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Solving Intuitionistic Fuzzy Linear Programming Problems by using Linear Programming

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Abstract

A new method namely, *separation and bound method* is proposed to find an optimal intuitionistic fuzzy (IF) solution to fully intuitionistic fuzzy linear programming (IFLP) problems, in which ranking functions are not used. The proposed method is based on the crisp linear programming (LP) technique. In the optimal solution of the fully IFLP problem obtained by the proposed method, the values of the decision variables do not contain any negative part. Separation and bound method is an appropriate method to apply for finding an optimal solution of IFLP problems occurring in real life situations.

Key words: Triangular intuitionistic fuzzy number, linear programming problems, intuitionistic fuzzy linear programming problems, separation and bound method.

1. Introduction

In operations research, linear programming (LP) is a one of the most important and applicable technique. In our daily life moments, we frequently deal with vague or imprecise information. Sometimes information available is inexact or insufficient. Vagueness is usually expressed by linguistic terms intervals, fuzzy numbers or intuitionistic fuzzy (IF) numbers. In 1965, Zadeh [8] proposed the fuzzy set theory for handling the vague data. After the introduction of the concept of fuzzy sets, Atanassov [1] introduced the concept of IF sets which is found to be highly useful to deal with vagueness. The major advantage of IF set over fuzzy set is that IF set separates the degree of membership and the degree of non-membership of an element in the set. In practice, it is realized that human expressions like perception, knowledge and behavior are represented by IF sets rather than fuzzy sets. IF set is applied in many fields such as medical diagnosis, decision making and logic programming etc...

IF optimization was introduced by Nehi [6].Dipti Dubey [2] solved the LP with Triangular IF numbers in which triangular IF numbers are converted into crisp set and solved. Parvathi and Malathi [7] solved IFLP problems by non-linear programming technique. Nachammai and Thangaraj [3] proposed a new method of ranking of generalized IF number and solved IFLP problem using the ranking function. Nagoorgani and Ponnalagu [4, 5] obtained an optimal solution to IFLP problem without converting it into crisp LP problem.

In the existing methods [2, 3, 4, 5, 6, 7] the optimal solution of some of the IF decision variables to the IFLP problem have negative part which depicts that quantity of the product may be negative. But the negative quantity of the product has no physical meaning. Therefore, the solution obtained in [2, 3, 4, 5, 6, 7] for IFLP problems are not realistic and not applicable.

In this paper, we develop a new method namely, separable and bound method for obtaining an IF optimal solution to a fully IFLP problem where all parameters are IF triangular numbers. The proposed method is based on algorithm of the crisp LP problem and provides non-negative optimal IF solution when compared with the existing methods [2, 3, 4, 5, 6, 7] for solving fully IFLP problem. Ranking functions are not used in the proposed method. With the help of numerical example, the proposed method of solving the fully IFLP problem is explained. The separable and bound method is an appropriate method for finding an applicable optimal solution to IFLP models for the real life problems.

2. Preliminaries

We need the following mathematical orientated definitions of IF set, triangular IF number and membership function and non-membership function of an IF set/number which can be found [2, 3, 4, 5, 6, 7].

Definition 2.1: Let X denote a universe of discourse and $A \subseteq X$. Then, an IF set of A in X, $\tilde{A}^I$ is defined as follows:

$\tilde{A}^I = \left\{ \left(x, \mu_{\tilde{A}^I}(x), \vartheta_{\tilde{A}^I}(x) \right); x \in X \right\}$

Where $\left(\mu_{\tilde{A}^I}(x), \vartheta_{\tilde{A}^I}(x) \right): X \to [0,1]$ are functions such that $0 \le \mu_{\tilde{A}^I}(x) + \vartheta_{\tilde{A}^I}(x) \le 1$, for all $x \in X$. For each x in X , $\mu_{\tilde{A}^I}(x)$ and $\vartheta_{\tilde{A}^I}(x)$ represent the degree of membership and non-membership values of x in the set $A \subseteq X$.

Definition 2.2: A fuzzy number $\tilde{a}^I$ is a triangular IF number denoted by $(a_2, a_3, a_4)(a_1, a_3, a_5)$ where a_1, a_2, a_3, a_4 and a_5 are real numbers such that $a_1 \le a_2 \le a_3 \le a_4 \le a_5$ and its membership function $\mu_{\tilde{A}^I}(x)$ and non-membership function $\vartheta_{\tilde{A}^I}(x)$ are given below: $\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a_2}{a_3-a_2} & ; \quad a_2 \le x \le a_3 \\ \dfrac{a_4-x}{a_4-a_3} & ; \quad a_3 \le x \le a_4 \\ 0 & ; \quad otherwise \end{cases}$ and

$$\vartheta_{\tilde{A}}(x) = \begin{cases} \dfrac{a_3-x}{a_3-a_1} & ; \quad a_1 \le x \le a_3 \\ \dfrac{x-a_5}{a_5-a_3} & ; \quad a_3 \le x \le a_5 \\ 0 & ; \quad otherwise \end{cases}.$$

Let $IF(R)$ be a set of all triangular IF numbers over R, a set of real numbers. Based on ordering relation in interval theory/fuzzy set theory, we define the following:

Definition 2.3: Let $\tilde{a}^I = (a_2, a_3, a_4)(a_1, a_3, a_5)$ and $\tilde{b}^I = (b_2, b_3, b_4)(b_1, b_3, b_5)$ be in $IF(R)$. Then,

(a) $\tilde{a}^I$ and $\tilde{b}^I$ are said to be equal if $a_i = b_i$, $i = 1,2,3,4,5$;

(b) $\tilde{a}^I$ is said to be less than or equal $\tilde{b}^I$ if $a_i \le b_i$, $i = 1,2,3,4,5$;

(c) $\tilde{a}^I$ is said to be greater than or equal $\tilde{b}^I$ if $a_i \ge b_i$, $i = 1,2,3,4,5$;

(d) $\tilde{a}^I$ is said to be equal $\tilde{b}^I$ if $a_i = b_i$, $i = 1,2,3,4,5$.

Definition 2.4: Let $\tilde{a}^I = (a_2, a_3, a_4)(a_1, a_3, a_5)$ be in $IF(R)$. Then, $\tilde{a}^I$ is said to be positive $\left(\tilde{a}^I \succeq \tilde{0}^I\right)$ if $a_i \ge 0$.

Consider the following fully IFLP problem

(IFLP) Max $\tilde{Z}^I = \sum_{j=1}^{n} \tilde{c}_j^{\,I} \otimes \tilde{x}_j^I$ subject to

$$\sum_{j=1}^{n} \tilde{a}_{ij}^{\,I} \otimes \tilde{x}_j^I \left(\preceq, \approx, \succeq\right) \tilde{b}_i^I, \tag{1}$$

$$\tilde{x}_j^I \succeq \tilde{0}, \text{ for } i = 1,2,...,m,\ j = 1,2,...,n, \tag{2}$$

where $\tilde{a}_{ij}^I$, $\tilde{c}_j^{\,I}$, $\tilde{b}_i^I$, $\tilde{x}_j^I$ are $(m \times n)$, $(1 \times n)$, $(m \times 1)$, $(n \times 1)$ triangular IF number.

A set of triangular IF numbers $\tilde{X}^I = \left\{ \left(x_j^{\,2}, x_j^{\,3}, x_j^{\,4}\right)\left(x_j^{\,1}, x_j^{\,3}, x_j^{\,5}\right), i = 1,2,...,m \text{ and } j = 1,2,...n \right\}$ is said to be a feasible IF solution to the problem (IFLP) if $\tilde{X}^I = \left\{ \left(x_j^{\,2}, x_j^{\,3}, x_j^{\,4}\right)\left(x_j^{\,1}, x_j^{\,3}, x_j^{\,5}\right), i = 1,2,...,m \text{ and } j = 1,2,...n \right\}$ satisfies the condition (1) and (2).

A feasible IF solution $\tilde{X}^I = \left\{ \left(x_j^{\,2}, x_j^{\,3}, x_j^{\,4}\right)\left(x_j^{\,1}, x_j^{\,3}, x_j^{\,5}\right), i = 1,2,...,m \text{ and } \mathrm{j} = 1,2,...\mathrm{n} \right\}$ of the problem (IFLP) is said to be an optimal IF solution to the problem (IFLP) if $Z\left(\tilde{X}^I\right) \le Z\left(\tilde{U}^I\right)$ for all feasible $\left(\tilde{U}^I\right)$ of the problem (IFLP).

Here, the parameters $\tilde{a}_{ij}^{\,I}, \tilde{c}_j^{\,I}, \tilde{x}_j^{\,I}$ and $\tilde{b}_i^I$ be the IF triangular numbers $\left(a_{ij}^{\,2}, a_{ij}^{\,3}, a_{ij}^{\,4}\right)\left(a_{ij}^{\,1}, a_{ij}^{\,3}, a_{ij}^{\,5}\right)$, $\left(c_j^{\,2}, c_j^{\,3}, c_j^{\,4}\right)\left(c_j^{\,1}, c_j^{\,3}, c_j^{\,5}\right)$, $\left(x_j^{\,2}, x_j^{\,3}, x_j^{\,4}\right)\left(x_j^{\,1}, x_j^{\,3}, x_j^{\,5}\right)$ and $\left(b_i^{\,2}, b_i^{\,3}, b_i^{\,4}\right)\left(b_i^{\,1}, b_i^{\,3}, b_i^{\,5}\right)$ respectively. Then, the problem (FILP) can be written as follows:

(IFLP) Maximize $\left(z_2, z_3, z_4\right)\left(z_1, z_3, z_5\right)$

$$\approx \sum_{j=1}^{n} \left(c_j^{\,2}, c_j^{\,3}, c_j^{\,4}\right)\left(c_j^{\,1}, c_j^{\,3}, c_j^{\,5}\right) \otimes \left(x_j^{\,2}, x_j^{\,3}, x_j^{\,4}\right)\left(x_j^{\,1}, x_j^{\,3}, x_j^{\,5}\right)$$

subject to

$$\sum_{j=1}^{n} \left(a_{ij}^{\,2}, a_{ij}^{\,3}, a_{ij}^{\,4}\right)\left(a_{ij}^{\,1}, a_{ij}^{\,3}, a_{ij}^{\,5}\right) \otimes \left(x_j^{\,2}, x_j^{\,3}, x_j^{\,4}\right)\left(x_j^{\,1}, x_j^{\,3}, x_j^{\,5}\right) \{\preceq, \approx, \succeq\} \left(b_i^{\,2}, b_i^{\,3}, b_i^{\,4}\right)\left(b_i^{\,1}, b_i^{\,3}, b_i^{\,5}\right),$$

$\left(x_j^{\,2}, x_j^{\,3}, x_j^{\,4}\right)\left(x_j^{\,1}, x_j^{\,3}, x_j^{\,5}\right) \succeq \tilde{0}$. for $i = 1,2,...,m$ and $j = 1,2,...,n$.

Now, since $\left(x_j^{\,2}, x_j^{\,3}, x_j^{\,4}\right)\left(x_j^{\,1}, x_j^{\,3}, x_j^{\,5}\right)$ is a triangular fuzzy intuitionistic number, then

$$x_j^{\,1} \le x_j^{\,2} \le x_j^{\,3} \le x_j^{\,4} \le x_j^{\,5} \qquad (3)$$

The relation (3) is called triangular IF number constraints.

Now, using the arithmetic operations and partial ordering relations, decompose the given IFLP problem as follows:

Maximize $z_1 = \sum_{j=1}^{n}$ first value of $\left(c_j^{\,2}, c_j^{\,3}, c_j^{\,4}\right)\left(c_j^{\,1}, c_j^{\,3}, c_j^{\,5}\right) \otimes \left(x_j^{\,2}, x_j^{\,3}, x_j^{\,4}\right)\left(x_j^{\,1}, x_j^{\,3}, x_j^{\,5}\right)$

Maximize $z_2 = \sum_{j=1}^{n}$ second value of $\left(c_j^{\,2}, c_j^{\,3}, c_j^{\,4}\right)\left(c_j^{\,1}, c_j^{\,3}, c_j^{\,5}\right) \otimes \left(x_j^{\,2}, x_j^{\,3}, x_j^{\,4}\right)\left(x_j^{\,1}, x_j^{\,3}, x_j^{\,5}\right)$

Maximize $z_3 = \sum_{j=1}^{n}$ third value of $\left(c_j^{\,2}, c_j^{\,3}, c_j^{\,4}\right)\left(c_j^{\,1}, c_j^{\,3}, c_j^{\,5}\right) \otimes \left(x_j^{\,2}, x_j^{\,3}, x_j^{\,4}\right)\left(x_j^{\,1}, x_j^{\,3}, x_j^{\,5}\right)$

Maximize $z_4 = \sum_{j=1}^{n}$ fourth value of $\left(c_j^{\,2}, c_j^{\,3}, c_j^{\,4}\right)\left(c_j^{\,1}, c_j^{\,3}, c_j^{\,5}\right) \otimes \left(x_j^{\,2}, x_j^{\,3}, x_j^{\,4}\right)\left(x_j^{\,1}, x_j^{\,3}, x_j^{\,5}\right)$

Maximize $z_5 = \sum_{j=1}^{n}$ fifth value of $\left(c_j^{\,2}, c_j^{\,3}, c_j^{\,4}\right)\left(c_j^{\,1}, c_j^{\,3}, c_j^{\,5}\right) \otimes \left(x_j^{\,2}, x_j^{\,3}, x_j^{\,4}\right)\left(x_j^{\,1}, x_j^{\,3}, x_j^{\,5}\right)$

subject to $\sum_{j=1}^{n}$ first value of $\left(a_{ij}^{\,2}, a_{ij}^{\,3}, a_{ij}^{\,4}\right)\left(a_{ij}^{\,1}, a_{ij}^{\,3}, a_{ij}^{\,5}\right) \otimes \left(x_j^{\,2}, x_j^{\,3}, x_j^{\,4}\right)\left(x_j^{\,1}, x_j^{\,3}, x_j^{\,5}\right) \{\preceq \approx, \succeq\} \left(b_i^{\,2}, b_i^{\,3}, b_i^{\,4}\right)\left(b_i^{\,1}, b_i^{\,3}, b_i^{\,5}\right)$;

$\sum_{j=1}^{n}$ second value of $\left(a_{ij}^{\,2}, a_{ij}^{\,3}, a_{ij}^{\,4}\right)\left(a_{ij}^{\,1}, a_{ij}^{\,3}, a_{ij}^{\,5}\right) \otimes \left(x_j^{\,2}, x_j^{\,3}, x_j^{\,4}\right)\left(x_j^{\,1}, x_j^{\,3}, x_j^{\,5}\right) \{\preceq, \approx, \succeq\} \left(b_i^{\,2}, b_i^{\,3}, b_i^{\,4}\right)\left(b_i^{\,1}, b_i^{\,3}, b_i^{\,5}\right)$;

$\sum_{j=1}^{n}$ third value of $\left(a_{ij}^{\,2}, a_{ij}^{\,3}, a_{ij}^{\,4}\right)\left(a_{ij}^{\,1}, a_{ij}^{\,3}, a_{ij}^{\,5}\right) \otimes \left(x_j^{\,2}, x_j^{\,3}, x_j^{\,4}\right)\left(x_j^{\,1}, x_j^{\,3}, x_j^{\,5}\right) \{\preceq, \approx, \succeq\} \left(b_i^{\,2}, b_i^{\,3}, b_i^{\,4}\right)\left(b_i^{\,1}, b_i^{\,3}, b_i^{\,5}\right)$;

$\sum_{j=1}^{n}$ fifth value of $\left(a_{ij}^{\,2}, a_{ij}^{\,3}, a_{ij}^{\,4}\right)\left(a_{ij}^{\,1}, a_{ij}^{\,3}, a_{ij}^{\,5}\right) \otimes \left(x_j^{\,2}, x_j^{\,3}, x_j^{\,4}\right)\left(x_j^{\,1}, x_j^{\,3}, x_j^{\,5}\right) \{\preceq, \approx, \succeq\} \left(b_i^{\,2}, b_i^{\,3}, b_i^{\,4}\right)\left(b_i^{\,1}, b_i^{\,3}, b_i^{\,5}\right)$;

$\sum_{j=1}^{n}$ fifth value of $\left(a_{ij}^{\,2}, a_{ij}^{\,3}, a_{ij}^{\,4}\right)\left(a_{ij}^{\,1}, a_{ij}^{\,3}, a_{ij}^{\,5}\right) \otimes \left(x_j^{\,2}, x_j^{\,3}, x_j^{\,4}\right)\left(x_j^{\,1}, x_j^{\,3}, x_j^{\,5}\right) \{\preceq, \approx, \succeq\} \left(b_i^{\,2}, b_i^{\,3}, b_i^{\,4}\right)\left(b_i^{\,1}, b_i^{\,3}, b_i^{\,5}\right)$;

for all $i = 1,2,...,m$, $j = 1,2,...,n$ and all decision variables are non-negative.
From the above decomposition problem, construct the following crisp LP problems namely, $(P3)$, $(P2)$, $(P4)$, $(P1)$ and $(P5)$ as follows:

$$(P3): \text{Maximize } z_3 = \sum_{j=1}^{n} \text{third value of } \left(c_j^{\,2}, c_j^{\,3}, c_j^{\,4}\right)\left(c_j^{\,1}, c_j^{\,3}, c_j^{\,5}\right) \otimes \left(x_j^{\,2}, x_j^{\,3}, x_j^{\,4}\right)\left(x_j^{\,1}, x_j^{\,3}, x_j^{\,5}\right)$$

subject to Constraints in the decomposition problem in which at least one decision variable of the $(P3)$ occurs and all decision variables are non-negative.

$$(P2): \text{Maximize } z_2 = \sum_{j=1}^{n} \text{second value of } \left(c_j^{\,2}, c_j^{\,3}, c_j^{\,4}\right)\left(c_j^{\,1}, c_j^{\,3}, c_j^{\,5}\right) \otimes \left(x_j^{\,2}, x_j^{\,3}, x_j^{\,4}\right)\left(x_j^{\,1}, x_j^{\,3}, x_j^{\,5}\right)$$

subject to $z_2 \le z_3^{\circ}$

Constraints in the decomposition problem in which at least one decision variable of the $(P2)$ occurs and are not used in $(P3)$; all variables in the constraints and objective function in $(P2)$ must satisfy the fuzzy triangular intuitionistic bounded constraints ; replacing all values of the decision variables which are obtained in $(P3)$ and all decision variables are non-negative; where z_3° is the optimal objective value of the problem $(P3)$.

$$(P4): \text{Maximize } z_4 = \sum_{j=1}^{n} \text{fouth value of } \left(c_j^{\,2}, c_j^{\,3}, c_j^{\,4}\right)\left(c_j^{\,1}, c_j^{\,3}, c_j^{\,5}\right) \otimes \left(x_j^{\,2}, x_j^{\,3}, x_j^{\,4}\right)\left(x_j^{\,1}, x_j^{\,3}, x_j^{\,5}\right)$$

subject to $z_4 \ge z_3^{\circ}$

Constraints in the decomposition problem in which at least one decision variable of the $(P4)$ occurs and are not used in $(P3)$ and $(P2)$; all variables in the constraints and objective function in $(P4)$ must satisfy the fuzzy triangular intuitionistic bounded constraints ; replacing all values of the decision variables which are obtained in $(P3)$ and all decision variables are non-negative; where z_3° is the optimal objective value of the problem $(P3)$.

$$(P1): \text{Maximize } z_1 = \sum_{j=1}^{n} \text{first value of } \left(c_j^{\,2}, c_j^{\,3}, c_j^{\,4}\right)\left(c_j^{\,1}, c_j^{\,3}, c_j^{\,5}\right) \otimes \left(x_j^{\,2}, x_j^{\,3}, x_j^{\,4}\right)\left(x_j^{\,1}, x_j^{\,3}, x_j^{\,5}\right)$$

subject to $z_1 \le z_2^{\circ}$

Constraints in the decomposition problem in which at least one decision variable of the $(P1)$ occurs and are not used in $(P2)$, $(P3)$ and $(P4)$; all variables in the constraints and objective function in $(P1)$ must satisfy the fuzzy triangular intuitionistic bounded constraints ; replacing all values of the decision variables which are obtained in $(P2)$, $(P3)$ and $(P4)$ and all decision variables are non-negative; where z_2° is the optimal objective value of the problem $(P2)$. and

$$(P5): \text{Maximize } z_5 = \sum_{j=1}^{n} \text{fifth value of } \left(c_j^{\,2}, c_j^{\,3}, c_j^{\,4}\right)\left(c_j^{\,1}, c_j^{\,3}, c_j^{\,5}\right) \otimes \left(x_j^{\,2}, x_j^{\,3}, x_j^{\,4}\right)\left(x_j^{\,1}, x_j^{\,3}, x_j^{\,5}\right)$$

subject to $z_5 \le z_4^{\circ}$

Constraints in the decomposition problem in which at least one decision variable of the $(P5)$ occurs and are not used in $(P1)$, $(P2)$, $(P3)$ and $(P4)$; all variables in the constraints and objective function in $(P5)$ must satisfy the fuzzy triangular intuitionistic bounded constraints ; replacing all values of the decision variables which are obtained in $(P1)$, $(P2)$, $(P3)$ and $(P4)$ and all decision variables are non-negative; where z_4° is the optimal objective value of the problem $(P4)$.

3. The Separation and Bound Method

We need the following theorem which is used in the proposed method namely, separation and bound method to solve the fully FILP problem.

THEOREM 3.1 Let $[x_j^{\circ}] = \{ x_j^{\circ k}, \text{j} = 1,2,..., \text{n and k} = 1,2,3,4,5 \}$ be an optimal solution of the problems $(P1)$, $(P2)$, $(P3)$, $(P4)$ and $(P5)$ respectively. Then $[\tilde{x}_j^{\circ I}] = \{ \tilde{x}_j^{\circ I} = (x_j^{\circ 2}, x_j^{\circ 3}, x_j^{\circ 4})(x_j^{\circ 1}, x_j^{\circ 3}, x_j^{\circ 5}), j = 1, 2, ..., n\}$ is an optimal IF solution to the given problem (IFLP).

PROOF: Since $[x_j^{\circ 1}], [x_j^{\circ 2}], [x_j^{\circ 3}], [x_j^{\circ 4}]$ and $[x_j^{\circ 5}]$ are feasible solution of $(P1)$, $(P2)$, $(P3)$, $(P4)$ and $(P5)$ respectively., $[\tilde{x}_j^{\circ I}] = \left\{ \tilde{x}_j^{\circ I} = \left(x_j^{\circ 2}, x_j^{\circ 3}, x_j^{\circ 4}\right)\left(x_j^{\circ 1}, x_j^{\circ 3}, x_j^{\circ 5}\right), j = 1, 2, ..., n \right\}$ is a feasible IF solution to the problem (IFLP).

Let $[\tilde{y}_j^{I}] = \left\{ \tilde{y}_j^{I} = \left(y_j^{2}, y_j^{3}, y_j^{4}\right)\left(y_j^{1}, y_j^{3}, y_j^{5}\right), j = 1, 2, ..., n \right\}$ be a feasible solution of (IFLP). Clearly, $[y_j^1], [y_j^2], [y_j^3], [y_j^4]$ and $[y_j^5]$ are feasible solution of $(P1)$, $(P2)$, $(P3)$, $(P4)$ and $(P5)$ respectively.

Now, since $[x_j^{\circ 1}], [x_j^{\circ 2}], [x_j^{\circ 3}], [x_j^{\circ 4}]$ and $[x_j^{\circ 5}]$ are optimal solution of $(P1)$, $(P2)$, $(P3)$, $(P4)$ and $(P5)$ respectively, we have

$Z_1([x_j^{\circ 1}]) \geq Z_1([y_j^1])$; $Z_2([x_j^{\circ 2}]) \geq Z_2([y_j^2])$; $Z_3([x_j^{\circ 3}]) \geq Z_3([y_j^3])$

$Z_4([x_j^{\circ 4}]) \geq Z_4([y_j^4])$ and $Z_5([x_j^{\circ 5}]) \geq Z_5([y_j^5])$

This implies that $Z([\tilde{x}_j^{\circ I}]) \geq Z([\tilde{y}_j^{\,I}])$, for all feasible solution of the problem (IFLP).

Therefore, $[\tilde{x}_j^{\circ I}] = \left\{ \tilde{x}_j^{\circ I} = \left(x_j^{\circ 2}, x_j^{\circ 3}, x_j^{\circ 4}\right)\left(x_j^{\circ 1}, x_j^{\circ 3}, x_j^{\circ 5}\right), j = 1, 2, ..., n \right\}$ is an optimal IF solution to the given problem (IFLP).

Hence the theorem.

Remark 3.1: In the optimal IF solution, the values of the decision variables $[\tilde{x}_j^{\circ I}] = \left\{ \tilde{x}_j^{\circ I} = \left(x_j^{\circ 2}, x_j^{\circ 3}, x_j^{\circ 4}\right)\left(x_j^{\circ 1}, x_j^{\circ 3}, x_j^{\circ 5}\right), j = 1, 2, ..., n \right\}$ are positive.

Now, we propose a new algorithm namely, Separation and Bound method for solving IFLP problem.

The proposed method proceeds as follows.

STEP 1: Construct $(P3)$, $(P2)$, $(P4)$, $(P1)$ and $(P5)$ problems from the given the fully IFLP problems.

STEP 2: Using existing linear programming technique, solve the problem $(P3)$, then the problems $(P2)$ and $(P4)$, then the problems $(P1)$ and $(P5)$ in the order only and obtain the values of all real decision variables $x_j^{\,2}, x_j^{\,3}, x_j^{\,4}$, $x_j^{\,1}$ and $x_j^{\,5}$ for $j = 1,2,...,n$ and the values of all objectives z_2, z_3, z_4, z_1 and z_5. Let the decision variables values be $x_j^{\,2\circ}, x_j^{\,3\circ}, x_j^{\,4\circ}$, $x_j^{\,1\circ}$ and $x_j^{\,5\circ}$ for $j = 1,2,...,n$ and the objective values be $z_2^{\,\circ}, z_3^{\,\circ}, z_4^{\,\circ}, z_1^{\,\circ}$ and $z_5^{\,\circ}$.

STEP 3: The optimal FI solution to the given IFLP problems is $\tilde{x}_j^{\circ I} = \left(x_j^{\,2\circ}, x_j^{\,3\circ}, x_j^{\,4\circ}\right)\left(x_j^{\,1\circ}, x_j^{\,3\circ}, x_j^{\,5\circ}\right)$, $j = 1,2,...,n$ and the maximum IF objective value is $\tilde{z}^I = \left(z_2^{\,\circ}, z_3^{\,\circ}, z_4^{\,\circ}\right)\left(z_1^{\,\circ}, z_3^{\,\circ}, z_5^{\,\circ}\right)$ (by the Theorem 3.1.).

Remark 3.2: Integer IFLP problem can be solved by using the separation and decomposition method after replacing the linear programming technique by integer linear programming technique in the solution procedures.
Now, the separation and bound method for solving IFLP problem is illustrated using the following numerical examples.

Example 3.1: Consider the following IFLP problem:
Maximize $\tilde{Z}^I \approx (1, 2, 3)(0.5, 2, 3.5) \otimes \tilde{x}_1^{\,I} \oplus (2, 3, 4)(1.5, 3, 4.5) \otimes \tilde{x}_2^{\,I}$
Subject to
$(0, 1, 2)(0, 1, 2.5) \otimes \tilde{x}_1^{\,I} \oplus (1, 2, 3)(0.5, 2, 3.5) \otimes \tilde{x}_2^{\,I} \preceq (1, 10, 27)(0.5, 10, 36)$
$(1, 2, 3)(0.5, 2, 3.5) \otimes \tilde{x}_1^{\,I} \oplus (0, 1, 2)(0, 1, 2.5) \otimes \tilde{x}_2^{\,I} \preceq (2, 11, 28)(1.5, 11, 38)$
$\tilde{x}_1^{\,\mathrm{I}}$, $\tilde{x}_2^{\,\mathrm{I}} \succeq \tilde{0}^I$.
Let $\tilde{x}_1^{\,I} = (x_2, x_3, x_4)(x_1, x_3, x_5)$, $\tilde{x}_2^{\,I} = (y_2, y_3, y_4)(y_1, y_3, y_5)$ and
$\tilde{Z}^I = (Z_2, Z_3, Z_4)(Z_1, Z_3, Z_5)$ be triangular fuzzy intuitionistic numbers.
Now, the problem $(P3)$ is given below:
$(P3)$: Maximize $z_3 = 2x_3 + 3y_3$
subject to
$x_3 + 2y_3 \le 10$; $2x_3 + y_3 \le 11$;
x_3, $y_3 \ge 0$.
Now, solving the problem $(P3)$ using simplex method, the optimal solution is $x_3 = 4$; $y_3 = 3$ and the maximum value of $z_3 = 17$.
Now, the problem $(P2)$ is given below:

$(P2)$: Maximize $z_2 = x_2 + 2y_2$
subject to
$0x_2 + y_2 \le 1$; $x_2 + 0y_2 \le 2$; $x_2 \le x_3$; $y_2 \le y_3$; $x_2 + 2y_2 \le z_3$;
x_2 , $y_2 \ge 0$.
Now, solving the problem $(P2)$ with $x_3 = 4$; $y_3 = 3$ and $z_3 = 17$ using simplex method, the optimal solution is $x_2 = 2$; $y_2 = 1$ and the maximum value of $z_2 = 4$.
Now, the problem $(P4)$ is given below:
$(P4)$: Maximize $z_4 = 3x_4 + 4y_4$
subject to
$2x_4 + 3y_4 \le 27$; $3x_4 + 2y_4 \le 28$; $x_4 \ge x_3$; $y_4 \ge y_3$; $3x_4 + 4y_4 \ge z_3$
$x_4, y_4 \ge 0$.
Now, solving the problem $(P4)$ with $x_3 = 4$; $y_3 = 3$ and $z_3 = 17$ using simplex method, the optimal solution is $x_4 = 6$; $y_4 = 5$ and the maximum value of $z_4 = 38$.
Now, the problem $(P1)$ is given below:
$(P1)$: Maximize $z_1 = 0.5x_1 + 1.5y_1$
subject to
$0x_1 + 0.5y_1 \le 0.5$; $0.5x_1 + 0y_1 \le 1.5$; $x_1 \le x_2$; $y_1 \le y_2$; $0.5x_1 + 1.5y_1 \le z_2$;
x_1 , $y_1 \ge 0$.
Now, solving the problem $(P1)$ with $x_2 = 2$; $y_2 = 1$ and $z_2 = 4$ using simplex method, the optimal solution is $x_1 = 2$; $y_1 = 1$ and the maximum value of $z_1 = 2.5$.
Now, the problem $(P5)$ is given below:
$(P5)$: Maximize $z_5 = 3.5x_5 + 4.5y_5$
subject to
$2.5x_5 + 3.5y_5 \le 36$; $3.5x_5 + 2.5y_5 \le 38$; $x_5 \ge x_4$; $y_5 \ge y_4$; $3.5x_5 + 4.5y_5 \ge z_4$
$x_5, y_5 \ge 0$.
Now, solving the problem $(P4)$ with $x_4 = 6$; $y_4 = 5$ and $z_4 = 38$, using simplex method, the optimal solution is $x_5 = 7.17$; $y_5 = 5.17$ and the maximum value of $z_5 = 48.33$.
Therefore, the optimal IF solution to the given IFLP problem is
$\tilde{x}_1^I = (x_2, x_3, x_4)(x_1, x_3, x_5)$ = $(2, 4, 6)(2, 4, 7.17)$;
$\tilde{x}_2^I = (y_2, y_3, y_4)(y_1, y_3, y_5)$ = $(1, 3, 5)(1, 3, 5.17)$;
and the maximum IF objective value is $\tilde{Z}^I = (Z_2, Z_3, Z_4)(Z_1, Z_3, Z_5) =$ $(4, 17, 38)(2.5, 17, 48.33)$.

Example 3.2: Consider the following IFLP problem:
Maximize $\tilde{Z}^I \approx (1, 2, 3)(-0.5, 2, 3.5) \otimes \tilde{x}_1^I \oplus (2, 3, 4)(1.5, 3, 4.5) \otimes \tilde{x}_2^I$
Subject to

$(0, 1, 2)(0, 1, 2.5) \otimes \tilde{x}_1^{\,I} \oplus (1, 2, 3)(-0.5, 2, 3.5) \otimes \tilde{x}_2^{\,I} \preceq (1, 10, 27)(0.5, 10, 36)$

$(1, 2, 3)(0.5, 2, 3.5) \otimes \tilde{x}_1^{\,I} \oplus (0, 1, 2)(0, 1, 2.5) \otimes \tilde{x}_2^{\,I} \preceq (2, 11, 28)(1.5, 11, 38)$

$\tilde{x}_1^{\,I}, \tilde{x}_2^{\,I} \succeq \tilde{0}^I$.

Let $\tilde{x}_1^{\,I} = (x_2, x_3, x_4)(x_1, x_3, x_5)$, $\tilde{x}_2^{\,I} = (y_2, y_3, y_4)(y_1, y_3, y_5)$ and $\tilde{Z}^I = (Z_2, Z_3, Z_4)(Z_1, Z_3, Z_5)$ be triangular fuzzy intuitionistic numbers.

Now, the problem $(P3)$ is given below:

$(P3)$: Maximize $z_3 = 2x_3 + 3y_3$

subject to

$x_3 + 2y_3 \le 10$; $2x_3 + y_3 \le 11$;

$x_3, y_3 \ge 0$.

Now, solving the problem $(P3)$ using simplex method, the optimal solution is $x_3 = 4$; $y_3 = 3$ and the maximum value of $z_3 = 17$.

Now, the problem $(P2)$ is given below:

$(P2)$: Maximize $z_2 = x_2 + 2y_2$

subject to

$0x_2 + y_2 \le 1$; $x_2 + 0y_2 \le 2$; $x_2 \le x_3$; $y_2 \le y_3$; $x_2 + 2y_2 \le z_3$;

$x_2, y_2 \ge 0$.

Now, solving the problem $(P2)$ with $x_3 = 4$; $y_3 = 3$ and $z_3 = 17$ using simplex method, the optimal solution is $x_2 = 2$; $y_2 = 1$ and the maximum value of $z_2 = 4$.

Now, the problem $(P4)$ is given below:

$(P4)$: Maximize $z_4 = 3x_4 + 4y_4$

subject to

$2x_4 + 3y_4 \le 27$; $3x_4 + 2y_4 \le 28$; $x_4 \ge x_3$; $y_4 \ge y_3$; $3x_4 + 4y_4 \ge z_3$

$x_4, y_4 \ge 0$.

Now, solving the problem $(P4)$ with $x_3 = 4$; $y_3 = 3$ and $z_3 = 17$ using simplex method, the optimal solution is $x_4 = 6$; $y_4 = 5$ and the maximum value of $z_4 = 38$.

Now, the problem $(P1)$ is given below:

$(P1)$: Maximize $z_1 = -0.5x_5 + 1.5y_1$

subject to

$0x_1 - 0.5y_5 \le 0.5$; $0.5x_1 + 0y_1 \le 1.5$; $x_1 \le x_2$; $y_1 \le y_2$; $0.5x_1 + 1.5y_1 \le z_2$;

$x_5 \ge x_4$; $y_5 \ge y_4$; $x_1, y_1, x_5, y_5 \ge 0$.

Now, solving the problem $(P1)$ with $x_2 = 2$; $y_2 = 1$; $x_4 = 6$; $y_4 = 5$ and $z_2 = 4$ using simplex method, the optimal solution is $x_1 = 0$; $y_1 = 1$; $x_5 = 6$; $y_5 = 5$ and the maximum value of $z_1 = -1.5$.

Now, the problem $(P5)$ is given below:

$(P5)$: Maximize $z_5 = 3.5x_5 + 4.5y_5$

subject to

$2.5x_5 + 3.5y_5 \le 36$; $3.5x_5 + 2.5y_5 \le 38$; $x_5 \ge x_4$; $y_5 \ge y_4$; $3.5x_5 + 4.5y_5 \ge z_4$

$x_5, y_5 \ge 0$.

Now, solving the problem $(P4)$ with $x_4 = 6$; $y_4 = 5$; $x_5 = 6$; $y_5 = 5$ and $z_4 = 38$, the maximum value of $z_5 = 43.5$.

Therefore, the optimal IF solution to the given IFLP problem is

$\tilde{x}_1^I = (x_2, x_3, x_4)(x_1, x_3, x_5) = (2, 4, 6)(0, 4, 6)$; $\tilde{x}_2^I = (y_2, y_3, y_4)(y_1, y_3, y_5) = (1, 3, 5)(1, 3, 5)$; and the maximum IF objective value is $\tilde{Z}^I = (Z_2, Z_3, Z_4)(Z_1, Z_3, Z_5) = (4, 17, 38)(-1.5, 17, 43.5)$.

4. Conclusion

The main advantage of the separable and bound method is that the decision values of an optimal IF solution to fully IFLP problems are non-negative IF numbers. Since the proposed method is based on the classical linear programming algorithm so it can be easy to compute and to apply. The proposed method provides a meaningful and an applicable solution to IFLP problems. The separable and bound method can serve an appropriate solving tool for decision makers while they are handling real life linear programming problems having IF parameters.

5. References

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