Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 10, Number 6 (2014), pp. 775-782

http://www.ripublication.com

Quantum Algorithm for Number Place Problem

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Abstract

A quantum algorithm for the number place problem and its example are reported. The rule of the number place problem is as follows. (i) A problem is given as an $n^2 \times n^2$ grid, which is divided into $n \times n$ squares with thick border lines. The value n is called order. (ii) Some cells are filled with an integer from 1 to n^2. (iii) The goal is to fill in all the blank cells so that each row, column and $n \times n$ square has each of integers from 1 through n^2 exactly once. A computational complexity of a classical computation is about $n^2!^{3n \times n}$. The computational complexity becomes about $8n^6$ by the quantum algorithm that uses quantum phase inversion gates and quantum inversion about mean gates. Therefore, a polynomial time process becomes possible.

AMS Subject Classification: Primary 81-08; Secondary 68R05, 68W40.

Keywords: Quantum algorithm, number place problem, computational complexity, polynomial time.

Introduction

The 3-SAT and the number place problems are known as the NP-complete problems which have been proposed by Cook [1-3]. When a computational complexity of a problem is large, a quantum computational method is applied for it [3-5]. The algorithms of the quantum computer by Deutsch-Jozsa [4-6], Shor [3-5, 7], Grover [4, 8, 9] and so on are known. A quantum algorithm for the 3-SAT problem by a numbering method has recently been reported by Fujimura [10]. Its computational complexity becomes a polynomial time. The number place problem is examined by the quantum computational method this time. Therefore, its result is reported.

Number Place Problem

The rule of the number place problem is as follows. (i) A problem is given as an $n^2 \times n^2$ grid, which is divided into $n \times n$ squares with thick border lines. The value n is called order. (ii) Some cells are filled with an integer from 1 to n^2. (iii) The goal is to fill in all the blank cells so that each row, column and $n \times n$ square has each of integers from 1 through n^2 exactly once [2].

Quantum Algorithm

Each element $f(i, j)$ [$1 \le i \le 3n^2$, $1 \le j \le n^2$. $1 \le f(i, j) \le n^2$. i, j, n and $f(i, j)$ are integers.] of an $n^2 \times n^2$ grid that is consisted by each row, column and $n \times n$ square is as follows. (I) Rows: They may be No. n^2 from No.1 from the top to the bottom [$1 \le i \le n^2$]. Elements in a row: They may be No. n^2 from No.1 from the left to the right [$1 \le j \le n^2$]. (II) Columns: They may be No.$2n^2$ from No.($n^2 + 1$) from the left to the right [$n^2 + 1 \le i \le 2n^2$]. Elements in a column: They may be No. n^2 from No.1 from the top to the bottom [$1 \le j \le n^2$].

(III) $n \times n$ squares: They may be No.$3n^2$ from No.($2n^2 + 1$) from the upper left → the upper right → the lower left to the lower right [$2n^2 + 1 \le i \le 3n^2$]. Elements in an $n \times n$ square: They may be No.n^2 from No.1 from the upper left → the upper right → the lower left to the lower right [$1 \le j \le n^2$].

(IV) The number of the already filled cells is M [$1 \le M \le n^4 - n^2$. M is an integer.]. Each value of them is $f_m(i_m, j_m) = f_m$((number of the row), (number of the order in the same row)) [$1 \le m \le M$. $1 \le i_m \le n^2 - 1$. $1 \le j_m \le n^2 - 1$. $1 \le f_m(i_m, j_m) \le n^2$. i_m, j_m, m and $f_m(i_m, j_m)$ are integers.]. For example, when there are $f(9, 5) = 1$ [$m = 18$], $f(9, 7) = 2$ [$m = 19$] and $f(9, 8) = 4$ [$m = 20$], they correspond to $f_{18}(9, 1) = 1$, $f_{19}(9, 2) = 2$ and $f_{20}(9, 3) = 4$, respectively.

First of all, quantum registers $|a(i, j)\rangle$ [$= |f(i, j) - 1\rangle$] and $|b\rangle$ are prepared. When P is the minimum integer that is $log_2\, n^2$ or more, each of $|a(i, j)\rangle$ is consisted of P quantum bits [=qubits]. States of $|a(i, j)\rangle$ and $|b\rangle$ are $a(i, j)$ and b, respectively.

Step 1: Each qubit of $|a(i, j)>$ and $|b>$ is set $|0>$.

Step 2: The Hadamard gate $\boxed{H}$ [4, 5] acts on each qubit of $|a(i, j)>$. It changes them for entangled states. The total states are $((2^P)^{n \times n})^{3n \times n}$.

Step 3: It is assumed that a quantum gate (A) changes $|b>$ for $|1>$ in $a(i, j) < n^2$, or it changes $|b>$ for $|0>$ in the others of $a(i, j)$. As a target state for $|b>$ is 1, quantum phase inversion gates (PI) and quantum inversion about mean gates (IM) [4, 8, 9] act on $|b>$. When Q is the minimum even integer that is $(2^P/n^2)^{1/2}$ or more, the total number that (PI) and (IM) act on $|b>$ is Q because they are a couple. Next, an observation gate (OB) observes $|b>$. There actions are repeated sequentially from $|a(1, 1)>$ to $|a(3n^2, n^2)>$. Therefore, each state of $|a(i, j)>$ is 0, 1, ⋯, $n^2 - 2$ or $n^2 - 1$, and the total states become $((n^2)^{n \times n})^{3n \times n}$.

Step 4: It is assumed that a quantum gate $(B(1, j),(1, g))$ [$1 \le j < g \le n^2$. g is an integer.] changes $|b>$ for $|1>$ in $a(1, j) \ne a(1, g)$, or it changes $|b>$ for $|0>$ at $a(1, j) = a(1, g)$. As the target state for $|b>$ is 1, (PI) and (IM) act on $|b>$. When R is the minimum even integer that is $((n^2 + 1 - j)/(n^2 - j))^{1/2}$ or more, the total number that (PI) and (IM) act on $|b>$ is R. Next, (OB) observes $|b>$. These actions are repeated sequentially from $|a(1, 1)>$ to $|a(1, n^2)>$. Therefore, the states from $|a(1, 1)>$ to $|a(1, n^2)>$ are each of integers from 0 through $n^2 - 1$ exactly once, and the total states become $n^2!((n^2)^{n\times n})^{3n\times n - 1}$. Moreover, these actions are repeated sequentially from $|a(2, 1)>$ to $|a(3n^2, n^2)>$, where $(B(i, j), (i, g))$ [$2 \le i \le 3n^2$. i is the integer.] is used. Similarly, the total states become $n^2!^{3n\times n}$ [$= W(0)$].

Step 5: It is assumed that a quantum gate $(C(i, j), (q_1 + n^2, h_1))$ [$1 \le i \le n^2$. $1 \le j \le n^2$. $1 \le q_1 \le n^2$. $1 \le h_1 \le n^2 - 1$. i, j, q_1 and h_1 are integers. $a(q_1 + n^2, h_1)$ in the column corresponds to $a(i, j)$ in the row.] changes $|b>$ for $|1>$ at $a(i, j) = a(q_1 + n^2, h_1)$, or it changes $|b>$ for $|0>$ in $a(i, j) \ne a(q_1 + n^2, h_1)$. As the target state for $|b>$ is 1, (PI) and (IM) act on $|b>$. When $T(v)$ [$1 \le v \le 2n^2(n^2 - 1)$. v is an integer.] is the minimum even integer that is $(W(v - 1)/W(v))^{1/2} = (W(v - 1)/(W(v - 1)/(n^2 + 1 - h_1)))^{1/2} = (n^2 + 1 - h_1)^{1/2}$ or more, the total number that (PI) and (IM) act on $|b>$ is $T(v)$. Next, (OB) observes $|b>$, and the total states become $W(v) = W(v - 1)/(n^2 + 1 - h_1)$.

$(C(i, j), (q_2 + 2n^2, h_2))$ [$1 \le q_2 \le n^2$. $1 \le h_2 \le n^2 - 1$. q_2 and h_2 are integers. $a(q_2 + 2n^2, h_2)$ in the $n \times n$ square corresponds to $a(i, j)$ in the row.] changes $|b>$ for $|1>$ at $a(i, j) = a(q_2 + 2n^2, h_2)$, or it changes $|b>$ for $|0>$ in $a(i, j) \ne a(q_2 + 2n^2, h_2)$. As the target state for $|b>$ is 1, (PI) and (IM) act on $|b>$. When $T(v + 1)$ is the minimum even integer that is $(W(v)/W(v + 1))^{1/2} = (W(v)/(W(v)/(n^2 + 1 - h_2)))^{1/2} = (n^2 + 1 - h_2)^{1/2}$ or more, the total number that (PI) and (IM) act on $|b>$ is $T(v + 1)$. Next, (OB) observes

$|b>$, and the total states become $W(v + 1) = W(v)/(n^2 + 1 – h_2)$.

These actions are repeated sequentially from $|a(1, 1)>$ to $|a(n^2, n^2)>$. And then, the total states become $n^2!^{n \times n}$ [$= W(2n^2(n^2 – 1))$].

Step 6: It is assumed that a quantum gate (D_m) changes $|b>$ for $|1>$ at $f_m(i_m, j_\mathrm{m}) = f(i, j) = a(i, j) + 1$, or it changes $|b>$ for $|0>$ in the others of $a(i, j)$. As the target state for $|b>$ is 1, (*PI*) and (*IM*) act on $|b>$. When $U(m)$ is the minimum even integer that is $(W(2n^2(n^2 − 1) + m − 1)/W(2n^2(n^2 – 1) + m))^{1/2} = (W(2n^2(n^2 – 1) + m – 1)/(W(2n^2(n^2 – 1) + m – 1)/(n^2 + 1 – j_m)))^{1/2} = (n^2 + 1 – j_m)^{1/2}$ or more, the total number that (*PI*) and (*IM*) act on $|b>$ is $U(m)$. Next, (*OB*) observes $|b>$, and the total states become $W(2n^2(n^2 − 1) + m) = W(2n^2(n^2 − 1) + m − 1)/(n^2 + 1 − j_m)$. These actions are repeated sequentially from $f_1(i_1, j_1)$ to $f_M(i_M, j_M)$. And then, the total states become $W(2n^2(n^2 − 1) + M)$. When m is M, (*OB*) observes $|a(i, j)>$ [$1 \le i \le 3n^2$. $1 \le j \le n^2$.] and $|b>$, and one of the data of $W(2n^2(n^2 − 1) + M)$ remains. After all, one of answers is $f(i, j) = a(i, j) + 1$ [$1 \le i \le n^2$. $1 \le j \le n^2$.].

Numerical Computation

It is assumed that there are $n = 3$ and $M = 20$ [$f_1(1, 1) = f(1, 2) = f(11, 1) = f(19, 2) = 6$, $f_2(1, 2) = f(1, 5) = f(14, 1) = f(20, 2) = 7$, $f_3(2, 1) = f(2, 4) = f(13, 2) = f(20, 4) = 1$, $f_4(2, 2) = f(2, 6) = f(15, 2) = f(20, 6) = 4$, $f_5(3, 1) = f(3, 3) = f(12, 3) = f(19, 9) = 2$, $f_6(3, 2) = f(3, 8) = f(17, 3) = f(21, 8) =1$, $f_7(4, 1) = f(4, 9) = f(18, 4) = f(24, 3) = 6$, $f_8(5, 1) = f(5, 1) = f(10, 5) = f(22, 4) = 9$, $f_9(5, 2) = f(5, 8) = f(17, 5) = f(24, 5) = 3$, $f_{10}(5, 3) = f(5, 9) = f(18, 5) = f(24, 6) = 7$, $f_{11}(6, 1) = f(6, 5) = f(14, 6) = f(23, 8) = 9$, $f_{12}(7, 1) = f(7, 2) = f(11, 7) = f(25, 2) =8$, $f_{13}(7, 2) = f(7, 7) = f(16, 7) = f(27, 1) = 3$, $f_{14}(7, 3) = f(7, 9) = f(18, 7) = f(27, 3) = 1$, $f_{15}(8, 1) = f(8, 4) = f(13, 8) = f(26, 4) = 8$, $f_{16}(8, 2) = f(8, 6) = f(15, 8) = f(26, 6) = 3$, $f_{17}(8, 3) = f(8, 9) = f(18, 8) = f(27, 6) = 9$, $f_{18}(9, 1) = f(9, 5) = f(14, 9) = f(26, 8) = 1$, $f_{19}(9, 2) = f(9, 7) = f(16, 9) = f(27, 7) = 2$, $f_{20}(9, 3) = f(9, 8) = f(17, 9) = f(27, 8) = 4$].

First of all, $|a(1, 1)>$, $|a(1, 2)>$, ···, $|a(27, 9)>$ and $|b>$ are prepared. When P is the minimum integer that is $log_2\ n^2 = 2log_2\ 3 \approx 3.2 \le 4 = P$, each of $|a(i, j)>$ is consisted of $P = 4$ qubits. States of $|a(i, j)>$ and $|b>$ are $a(i, j)$ and b, respectively.

Step 1: Each qubit of $|a(i, j)>$ and $|b>$ is set $|0>$.

Step 2: $\boxed{\mathrm{H}}$ acts on each qubit of $|a(i, j)>$. It changes them for entangled states. The total states are $((2^P)^{n \times n})^{3n \times n} = ((2^4)^9)^{27} = 16^{9 \times 27}$.

Step 3: (*A*) changes $|b>$ for $|1>$ in $a(i, j) < n^2 = 9$, or it change $|b>$ for $|0>$ in the others of $a(i, j)$. As a target state for $|b>$ is 1, (*PI*) and (*IM*) act on $|b>$. When Q is the minimum even integer that is $(2^P/n^2)^{1/2} = (16/9)^{1/2} \approx 1.3 \leq 2 = Q$, the total number that (*PI*) and (*IM*) act on |*b*> is $Q = 2$. Next, (*OB*) observes |*b*>. There actions are repeated sequentially from |*a*(1, 1)> to |*a*(27, 9)>. Therefore, each state of |*a*(*i*, *j*)> is 0, 1, ···, 7 or 8, and the total states become $((n^2)^{n \times n})^{3n \times n} = 9^{9 \times 27}$.

Step 4: (*B*(1, *j*),(1, *g*)) [$1 \leq j < g \leq 9$. *g* is the integer.] changes |*b*> for |1> in $a(1, j) \neq a(1, g)$, or it changes |b> for |0> at $a(1, j) = a(1, g)$. As the target state for |*b*> is 1, (*PI*) and (*IM*) act on |*b*>. When *R* is the minimum even integer that is $((n^2 + 1 - j)/(n^2 - j))^{1/2} = ((10 - j)/(9 - j))^{1/2} \leq 2 = R$, the total number that (*PI*) and (*IM*) act on |*b*> is $R = 2$. Next, (*OB*) observes |*b*>. These actions are repeated sequentially from |*a*(1, 1)> to |*a*(1, 9)>. Therefore, the states from |*a*(1, 1)> to |*a*(1, 9)> are each of integers from 0 through 8 exactly once, and the total states become $n^2!((n^2)^{n \times n})^{3n \times n - 1} = 9!(9^9)^{26}$. Moreover, these actions are repeated sequentially from |*a*(2, 1)> to |*a*(27, 9)>, where (*B*(*i*, *j*), (*i*, *g*)) [$2 \leq i \leq 27$. *i* is the integer.] is used. Similarly, the total states become $(9!)^{27}$ [= *W*(0)].

Step 5: ($C(i, j)$, $(q_1 + 9, h_1)$) [$1 \leq i \leq 9$. $1 \leq j \leq 9$. $1 \leq q_1 \leq 9$. $1 \leq h_1 \leq 8$. *i*, *j*, q_1 and h_1 are integers. $a(q_1 + 9, h_1)$ in the column corresponds to $a(i, j)$ in the row.] changes |*b*> for |1> at $a(i, j) = a(q_1 + 9, h_1)$, or it changes |*b*> for |0 > in $a(i, j) \neq a(q_1 + 9, h_1)$. As the target state for |*b*> is 1, (*PI*) and (*IM*) act on |*b*>. When $T(v)$ [$1 \leq v \leq 144$. *v* is the integer.] is the minimum even integer that is $(W(v - 1)/W(v))^{1/2} = (W(v - 1)/(W(v - 1)/(10 - h_1)))^{1/2} = (10 - h_1)^{1/2} \leq T(v)$, the total number that (*PI*) and (*IM*) act on |*b*> is $T(v)$. Next, (*OB*) observes |*b*>, and the total states become $W(v) = W(v - 1)/(10 - h_1)$.

($C(i, j)$, $(q_2 + 18, h_2)$) [$1 \leq q_2 \leq 9$. $1 \leq h_2 \leq 8$. q_2 and h_2 are integers. $a(q_2 + 18, h_2)$ in the 3 × 3 square corresponds to $a(i, j)$ in the row.] changes |*b*> for |1> at $a(i, j) = a(q_2 + 18, h_2)$, or it changes |*b*> for |0 > in $a(i, j) \neq a(q_2 + 18, h_2)$. As the target state for |*b*> is 1, (*PI*) and (*IM*) act on |*b*>. When $T(v + 1)$ is the minimum even integer that is $(W(v)/W(v + 1))^{1/2} = (W(v)/(W(v)/(10 - h_2)))^{1/2} = (10 - h_2)^{1/2} \leq T(v + 1)$, the total number that (*PI*) and (*IM*) act on |*b*> is $T(v + 1)$. Next, (*OB*) observes |*b*>, and the total states become $W(v + 1) = W(v)/(10 - h_2)$.

These actions are repeated sequentially from |*a*(1, 1)> to |*a*(9, 9)>. And then, the total states become $9!^9$ [= *W*(144)].

Step 6: (D_m) changes |*b*> for |1> at $f_m(i_m, j_{\mathrm{m}}) = f(i, j) = a(i, j) + 1$, or it changes |*b*> for |0> in the others of $a(i, j)$. As the target state for |*b*> is 1, (*PI*) and (*IM*) act on |*b*>. When $U(m)$ is the minimum even integer that is $(W(144 + m - 1)/W(144 + m))^{1/2} = (W(144 + m - 1)/(W(144 + m - 1)/(10 - j_m)))^{1/2} = (10 - j_m)^{1/2} \leq U(m)$, the total number that (*PI*) and (*IM*) act on |*b*> is *U*(*m*). Next, (*OB*) observes |*b*>, and the total states become *W*(144 + *m*) = *W*(144 + *m* − 1)/(10 − j_m). These actions are repeated sequentially from f_1(1, 1) to f_{20}(9, 3). And then, the total states become *W*(164) = $8!^2 7!^3 6!^4$. When *m* is *M* = 20, (*OB*) observes |*a*(*i*, *j*)> [1 ≤ *i* ≤ 27. 1 ≤ *j* ≤ 9.] and |*b*>, and one of the data of *W*(164) remains. For example, there are *a*(1, 1) = *a*(10, 1) = *a*(19, 1) = 0, *a*(1, 2) = *a*(11, 1) = *a*(19, 2) = 5, *a*(1, 3) = *a*(12, 1) = *a*(19, 3) = 3, *a*(1, 4) = *a*(13, 1) = *a*(20, 1) = 4, *a*(1, 5) = *a*(14, 1) = *a*(20, 2) = 6, *a*(1, 6) = *a*(15, 1) = *a*(20, 3) = 1, *a*(1, 7) = *a*(16, 1) = *a*(21, 1) = 7, *a*(1, 8) = *a*(17, 1) = *a*(21, 2) = 8, *a*(1, 9) = *a*(18, 1) = *a*(21, 3) = 2, *a*(2, 1) = *a*(10, 2) = *a*(19, 4) = 7, *a*(2, 2) = *a*(11, 2) = *a*(19, 5) = 8, *a*(2, 3) = *a*(12, 2) = *a*(19, 6) = 6, *a*(2, 4) = *a*(13, 2) = *a*(20, 4) = 0, *a*(2, 5) = *a*(14, 2) = *a*(20, 5) = 2, *a*(2, 6) = *a*(15, 2) = *a*(20, 6) = 3, *a*(2, 7) = *a*(16, 2) = *a*(21, 4) = 5, *a*(2, 8) = *a*(17, 2) = *a*(21, 5) = 1, *a*(2, 9) = *a*(18, 2) = *a*(21, 6) = 4, *a*(3, 1) = *a*(10, 3) = *a*(19, 7) = 2, *a*(3, 2) = *a*(11, 2) = *a*(19, 8) = 4, *a*(3, 3) = *a*(12, 3) = *a*(19, 9) = 1, *a*(3, 4) = *a*(13, 3) = *a*(20, 7) = 5, *a*(3, 5) = *a*(14, 3) = *a*(20, 8) = 7, *a*(3, 6) = *a*(15, 3) = *a*(20, 9) = 8, *a*(3, 7) = *a*(16, 3) = *a*(21, 7) = 6, *a*(3, 8) = *a*(17, 3) = *a*(21, 8) = 0, *a*(3, 9) = *a*(18, 3) = *a*(21, 9) = 3, *a*(4, 1) = *a*(10, 4) = *a*(22, 1) = 6, *a*(4, 2) = *a*(11, 4) = *a*(22, 2) = 1, *a*(4, 3) = *a*(12, 4) = *a*(22, 3) = 7, *a*(4, 4) = *a*(13, 4) = *a*(23, 1) = 2, *a*(4, 5) = *a*(14, 4) = *a*(23, 2) = 3, *a*(4, 6) = *a*(15, 4) = *a*(23, 3) = 0, *a*(4, 7) = *a*(16, 4) = *a*(24, 1) = 8, *a*(4, 8) = *a*(17, 4) = *a*(24, 2) = 4, *a*(4, 9) = *a*(18, 4) = *a*(24, 3) = 5, *a*(5, 1) = *a*(10, 5) = *a*(22, 4) = 8, *a*(5, 2) = *a*(11, 5) = *a*(22, 5) = 3, *a*(5, 3) = *a*(12, 5) = *a*(22, 6) = 4, *a*(5, 4) = *a*(13, 5) = *a*(23, 4) = 1, *a*(5, 5) = *a*(14, 5) = *a*(23, 5) = 5, *a*(5, 6) = *a*(15, 5) = *a*(23, 6) = 7, *a*(5, 7) = *a*(16, 5) = *a*(24, 4) = 0, *a*(5, 8) = *a*(17, 5) = *a*(24, 5) = 2, *a*(5, 9) = *a*(18, 5) = *a*(24, 6) = 6, *a*(6, 1) = *a*(10, 6) = *a*(22, 7) = 5, *a*(6, 2) = *a*(11, 6) = *a*(22, 8) = 2, *a*(6, 3) = *a*(12, 6) = *a*(22, 9) = 0, *a*(6, 4) = *a*(13, 6) = *a*(23, 7) = 6, *a*(6, 5) = *a*(14, 6) = *a*(23, 8) = 8, *a*(6, 6) = *a*(15, 6) = *a*(23, 9) = 4, *a*(6, 7) = *a*(16, 6) = *a*(24, 7) = 3, *a*(6, 8) = *a*(17, 6) = *a*(24, 8) = 7, *a*(6, 9) = *a*(18, 6) = *a*(24, 9) = 1, *a*(7, 1) = *a*(10, 7) = *a*(25, 1) = 1, *a*(7, 2) = *a*(11, 7) = *a*(25, 2) = 7, *a*(7, 3) = *a*(12, 7) = *a*(25, 3) = 8, *a*(7, 4) = *a*(13, 7) = *a*(26, 1) = 3, *a*(7, 5) = *a*(14, 7) = *a*(26, 2) = 4, *a*(7, 6) = *a*(15, 7) = *a*(26, 3) = 6, *a*(7, 7) = *a*(16, 7) = *a*(27, 1) = 2, *a*(7, 8) = *a*(17, 7) = *a*(27, 2) = 5, *a*(7, 9) = *a*(18, 7) = *a*(27, 3) = 0, *a*(8, 1) = *a*(10, 8) = *a*(25, 4) = 3, *a*(8, 2) = *a*(11, 8) = *a*(25, 5) = 0, *a*(8, 3) = *a*(12, 8) = *a*(25, 6) = 5, *a*(8, 4) = *a*(13, 8) = *a*(26, 4) = 7, *a*(8, 5) = *a*(14, 8) = *a*(26, 5) = 1, *a*(8, 6) = *a*(15, 8) = *a*(26, 6) = 2, *a*(8, 7) = *a*(16, 8) = *a*(27, 4) = 4, *a*(8, 8) = *a*(17, 8) = *a*(27, 5) = 6, *a*(8, 9) = *a*(18, 8) = *a*(27, 6) = 8, *a*(9, 1) = *a*(10, 9) = *a*(25, 7) = 4, *a*(9, 2) = *a*(11, 9) = *a*(25, 8) = 6, *a*(9, 3) = *a*(12, 9) = *a*(25, 9) = 2, *a*(9, 4) = *a*(13, 9) =

$a(26, 7) = 8$, $a(9, 5) = a(14, 9) = a(26, 8) = 0$, $a(9, 6) = a(15, 9) = a(26, 9) = 5$, $a(9, 7) = a(16, 9) = a(27, 7) = 1$, $a(9, 8) = a(17, 9) = a(27, 8) = 3$, $a(9, 9) = a(18, 9) = a(27, 9) = 7$ and $b = 1$.

Therefore, it is obtained that one of answers is $f(i, j) = a(i, j) + 1$ [$1 \le i \le 9$. $1 \le j \le 9$.].

Discussion and Summary

The computational complexity of this quantum algorithm [= S] becomes the following. In the order of the actions by the gates, the number of them is $3Pn^4$ at $\boxed{H}$, $3n^4$ at (A), $6n^4$ (PI) and (IM), $3n^4$ at (OB), $3n^4(n^2 - 1)$ at ($B(i, j)$, (i, g)) [$1 \le i \le 3n^2$. $1 \le j < g \le n^2$. i, j and g are integers.], $3n^4(n^2 - 1)$ at (PI) and (IM), $(3/2)n^4(n^2 - 1)$ at (OB), $4n^2(n^2 - 1)$ at ($C(i, j)$, $(q_1 + n^2, h_1)$) [$1 \le i \le n^2$. $1 \le j \le n^2$. $1 \le q_1 \le n^2$. $1 \le h_1 \le n^2 - 1$. i, j, q_1 and h_1 are integers.] and ($C(i, j)$, $(q_2 + 2n^2, h_2)$) [$1 \le q_2 \le n^2$. $1 \le h_2 \le n^2 - 1$. q_2 and h_2 are integers.], $\Sigma_{v = 1 \to 2n \times n(n \times n - 1)}\, T(v)$ at (PI) and (IM), $2n^2(n^2 - 1)$ at (OB), M at (D_m) [$1 \le m \le M$. m is the integer.], $\Sigma_{m = 1 \to M}\, U(m)$ at (PI) and (IM), and M at (OB). Therefore, S becomes $7.5n^6 + (3P + 10.5)n^4 - 6n^2 + 2M + \Sigma_{v = 1 \to 2n \times n(n \times n - 1)}\, T(v) + \Sigma_{m = 1 \to M}\, U(m)$. In the example of the section 4, S is 7824. The computational complexity of the classical computation [= Z] is $W_0 = n^2!^{3n \times n} = 9!^{27} \approx 10^{150}$. After all, S/Z becomes about $1/10^{146}$. When n is large enough, S becomes about $8n^6$, where P is about $log_2\, n^2$. And then, S/Z is about $8n^6/\, n^2!^{3n \times n}$. For example, as for $n = 10$, S/Z is about $8 \times 10^6/100!^{300} \approx 1/10^{17393}$, where $N!$ is about $N^N e^{-N}(2\pi N)^{1/2}$ [Stirling's formura].

Therefore, the polynomial time process becomes possible.

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