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Some Conditions for a Generalized Variational Inequalities Involving the Different Nonlinear Operators Related to Neural Network in Hilbert Spaces

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Abstract

This paper establishes necessary and sufficient conditions for the existence of solutions to the general inverse mixed variation equation. Furthermore, the Wiener–Hopf equation is investigated, and it is shown that its solution is equivalent to that of a general variational inequality involving different operators.
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1. INTRODUCTION

A neural network is the problem related to time and is a powerful tool which is used to apply in the signal processing, pattern recognition, associative memory and other engineering or scientific field, see.[5–9] By the characteristic of nature of parallelization and distributed information process, the neural networks have served as the promising computational models for real time applications. So, the neural networks have been designed to solve the mathematics programming and the related optimization problems, see[10–12] and the reference therein.

From the foregoing, it will be interesting to study and develop the neural network further and can be also seen from the continuous development of research in artificial neural networks such as the following research: In 1996, A. Nagurney[13] studied the projected dynamical system and variational inequalities and also presented some applications of these problems in economics and transportation.

In 2002, Xia et al.[14] introduced a single-layer neural network that is well suited for parallel implementation. They established the equivalence between the neural network model and variational inequality formulations for solving nonlinear problems and analyzed the stability of the network. In 2015, Zou et al.[15] proposed a simple one-layer neural network for addressing inverse variational inequality problems, proved its stability, and provided numerical examples. In the same year, M. A. Noor et al.[16] developed dynamical systems for extended general quasi-variational inequalities and demonstrated the global exponential convergence of the system.

In 2021, Vuong et al.[17] investigated a projected neural network for solving inverse variational inequalities, established its stability, and illustrated applications in transportation science. Subsequently, in 2022, D. Hu et al.[18] applied a modified projection neural network to optimization problems, particularly for solving nonsmoothed, nonlinear, and constrained convex optimization models. Numerous other studies have also explored the application of neural networks to optimization problems.[20–22]

In 2023, J. Tangkhawiwetkul[19] studied and analyzed the generalized inverse mixed variational inequality, establishing the existence and uniqueness of its solution. A neural network associated with this problem was formulated, and the Wiener–Hopf equation was considered, whose solution is equivalent to that of the generalized inverse mixed variational inequality. These developments highlight the importance and potential of neural networks as powerful tools for further research and applications.

Motivated by these works, this paper establishes necessary and sufficient conditions for the existence of solutions to the general inverse mixed variation equation. In addition, the Wiener–Hopf equation is examined, whose solution is shown to be equivalent to that of a general variational inequality involving different operators.

2. PRELIMINARIES

In this section, we let H be a real Hilbert space whose inner product and norm are denoted by $\langle \cdot\,, \cdot \rangle$ and $\|\cdot\|$ respectively. Let 2^H be denoted for the class of all nonempty subset of H and K be a nonempty closed and convex subset of H, and we will introduce the concept of the generalized f -projector operator and some knows properties.

[23] Let H be a real Hilbert space and K be a nonempty closed and convex subset of H: We say that $P_K^{f,\rho} : H \to 2^K$ is a generalized f -projection operator if

$$P_K^{f,\rho}(x) = \Big\{u \in K \Big| G(x,u) = \inf_{\xi \in K} G(x,\xi)\Big\}, \forall x \in H.$$

where $G : H \times K \to R \cup \{+\infty\}$ is a functional defined as follows:

$$G(x,\xi) = \|x\|^2 - 2\langle x, \xi\rangle + \|\xi\|^2 + 2\rho f(\xi),$$

with $x \in H, \xi \in K$, ρ is a positive number and $f : K \to R \cup \{+\infty\}$ is a proper, convex and lower semicontinuous function for the set of real numbers denoted by R.

Later, in 2014, Li et al.[24] presented the properties of the operator $P_K^{f,\rho}$ in Hilbert spaces as follows:[24] Let H be a real Hilbert space and K be a nonempty closed and convex subset of H: Then, the following statements hold:

1. $P_K^{f,\rho}(x)$ is nonempty and $P_K^{f,\rho}$ is a single valued mapping:
2. for all $x \in H$, $x^* = P_K^{f,\rho}(x)$ if and only if
$$\langle x - x^*, y - x^*\rangle + \rho f(y) - \rho f(x^*) \geq 0, \forall y \in K :$$
3. $P_K^{f,\rho}$ is continuous.

[3] Let H be a real Hilbert space and K be a nonempty closed and convex subset of H. Let $f : K \to R \cup \{+\infty\}$ be a proper, convex and lower semicontinuous function. Then, the following statements hold:

$$\|(v - P_K^{f\rho}(v)) - (u - P_K^{f\rho}(u))\|^2 \leq \|v - u\|^2 - \|P_K^{f,\rho}(v) - P_K^{f,\rho}(u)\|^2,$$

and

$$\|(v - P_K^{f\rho}(v)) - (u - P_K^{f\rho}(u))\| \leq \|v - u\|,$$

for all $u, v \in H$. [1] Let H be a real Hilbert space and $g, A : H \to H$ be two single valued mappings.

1. A is said to be a $\lambda-$ strongly monotone on H if there exists a constant $\lambda > 0$ such that
$$\langle Ax - Ay, x - y\rangle \geq \lambda\|x - y\|^2, \ \forall x, y \in H.$$
2. A is said to be a $\gamma-$Lipschitz continuous on H if there exists a constant $\gamma > 0$ such that
$$\|Ax - Ay\| \leq \gamma\|x - y\|, \ \forall x, y \in H.$$

3. (A, g) is said to be a $mu-$strongly monotone couple on H if there exists a positive constant $\mu > 0$ such that

$$\langle Ax - Ay, gx - gy\rangle \geq \mu\|x - y\|^2, \ \forall x, y \in H.$$

In 2023, J. Tangkhawiwetkulwe,[19] propose the generalized inverse mixed variational inequality. Let $g; A : H \to H$ be two continuous mappings and $f : K \to R \cup \{+\infty\}$ be a proper, convex and lower semicontinuous function. The generalized inverse mixed variational inequality is: to find an $x^* \in H$ such that $A(x^*) \in K$ and

$$\langle g(x^*), y - A(x^*)\rangle + \rho f(y) - \rho f(A(x^*)) \geq 0, \ \text{for all } y \in K. \tag{2.1}$$

[4] (Gronwall) Let $\hat{u}$ and $\hat{v}$ be real valued nonnegative continuous functions with domain $\{t|t \geq t_0\}$ and let $\alpha(t) = \alpha_0(|t - t_0|)$, where α_0 is a monotone increasing function. If for all $t \geq t_0$,

$$\hat{u}(t) \leq \alpha(t) + \int_{t_0}^{t} \hat{u}(s)\hat{u}(s)ds,$$

then,

$$\hat{u}(t) \leq \alpha(t) \exp^{\int_{t_0}^{t} \hat{u}(s)ds}$$

Now, we consider some examples: Let $f, g : R \to R$ defined $f(x) = x$ and $g(x) = -2x$. Put $F(x) = f(x) + g(x)$. We must show that a mapping F is a Lipschitz continuous but is not strongly monotone. From example we see that

$$\|f(x) - f(y)\| = |x - y|, \tag{2.2}$$

and

$$\langle f(x) - f(y), x - y\rangle = (f(x) - f(y))(x - y) = (x - y)^2 \geq 1 \cdot |x - y|^2. \tag{2.3}$$

From (2.2) and (2.3), we see that f is 1-Lipschitz and 1-strongly monotone. Consider,

$$\|g(x) - g(y)\| = | - 2x + 2y| = 2|x - y|, \tag{2.4}$$

but

$$\langle g(x) - g(y), x - y\rangle = (-2x + 2y)(x - y) = (-2)(x - y)^2 \leq 0. \tag{2.5}$$

Fron (2.4) and (2.5), we have g is a 2-Lipschitz but is not strongly monotone.

Next, we consider, $F(x) = f(x) + g(x) = x - 2x = -x$ we see that

$$\|F(x) - F(y)\| = | - x + y| = |x - y|, \tag{2.6}$$

But

$$\langle F(x) - F(y), x - y\rangle = (-x + y)(x - y) = (-1)(x - y)^2 \leq 0. \tag{2.7}$$

From (2.6) and (2.7), we see that F is a 1-Lipschitz but is not strongly monotone.

From the example above, it can be seen that there are still functions that Lipschitz continuous but is not strongly monotone. Therefore, this research is interested in studying functions that have such conditions.

3. MAIN RESULTS

In this section, we will propose the generalized inverse mixed variational inequality (2.1) under some necessary and sufficient conditions of mappings to the unique solution of the generalized inversed mixed variational inequality (2.1).[19] Let H be a real Hilbert space and K be a nonempty closed and convex subset of H. Let $f : K \to R \cup \{+\infty\}$ be a proper, convex and lower semicontinuous function. Then x^* is a solution of the generalized inverse mixed variational inequality (2.1) if and only if x^* satisfies

$$A(x^*) = P_K^{f,\rho}[A(x^*) - g(x^*)]. \tag{3.1}$$

The next theorem, we consider the necessary and sufficient conditions for proved the existence and uniqueness of the generalized inverse mixed variational inequality (2.1) as follows. Let H be a real Hilbert space and K be a nonempty closed convex subset of H, $A : H \to H$ be Lipschitz continuous on H with constant α. Let $f : K \to R\cup\{+\infty\}$ be a proper convex and lowersemicontinuous function, $g_1, g_2 : H \to H$ with $g = g_1 + g_2$. Assume that

1. g_1 are both β_1-Lipschitz continuous and $\lambda-$strongly monotone and g_2 is a β_2-Lipschitz continuous.
2. Assume that, $\alpha \in (0, \frac{1}{2}]$ and $0 < \beta_1, \beta_2 \leq \frac{\alpha}{4}$ which $1 - \alpha - \beta_1 - 2\beta_2 > 0$ and

$$\sqrt{1 - 2\lambda + \beta_1^2} < 1 - \alpha - \beta_1 - 2\beta_2, \tag{3.2}$$

where $\frac{1+\beta_1^2-\delta^2}{2} < \lambda < \frac{1+\beta_1^2}{2}$ and $1 - \alpha - \beta_1 - 2\beta_2 = \delta$.

Then, the generalized inverses mixed variational inequality has a unique solution in H. Let $F : H \to H$ be defined as follows: for any $x \in H$,

$$F(x) = x - Ax + P_K^{f,\rho}(Ax - g(x)). \tag{3.3}$$

For any $x, y \in H$. Put $x' = Ax - g(x)$ and $y' = Ay - g(y)$, we have

$$\begin{aligned}
\|F(x) - F(y)\| &= \Big\|(x - Ax + P_K^{f,\rho}(Ax - g(x))) - (y - Ay + P_K^{f,\rho}(Ay - g(y)))\Big\| \\
&= \Big\|(x - Ax + g(x) - g(x) + P_K^{f,\rho}(Ax - g(x))) \\
&\quad -(y - Ay + g(y) - g(y) + P_K^{f,\rho}(Ay - g(y)))\Big\| \\
&= \Big\|(x - g(x) - (Ax - g(x)) + P_K^{f,\rho}(Ax - g(x))) \\
&\quad -(y - g(y) - (Ay - g(y)) + P_K^{f,\rho}(Ay - g(y)))\Big\| \\
&= \Big\|(x - g(x) - y + g(y)) - (x' - P_K^{f,\rho}(x') - y' + P_K^{f,\rho}(y'))\Big\| \\
&\le \Big\|(x - y) - (g(x) + g(y))\Big\| + \Big\|x' - P_K^{f,\rho}(x') - (y' - P_K^{f,\rho}(y'))\Big\| \quad (3.4)
\end{aligned}$$

Since, $g = g_1 + g_2$ we obtain

$$\begin{aligned}
\Big\|(x - y) - (g(x) + g(y))\Big\| &= \Big\|(x - y) - ((g_1 + g_2)(x) + (g_1 + g_2)(y))\Big\| \\
&\le \Big\|(x - y) - (g_1(x) - g_1(y))\Big\| + \Big\|g_2(x) - g_2(y)\Big\| \quad (3.5)
\end{aligned}$$

From g_1 are both β_1-Lipschitz continuous and $\lambda-$strongly monotone. By Definition 2, we have

$$\begin{aligned}
\Big\|(x - y) - (g(x) + g(y))\Big\|^2 &= \Big\|x - y\Big\|^2 - 2\langle g_1(x) - g_1(y), x - y\rangle + \Big\|g_1(x) - g_1(y)\Big\|^2 \\
&\le \Big\|x - y\Big\|^2 - 2\lambda\Big\|x - y\Big\|^2 + \beta_1^2\Big\|x - y\Big\|^2 \\
&= (1 - 2\lambda + \beta_1^2)\Big\|x - y\Big\|^2. \quad (3.6)
\end{aligned}$$

Since, g_2 is a β_2-Lipschitz continuous, we have

$$\Big\|g_2(x) - g_2(y)\Big\| \le \beta_2\Big\|x - y\Big\|. \quad (3.7)$$

From (3.5), (3.6) and (3.7), we have

$$\begin{aligned}
\Big\|(x - y) - (g(x) + g(y))\Big\| &\le \Big(\sqrt{1 - 2\lambda + \beta_1^2}\Big)\Big\|x - y\Big\| + \beta_2\Big\|x - y\Big\| \\
&= \Big(\sqrt{1 - 2\lambda + \beta_1^2} + \beta_2\Big)\Big\|x - y\Big\|. \quad (3.8)
\end{aligned}$$

Since, A, g_1, g_2 are Lipschitz continuous with constants α, β_1, β_2 and by Theroem 2, we obtain that

$$
\begin{aligned}
\left\|x' - P_K^{f,\rho}(x') - (y' - P_K^{f,\rho}(y'))\right\| &\leq \left\|x' - y'\right\| \\
&= \left\|Ax - g(x) - Ay + g(y)\right\| \\
&\leq \left\|Ax - Ay\right\| + \left\|g(x) - g(y)\right\| \\
&\leq \left\|Ax - Ay\right\| + \left\|g_1(x) + g_2(x) - g_1(y) - g_2(y)\right\| \\
&\leq \left\|Ax - Ay\right\| + \left\|g_1(x) - g_1(y)\right\| + \left\|g_2(x) - g_2(y)\right\| \\
&\leq \alpha\left\|x - y\right\| + \beta_1\left\|x - y\right\| + \beta_2\left\|x - y\right\| \\
&= \left(\alpha + \beta_1 + \beta_2\right)\left\|x - y\right\|. \qquad (3.9)
\end{aligned}
$$

Replacing (3.8) and (3.9) in (3.4), it follows that

$$
\begin{aligned}
\|F(x) - F(y)\| &\leq \left(\sqrt{1 - 2\lambda + \beta_1^2} + \beta_2\right)\left\|x - y\right\| + \left(\alpha + \beta_1 + \beta_2\right)\left\|x - y\right\| \\
&= \left(\sqrt{1 - 2\lambda + \beta_1^2} + \alpha + \beta_1 + 2\beta_2\right)\left\|x - y\right\| \\
&= \mu\left\|x - y\right\|, \qquad (3.10)
\end{aligned}
$$

where $\mu = \left(\sqrt{1 - 2\lambda + \beta_1^2} + \alpha + \beta_1 + 2\beta_2\right)$. From condition 2, it implies that $0 < \mu < 1$. This implies that F is a contraction mapping in H. So, F has a unique fixed point in H.

Next, we can show some conditions for support the Theorem 3 exists. Assume that $\alpha = 0.5, \beta_1 = 0.125$ and $\beta_2 = 0.125$ we have $\alpha + \beta_1 + 2\beta_2 = 0.875 < 1$. It follows that $\delta = 0.125$,

$$0.500 < \lambda < 0.507.$$

Put $\lambda = \frac{0.500+0.507}{2} = 0.5035$, it implies that

$$\sqrt{1 - 2\lambda + \beta_1^2} < 1 - \alpha - \beta_1 - 2\beta_2.$$

4. NEURAL NETWORK

In this section we present the Wiener-Hopf inequality, which is represented by an equivalent solution to the general inverse variation inequality (2.1). We then present a neural network relating to the general inverse variation inequality.

In 2023, J. Tangkhawiwetkulwe,[19] consider $g; A : H \to H$ be two continuous mappings and K be a nonempty closed and convex subset of H. Let $f : K \to R \cup \{+\infty\}$ be a proper, convex and lower semicontinuous function. The Wiener Hopf Equation which is equivalent to the generalized inverse mixed variational inequality (2.1) as follows: find $x^* \in H$ such that

$$Q_K(A(x^*) - g(x^*)) + g(x^*) = 0, \tag{4.1}$$

where $Q_K = I - P_K^{f\rho}$ with I is an identity operator.

Then, J. Tangkhawiwetkulwe,[19] in Lemma 4.1, present the equivalent solution of the Wiener Hopf Equation (4.1) and the generalized inverse mixed variational inequality (2.1) problem.[19] x^* is a solution of the generalized inverse mixed variational inequality (2.1) if and only if x^* is a solution of the Wiener Hopf Equation (4.1). Let $A : H \to K$ be a Lipschitz continuous with constants α. Let $f : K \to R \cup \{+\infty\}$ be a proper, convex and lower semicontinuous function. Assume that all of assumption of Theorem 3 holds. Then, for each $x_0 \in H$, there exists the unique continuous solution $x(t)$ of the neueal network associated with the generalized inverse mixed variational inequality (4.4) with $x(t_0) = x_0$ over the interval $[t_0, \infty)$. Let η be a positive constant and define the mapping $F : H \to K$ by

$$F(x) = \eta\Big\{P_K^{f,\rho}[A(x) - g(x)] - A(x)\Big\}, \tag{4.2}$$

for all $x \in H$. By using Theorem 2 and (4.2), we obtain

$$\begin{aligned}
\|F(x) - F(y)\| &= \Big\|\eta\Big\{P_K^{f,\rho}[A(x) - g(x)] - A(x)\Big\} - \eta\Big\{P_K^{f,\rho}[A(y) - g(y)] - A(y)\Big\}\Big\| \\
&= \eta\Big\|\Big\{P_K^{f,\rho}[A(x) - g(x)] - A(x)\Big\} - \Big\{P_K^{f,\rho}[A(y) - g(y)] - A(y)\Big\}\Big\| \\
&= \eta\Big\|\Big[A(y) - g(y) - P_K^{f,\rho}[A(y) - g(y)]\Big] - \Big[A(x) - g(x) - P_K^{f,\rho}[A(x) - g(x)]\Big] \\
&\quad + g(y) - g(x)\Big\| \\
&\leq \eta\Big\{\Big\|\Big[A(y) - g(y) - P_K^{f,\rho}[A(y) - g(y)]\Big] - \Big[A(x) - g(x) - P_K^{f,\rho}[A(x) - g(x)]\Big]\Big\| \\
&\quad + \Big\|g(y) - g(x)\Big\| \\
&\leq \eta\Big\{\Big\|(A(y) - g(y)) - (A(x) - g(x))\Big\| + \Big\|g(y) - g(x)\Big\|\Big\}.
\end{aligned}$$

Since $g = g_1 + g_2$, we obtain

$$\begin{aligned}
\Big\|g(x) - g(y)\Big\| &= \Big\|(g_1 + g_2)(x) - (g_1 + g_2)(y)\Big\| \\
&= \Big\|(g_1(x) - g_1(y)) + (g_2(x) - g_2(y))\Big\| \\
&\leq \Big\|g_1(x) - g_1(y)\Big\| + \Big\|g_2(x) - g_2(y)\Big\| \\
&\leq \beta_1\|x - y\| + \beta_2\|x - y\| \\
&= (\beta_1 + \beta_2)\|x - y\|. \qquad (4.3)
\end{aligned}$$

From (3.9) and (4.3) we have

$$\begin{aligned}\Big\|F(x)-F(y)\Big\| &\le \eta\Big\{(\alpha+\beta_1+\beta_2)\|x-y\|+(\beta_1+\beta_2)\|X-y\|\Big\}\\ &\le \eta(\alpha+2\beta_1+2\beta_2)\|x-y\|.\end{aligned}$$

By the assumption (2) of Theorem 3, we have $\eta(\alpha+2\beta_1+2\beta_2)>0$. Then, F is a Lipschitz continuous. This implies that, for each $x_0\in H$, there exists a unique continuous solution $x(t)$ of (**??**), defined in initial $t_0\le t<\xi$ with the initial condition $x(t_0)=x_0$.

Let $[t_0,\xi)$ be its maximal initial of existance, we will show that $\xi=\infty$. Under the assumption, we obtain that (2.1) has a unique solution(say x^*) such that $A(x^*)\in K$ and

$$A(x^*)=P_K^{f,\rho}[A(x^*)-g(x^*)]. \tag{4.4}$$

Let $x\in H$, from (4.2) and (4.4) we have

$$\begin{aligned}\|F(x)\| &= \Big\|\eta\Big\{P_K^{f,\rho}\Big[A(x)-g(x)\Big]-A(x)\Big\}\Big\|\\ &= \Big\|\eta\Big\{P_K^{f,\rho}\Big[A(x)-g(x)\Big]-A(x^*)+A(x^*)-A(x)\Big\}\Big\|\\ &= \eta\Big\|P_K^{f,\rho}\Big[A(x)-g(x)\Big]-P_K^{f,\rho}\Big[A(x^*)-g(x^*)\Big]+A(x^*)-A(x)\Big\|\\ &= \eta\Big\|P_K^{f,\rho}\Big[A(x)-g(x)\Big]-P_K^{f,\rho}\Big[A(x^*)-g(x^*)\Big]+A(x^*)-g(x^*)-A(x)+g(x)\\ &\quad +g(x^*)-g(x)\Big\|\\ &\le \eta\Big\{\Big[A(x^*)-g(x^*)\Big]-P_K^{f,\rho}\Big[A(x^*)-g(x^*)\Big]-\Big[(A(x)-g(x))-P_K^{f,\rho}\Big[A(x)-g(x)\Big]\Big]\Big\|\\ &\quad +\Big\|g(x^*)-g(x)\Big\|Big\}\\ &\le \eta\Big\{A(x^*)-g(x^*)-A(x)+g(x)\Big\|+\Big\|g(x^*)-g(x)\Big\|\\ &\le \eta\Big\{(\alpha+\beta_1+\beta_2)\|x^*-x\|+(\beta_1+\beta_2)\|x^*-x\|\Big\}\\ &= \eta(\alpha+2\beta_1+2\beta_2)\|x^*-x\|\\ &\le \eta(\alpha+2\beta_1+2\beta_2)\|x^*\|+\eta(\alpha+2\beta_1+2\beta_2)\|x\|.\end{aligned} \tag{4.5}$$

Hence,

$$\begin{aligned}\|x(t)\| &\leq \|x(t_0)\| + \int_{t_0}^{t} \|F(s)\|ds \\ &\leq \|x(t_0)\| + \int_{t_0}^{t} \eta(\alpha + 2\beta_1 + 2\beta_2)\|x^*\|ds + \int_{t_0}^{t} \eta(\alpha + 2\beta_1 + 2\beta_2)\|x(s)\|ds \\ &= \|x(t_0)\| + \eta(\alpha + 2\beta_1 + 2\beta_2)\|x^*\|(t - t_0) + \eta(\alpha + 2\beta_1 + 2\beta_2) \int_{t_0}^{t} \|x(s)\|ds \\ &= \|x(t_0)\| + k_1(t - t_0) + k_2 \int_{t_0}^{t} \|x(s)\|ds, \end{aligned} \tag{4.6}$$

where $k_1 = \eta(\alpha+2\beta_1+2\beta_2)\|x^*\|$ and $k_2 = \eta(\alpha+2\beta_1+2\beta_2)$. By Lemma 2, Gronwall's Lemma, we obtain that

$$\|x(t)\| \leq \Big\{\|x(t_0)\| + k_1(t - t_0)\Big\} \exp^{k_2(t-t_0)},$$

where $t \in [t_0, \xi)$. Therefor, the solution $x(t)$ is bounded on $[t_0, \xi)$. If ξ is finite, we conclude that $\xi = \infty$.

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