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Quantum Algorithm for Minimum Multiprocessor Scheduling Problem by Numbering Method

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Abstract

A quantum algorithm for the minimum multiprocessor scheduling problem by a numbering method and its example are reported. When n tasks are parted by m processors, and a sum of length of each task in the k-th processor [$0 \leq k \leq m - 1$. k is an integer.] is t_k, it is decided whether t_k is a finish time D or less or not. A computational complexity of a classical computation is m^n. The computational complexity becomes about $3(log_2\ m)n$ by the quantum algorithm that uses quantum phase inversion gates, quantum inversion about mean gates and the numbering method. Therefore, a polynomial time process becomes possible.

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***Keywords*:** Quantum algorithm, minimum multiprocessor scheduling problem, numbering method, computational complexity, polynomial time.

1. Introduction

The methods for the very first steps towards building a quantum computer were developed by Haroche and Wineland [1]. Deutsch-Jozsa's algorithm for the rapid solution [2−4], Shor's algorithm for the factorization [3−5], Grover's algorithms for the database search [3, 6, 7] and so on are known. A quantum algorithm for the traveling salesman problem by a numbering method has recently been reported by Fujimura [8]. Its computational complexity becomes a polynomial time. The minimum multiprocessor scheduling problem [9, 10] is examined by the numbering method this time. Therefore, its result is reported.

2. Minimum Multiprocessor Scheduling Problem

When n tasks are parted by m processors, and a sum of length of each task in the k-th processor [$0 \leq k \leq m - 1$. k is an integer.] is t_k, it is decided whether t_k is a finish time D or less or not.

3. Quantum Algorithm

It is assumed that n tasks are x_0, x_1, ⋯, x_{n-2} and x_{n-1} that are lengths, and when they are parted by m processors [$1 \leq m \leq n$. m is an integer.] and a sum of length of each task in the k-th processor [$0 \leq k \leq m - 1$. k is the integer.] is t_k, it is decided whether t_k is the finish time D or less or not. When the number of the n times repeated permutation of 0, 1, ⋯, $m - 2$ and $m - 1$ is m^n, $a_0 m^{n-1} + a_1 m^{n-2} + \cdots + a_{n-1} m^0 = \sum_{f=0\to n-1} a_f m^{n-1-f} = U$ is the numbering datum from 0 to $m^n - 1$ [The 0-th datum is 0, 0, ⋯, 0 and 0. The ($m^n - 1$)-th datum is ($m - 1$), ($m - 1$), ⋯, ($m - 1$) and ($m - 1$).]. This method is named the numbering method for this problem. g is the minimum integer that follows $m^n/m! \leq 4^g = 2^{2g}$, because a number of combinations of an answer is at least $m!$.

First of all, quantum registers $|a_0>$, $|a_1>$, ⋯, $|a_{n-1}>$, $|b_1>$, $|b_2>$, $|c_0>$, $|c_1>$, ⋯, $|c_{m-1}>$, $|d>$, $|e_1>$ and $|e_2>$ are prepared. When P is the minimum integer that is $log_2 m$ or more, each of $|a_f>$ that f is an integer from 0 to $n-1$ is consisted of P quantum bits [= qubits]. States of $|a_f>$, $|b_1>$, $|b_2>$, $|c_k>$, $|d>$, $|e_1>$ and $|e_2>$ are a_f, b_1, b_2, c_1, c_k, d, e_1 and e_2, respectively.

Step 1: Each qubit of $|a_f>$, $|b_1>$, $|b_2>$, $|c_k>$, $|d>$, $|e_1>$ and $|e_2>$ is set $|0>$.

Step 2: The Hadamard gate $\boxed{H}$ [3, 4] acts on each qubit of $|a_f>$. It changes them for entangled states. The total states are $(2^P)^n$.

Step 3: It is assumed that a quantum gate (A) changes $|b_1>$ for $|1>$ in $a_f < m$, or it changes $|b_1>$ for $|0>$ in the others of a_f, it changes $|b_2>$ for $|b_2 + a_f m^{n-1-f}>$ at $|a_f>$, and it changes $|c_k>$ for $|c_k + x_f>$ at $a_f = k$. As a target state for $|b_1>$ is 1, quantum phase inversion gates (PI) and quantum inversion about mean gates (IM) [3, 6, 7] act on $|b_1>$. When Q is the minimum even integer that is $(2^P/m)^{1/2}$ or more, the total number that (PI) and (IM) act on $|b_1>$ is Q because they are a couple. Next, an observation gate (OB) observes $|b_1>$. These actions are repeated sequentially from $|a_0>$ to $|a_{n-1}>$. Therefore, each state of $|a_f>$ is 0, 1, ⋯, $m - 2$ and $m - 1$, and the total states become m^n [$=W_0$].

Step 4: It is assumed that a quantum gate (B) changes $|d>$ for $|d + 1>$ in c_k ($=t_k$) $\leq D$, or it doesn't change $|d>$ in the others of c_k. These actions are repeated sequentially from $|c_0>$ to $|c_{m-1}>$.

Step 5: It is assumed that a quantum gate (C) changes $|e_1>$ for $|e_1 + 0>$ at $d = m$, or it changes $|e_1>$ for $|e_1 + 1 + b_2>$ in the others of d.
Step 6: It is assumed that a quantum gate (D_1) changes $|e_2>$ for $|1>$ in $0 \le e_1 \le (m^n/4) - m!$, or it changes $|e_2>$ for $|0>$ in the others of e_1. As the target state for $|e_2>$ is 1, (PI) and (IM) act on $|e_2>$. The number of the data that is included in $0 \le e_1 \le (m^n/4) - m!$ is $W_1 \approx m^n/4$. When R_1 is the minimum even integer that is $(W_0/W_1)^{1/2} \approx (m^n/(m^n/4))^{1/2}$ or more, the total number that (PI) and (IM) act on $|e_2>$ is $R_1 \approx 2$. Next, (OB) observes $|e_2>$, and the data of W_1 remain. Similarly, (D_j) [$2 \le j \le g - 1$. j is an integer.] changes $|e_2>$ for $|1>$ in $0 \le e_1 \le (m^n/4^j) - m!$, or it changes $|e_2>$ for $|0>$ in the others of e_1. As the target state for $|e_2>$ is 1, (PI) and (IM) act on $|e_2>$. The number of the data that is included in $0 \le e_1 \le (m^n/4^j) - m!$ is $W_j \approx m^n/4^j$. When R_j is the minimum even integer that is $(W_{j-1}/W_j)^{1/2} \approx ((m^n/4^{j-1})/(m^n/4^j))^{1/2}$ or more, the total number that (PI) and (IM) act on $|e_2>$ is $R_j \approx 2$. Next, (OB) observes $|e_2>$, and the data of W_j remain. These actions are repeated sequentially from 2 to $g - 1$ at j. (D_g) changes $|e_2>$ for $|1>$ at $e_1 = 0$, or it changes $|e_2>$ for $|0>$ in the others of e_1. As the target state for $|e_2>$ is 1, (PI) and (IM) act on $|e_2>$. The number of the data that is included at $e_1 = 0$ is $W_g \approx m! \approx m^n/4^g$. When R_g is the minimum even integer that is $(W_{g-1}/W_g)^{1/2} \approx ((m^n/4^{g-1})/(m^n/4^g))^{1/2}$ or more, the total number that (PI) and (IM) act on $|e_2>$ is $R_g \approx 2$. Next, (OB) observes $|a_f>$, $|b_1>$, $|b_2>$, $|c_k>$, $|d>$, $|e_1>$ and $|e_2>$, and one of the data of W_g remains. Therefore, one example of combinations that are $c_k \le D$ is obtained.

4. Numerical Computation

It is assumed that there are $n = 6$, $x_0 = 5$, $x_1 = 3$, $x_2 = 8$, $x_3 = 7$, $x_4 = 6$, $x_5 = 1$, $m = 3$, $D = 11$, $g = 4$, $0 \le f \le 5$ [f is the integer.] and $0 \le k \le 2$ [k is the integer.].

First of all, $|a_f>$, $|b_1>$, $|b_2>$, $|c_k>$, $|d>$, $|e_1>$ and $|e_2>$ are prepared. When P is the minimum integer that is $log_2 3 \approx 1.6 \le 2 = P$, each of $|a_f>$ that f is the integer from 0 to 5 is consisted of 2 qubits. States of $|a_f>$, $|b_1>$, $|b_2>$, $|c_k>$, $|d>$, $|e_1>$ and $|e_2>$ are a_f, b_1, b_2, c_k, d, e_1, and e_2, respectively.
Step 1: Each qubit of $|a_f>$, $|b_1>$, $|b_2>$, $|c_k>$, $|d>$, $|e_1>$ and $|e_2>$ is set $|0>$.
Step 2: $\boxed{H}$ acts on each qubit of $|a_f>$. It changes them for entangled states. The total states are $(2^2)^6$.
Step 3: (A) changes $|b_1>$ for $|1>$ in $a_f < 3$, or it changes $|b_1>$ for $|0>$ in the others of a_f, it changes $|b_2>$ for $|b_2 + a_f 3^{5-f}>$ at $|a_f>$, and it changes $|c_k>$ for $|c_k + x_f>$ at $a_f = k$. As the target state for $|b_1>$ is 1, (PI) and (IM) act on $|b_1>$. When Q is the minimum even integer that is $(2^2/3)^{1/2} \approx 1.2 \le 2 = Q$, the total number that (PI) and (IM) act on $|b_1>$ is $Q \approx 2$. Next, (OB) observes $|b_1>$. These actions are repeated sequentially from $|a_0>$ to $|a_5>$. Therefore, each state of $|a_f>$ is 0, 1 or 2, and the total states become 3^6 [$= W_0$].
Step 4: (B) changes $|d>$ for $|d + 1>$ in c_k ($=t_k$) ≤ 11, or it doesn't change $|d>$ in the others of c_k. These actions are repeated sequentially from $|c_0>$ to $|c_2>$.

Step 5: (C) changes $|e_1>$ for $|e_1 + 0>$ at $d = 5$, or it changes $|e_1>$ for $|e_1 + 1 + b_2>$ in the others of d.
Step 6: (D_1) changes $|e_2>$ for $|1>$ in $0 \le e_1 \le (3^6/4) - 3!$, or it changes $|e_2>$ for $|0>$ in the others of e_1. As the target state for $|e_2>$ is 1, (PI) and (IM) act on $|e_2>$. The number of the data that is included in $0 \le e_1 \le (3^6/4) - 3!$ is $W_1 \approx 3^6/4$. When R_1 is the minimum even integer that is $(W_0/W_1)^{1/2} \approx (3^6/(3^6/4))^{1/2} \approx 2 \le 2 = R_1$, the total number that (PI) and (IM) act on $|e_2>$ is $R_1 \approx 2$. Next, (OB) observes $|e_2>$, and the data of W_1 remain. Similarly, (D_j) [$2 \le j \le 3$. j is the integer.] changes $|e_2>$ for $|1>$ in $0 \le e_1 \le (3^6/4^j) - 3!$, or it changes $|e_2>$ for $|0>$ in the others of e_1. As the target state for $|e_2>$ is 1, (PI) and (IM) act on $|e_2>$. The number of the data that is included in $0 \le e_1 \le (3^6/4^j) - 3!$ is $W_j \approx 3^6/4^j$. When R_j is the minimum even integer that is $(W_{j-1}/W_j)^{1/2} \approx ((3^6/4^{j-1})/(3^6/4^j))^{1/2} \approx 2 \le 2 = R_j$, the total number that (PI) and (IM) act on $|e_2>$ is $R_j \approx 2$. Next, (OB) observes $|e_2>$, and the data of W_j remain. These actions are repeated sequentially from 2 to 3 at j. (D_4) changes $|e_2>$ for $|1>$ at $e_1 = 0$, or it changes $|e_2>$ for $|0>$ in the others of e_1. As the target state for $|e_2>$ is 1, (PI) and (IM) act on $|e_2>$. The number of the data that is included at $e_1 = 0$ is $W_4 \approx 3! \approx 3^6/4^4$. When R_4 is the minimum even integer that is $(W_3/W_4)^{1/2} \approx ((3^6/4^3)/(3^6/4^4))^{1/2} \approx 2 \le 2 = R_4$, the total number that (PI) and (IM) act on $|e_2>$ is $R_4 \approx 2$. Next, (OB) observes $|a_f>$, $|b_1>$, $|b_2>$, $|c_k>$, $|d>$, $|e_1>$ and $|e_2>$, and one of the data of W_4 remains. For example, when a_0, a_1, a_2, a_3, a_4, a_5, b_1, b_2, c_0, c_1, c_2, d, e_1 and e_2 are 0, 1, 2, 1, 0, 2, 1, 146, 11, 10, 9, 3, 0 and 1, respectively, it is obtained that 3 combinations are (5, 6), (3, 7) and (8, 1).

5. Discussion and Summary

The computational complexity of this quantum algorithm [= S] becomes the following. In the order of the actions by the gates, the number of them is Pn at $\boxed{H}$, n at (A), $Qn \approx 2n$ at (PI) and (IM), n at (OB), m at (B), 2 at (C), g at (D_j) [$1 \le j \le g$. j is the integer.], $\sum_{j=1\to g} R_j \approx 2g$ at (PI) and (IM), and g at (OB). Therefore, S becomes $(P + 4)n + m + 2 + 4g$. In the example of the section 4, S is 57. The computational complexity of the classical computation [= Z] is $m^n = 3^6 = 729$. After all, S/Z becomes about 1/13. When n is large enough, S becomes about $3(log_2\ m)n$, where P is about $log_2\ m$, g is about $(1/2)log_2\ (m^n/m!) \approx (n/2)log_2\ m$, and $m!$ is about $m^m e^{-m}(2m)^{1/2}$ [Stirling's formula]. And then, S/Z is about $3(log_2\ m)n/m^n \approx n/m^n$. For example, as for $n = 100$ and $m = 5$, S/Z is about $100/5^{100} \approx 1/10^{68}$.

Therefore, the polynomial time process becomes possible.

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