$ then $N_1(t)$ is an increasing function of $N_2(t)$ and the trajectories move down and left (The perturbations in the commensal flourishes where as in the host species declines with time t).

Region IV: Here $\frac{dN_1}{dt} > 0$ and $\frac{dN_2}{dt} < 0$ $\Rightarrow \frac{dN_1}{dN_2} < 0$ then $N_1(t)$ is a decreasing function of $N_2(t)$ and the trajectories move down and right (The perturbations in both the commensal and host species decline each other with time t).

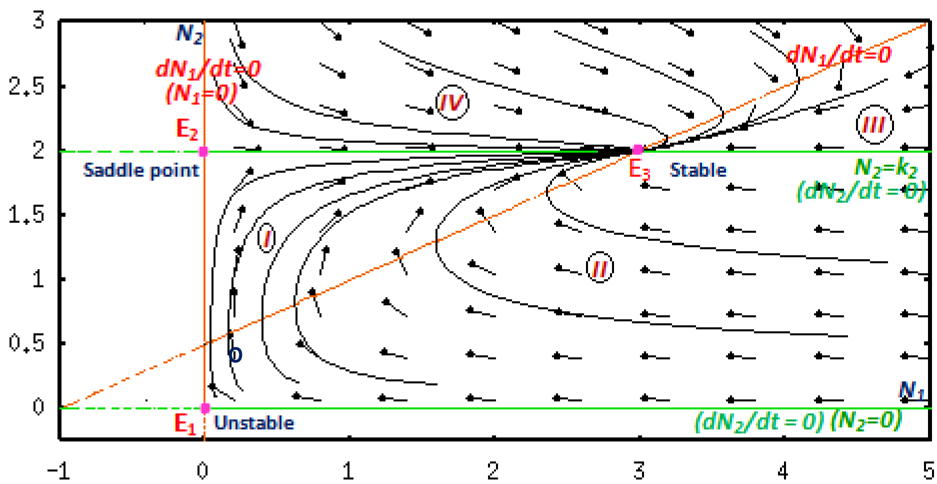

Figure 2: Threshold Diagram

Conclusions

In the above all four regions, all the solutions (or trajectories) which start in respective regions and they will finally approach the equilibrium point E_3 as shown in the Figure-2. It explicates that the state corresponding to the equilibrium point E_3 is stable whereas other two equilibrium points E_1, E_2 are unstable. Also it is concluded that for the maximum possible values of the mortal coefficient e_1 of the commensal species, the commensal species is extinct intially and then no interaction between the two species are occurred.

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