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Application of Equation of State of Ideal Gas Law in Investigating LRS Bianchi Type-I Perfect Fluid Cosmological Model

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Abstract

Here in this present paper, a homogeneous plane symmetric cosmological model occupied with an unconventional Bianchi Type-I Perfect Fluid be inflicted with the Equation of State of Ideal Gas is deliberated and investigated the effects of energy density, pressure and other imperative cosmological parameters like Hubble Constant etc. have been investigated with respect to the time factor and the subsistence of dark energy in this present cosmological epoch have been inspected.

Keywords: Bianchi Type-I Perfect Fluid, Dark Energy, Early Universe, energy Density, Ideal Gas.

1.0 Introduction:

1.1 Visualizing the Concept of Exceedingly Rising Temperature Admittance at the Time of Formation of the Cosmos:

The Big Bang Cosmology is the most mysterious yet explanatory cosmological speculative mechanism explicated in the cosmos which in a simpler sense is lengthening far beyond the degree of human captivating boundaries. It is for such proffering mechanisms incurred in the cosmos, Astrophysicists and space scientists had been putting vigorous mathematically aggrandized analytical computational and scientific interpretations to through light on the super extreme environments and primary early time epochs which might be necessarily standing as additionally intriguingly exploratory. George Lemaitre had nomenclature this initial early time eon as primeval atom while another Astrophysicist named George Gamow, a Soviet and American polymath, theoretical physicist and cosmologist whose name can be uttered as an early provocatory and developer of Lemaitre's Research promoter on Big Bang Cosmology, as ylem.

The very Equations of Classical General Relativity had been operational in pervading the mechanisms functioned during the evolution of Big Bang Cosmology. The postulates or generalized presumptions are not implicational at the very early time instances of cosmic time in which the temperature of the cosmos escalated to reach the measured demarcation of Planck Scale.

In the subject arena of cosmology, the Planck temperature scale refers to a theoretically acclaimed maximum attainment of temperature, considered to be the surprisingly hottest reachable point of temperature scale which has been estimated to be nearly reaching an appraise of temperature elevation as $1.416808 \text{ x } 10^{32}$ Kelvin and this has been prioritized to get assembled with the cosmic ambience of ultimate initial epoch of the evolution of the universe, just after a fraction of second after the Big Bang cosmological phenomena, at such instance of which all the forces of the cosmos might have been integrated as a single force and rendering enormous heat and energy.

Some Exceptionally Fascinating Key Essentials Regarding the Implicational Planck Temperature at the extremely early Epoch of the Cosmos:

- **Extravagant heat:**

The Germany based physicist named Max Planck enunciating his revolutionary grounding theory of Quantum Physics, won the Nobel Prize in Physics in 1918. According to Max Planck's proposition, energy can be quantized. The Planck's Constant, Planck length, Planck time, and Planck temperature, characterize the scales at which classical physics is replenished by Quantum Cosmology where traditional physics substantiates a standstill shock and the cosmic theory acculturates. The Planck temperature is almost nearly quantified as 1.416808×1032 Kelvin [1]. At this supreme essence of temperature, the persistent heat is so escalating that the constituent particles even become unable to hold tight their normal criterions and eventually gravity and electromagnetic properties of the constituting particles are rigorously presumed to cluster into a single integrated force. This unification could offer insights into the universe's birth, just fractions of a second after the Big Bang, but currently remains a realm only accessible through theoretical physics. This temperature is such an extremely elevated that the constituent of an atomic structure turns into incompetent of pertaining their normal criteria and eventually the essential forces such as gravity and electromagnetics may possibly be assimilating into a single incorporated force.

The Supreme Functional Portrayed with the Essence of Planck Temperature in Investigating the Big Bang Cosmology:

The Big Bang theory provokes regarding the evolution of the universe that it got started from an extremely dense and blisteringly point structure.

The Planck Temperature proposes latent perceptions into the following cosmic phenomena:

- **Concerning the Preliminary Settings of the Evolving Stage of the Universe:**

The Plank temperature supports the astronomers in explaining the homogeneity, isentropic nature and an intensifying streaming out structure of the expanding cosmos that has been still stretching at a hurried extent.

- **The Conceivable Endurance of a Pre-Big Bang Intensified Ambiance**:

This very scenario fundamentally signifies that the temperatures might have dropped from directly above the Planck Temperature value. It also stands as a mythological symbol in the world's dispersed cultural and civilizational scriptures of a few cultural antiquities such as Hindu, Egypt and Finland where the Cosmic Egg symbolizes the interrelatedness of all living organisms, the cyclic nature of the universe, inventive supremacy, competency and as a whole can be rendered as the core driving force for the manifestation of the universe [1], [2]. The Cosmic Egg with its ancient roots and universal symbolism serves as a powerful remainder of the Continuous cycle of rebirth and promise of new beginnings [3].

Significant Facts Regarding the Pre-Big Bang Cosmology:
Prodigious good numerals of the Big Bang cosmological models redirect that prior to the execution of the cosmic episode, all matter and energy contained in the universe were condensed in an unremittingly dense point acknowledged as a singularity.
Cosmological Scenario Exterior to this Singularity:
The possible upshots embrace the following cosmic endeavours:

- **Quantum Fluctuations:**

These engraining tiny fluctuations or perturbations experienced in the quantum vacuum which are prominent to steer up the formation of the cosmos [2].

- **Brane Cosmology:**

The perception of today's discernible cosmos raises as a membrane (brane) in an advanced dimensional space where the forthcoming collisions between branes possibly will initiation a Big Bang Cosmology [2].

- **Inflationary Epoch:**

It depicts a phase of fast- track expansion instantly succeeding the Big Bang, feasibly illuminating the uniformity of the cosmos.

- **Quantum Gravitational Effects:**

This very effect is supposed to reveal the underlying perplexities of dark matter and energy and the ultimate fate of the cosmos.

1.2 Conceptualizing the Concept of Equation of State parameter ω and the Modified Generalized Chaplygin Gas (MGCG):

There have been quite a few established dark energy models that have their own characteristic features represented by their Equation of state $\omega = \frac{pDE}{\rho DE}$. Astronomical data designate regarding the value perceived by the Equation of State parameter ω which is very close to -1. The condition $\omega = -1$ acquiesces to the Cosmological Constant. This is nothing but the form of vacuum energy abundant in the cosmos and is the modest and the utmost prevalent aspirant for dark energy as per the data revealed from Weinberg (1999) [4], Sahni and Starobinsky (2000) [5], Peebles and Ratra (2003) [6], Padmanabhan (2003) [7], Padmanabhan (2005) [8]. For the condition entailing $\omega < -1$, the Phantom Energy, described in the research papers of Caldwell (2002) [9], Nojiri and Odintsov (2003) [10], Wei and Tian (2004) [11], Onemli and Woodard (2004) [12], Setare (2007) [13]. For the residual condition pertaining to the Equation of State parameter ω where $\omega \in \left]-1, -\frac{1}{3}\right[$is $-1 < \omega < -\frac{1}{3}$. The dark energy has been referred to as Quintessence, Ratra and Peebles (1988) [14], Wetterich (1988) [15], Caldwell et al. (1998) [16], Zlatev et al. (1999) [17]. Majority of the dark energy models credence scalar field(s) as dark components(s) validating to a dynamical Equation of State.

The present paper is an effort to explicate the late-time acceleration experienced in the universe with the insinuation of a Chaplygin Gas Model in an anisotropic background. Chaplygin Gas (CG) is a uncharacteristic fluid interpreted by the Equation of state:

$$p = -\frac{B}{\rho} \qquad \ldots\ldots\ldots \quad (1)$$

where ρ and p are respectively pressure and energy density in comoving reference frame, with $\rho > 0$; B is a positive constant. It emerges as an effective fluid associated with d- branes, illustrated by Bordermann and Hoppe (1993) [18], Fabris et al. (2002) [19] and can be derived from Born-Infeld type Lagrangians, Bento et al. (2002) [20], Novello et al. (2005) [21]. One of its supreme extraordinary strengths is that it designates a transition from a decelerated cosmological expansion to a stage of cosmic acceleration. The Equation of state described in Equation (1) was oversimplified by Bento et al. (2002) [20] to

$$p = -\frac{B}{\rho^{\alpha}} \qquad \ldots\ldots\ldots \quad (2)$$

with the property of $0 < \alpha \leq 1$. In this connection, certain specific cosmological models have been explored with the application of Generalized Chaplygin Gas (GCG) Equation of State, illustrated by Gorini et al. (2003) [22], Alam et al. (2003) [23], Sri Ram et al. (2009) [24]. Benaoum (2002) [25], a step forward improved the Generalized Chaplygin Gas Equation of State which takes the form as

$$p = A\rho - \frac{B}{\rho^{\alpha}}$$

in which $A > 0$ is a positive constant.

This aforementioned Equation is known as the Modified Generalized Chaplygin Gas (MGCG).

This Equation of State represents the evolution of the universe starting from a Radiation-dominated Era (when $A = 1/3$) to the ΛCDM model with a Dust-dominated Era (when $A = 0$) in between these two epochs. At all other stages, it brings about the demonstrations of a mixture. The names of the astrophysicists uttered in this context are Guo and Zhang (2007) [26] who had projected a variable Chaplygin Gas (VGCG) model, where the constant B appearing in (2) is taken to be a positive function of scale factor that is further analyzed by numerous space researchers viz., Guo and Zhang (2005) [27], Sethi et al. (2006) [28], Yang et al. (2007) [29], Debnath (2007) [30]. Likewise, a new generalized Chaplygin Gas (NGCG) model was deliberated by Zhang et al. [31] in the year 2006.

1.3 Acquainting with the Ideal Gas Law:

Here we have considered the Equation of state which is taken from the Ideal Gas Law as

$$p = \rho(\gamma - 1)e - \gamma p^0 \qquad \dots\dots\dots (3)$$

where e = Internal energy per unit mass and

$e = c_v T$,

where c_v is the specific heat constant volume and T is the temperature in Kelvin Scale

$c_v = \left(\frac{\partial U}{\partial T}\right)_v$, It measures the rate of change of internal energy U

with respect to temperature in Kelvin Scale T

$\gamma = 6.1$ (Constant)

p^0 is a constant

In the present paper, an LRS Bianchi Type- I space-time with plane symmetry (one of the modest models of an anisotropic universe which is homogeneous and spatially flat) filled with a Modified Generalized Chaplygin Gas (MGCG) be laid up with presentation of the Equation of State as specified in the aforementioned Equation (3). The various cosmological parameters have been analytically studied in congruence with the subsistence of dark energy in the cosmos in the early as well as present and future epochs have been investigated with the implication of the Equation of State of Ideal Gas in the universe.

1.4 Field Equations:

In order to explicate the analytical investigations, a space-time transpiring a Bianchi Type- I group of motions and possessing plane symmetry, Abdussattar (1983, 1984), Viswakarma et al. (1999), having denotations as under:

$$ds^2 = -dt^2 + R^2(t)\left[dx^2 + dy^2 + \left(1 + \beta \int \frac{dt}{R^3}\right)^2 dt^2\right] \quad \dots\dots\dots \quad (4)$$

where β is a positive constant and R stands for some function of cosmic time t. If the value of β is considered to be zero, the metric (4) reduces to the Friedmann models with flat space- sections. The cosmos is hypothesized to be occupied with a distribution of matter presented by the Equation of energy- momentum tensor of a perfect fluid as follows:

$$T_{ij} = (\rho + p).U_i U_j + p.g_{ij} \qquad \dots\dots\dots \qquad (5)$$

where ρ is the energy density of cosmic matter and specifically the cosmic fluid considered for the model universe under investigation, p stands for its isotropic pressure, U^i is the fluid flow vector field that is responsible for constructing an expanding geodesic and hypersurface- orthogonal congruence. The geometry of the space-time is governed by the gravitational effects of the cosmic matter or more significantly the cosmic fluid obeying the Equation of State of the Ideal Gas with the help of Einstein's Field Equations where the units have been considered as $8\pi G = c = 1$

For this, the Tensor Equation has been written as

$$R_{ij} - \frac{1}{2} R_k^k g_{ij} = -T_{ij} \qquad \dots\dots\dots \qquad (6)$$

The Einstein's Field equations (6) in connecting the metric (4) with T_{ij} given by (5), generate the two independent Equations viz.,

$$\rho = 3\frac{\dot{R}^2}{R^2} + \frac{2\beta\dot{R}}{R^4\left(1+\beta\int\frac{dt}{R^3}\right)^{\sigma}} \qquad \dots\dots\dots \qquad (7)$$

and

$$p = 2\frac{\ddot{R}}{R} - \frac{\dot{R}^2}{R^2} \qquad \dots\dots\dots \qquad (8)$$

Introducing the volume expansion $\theta \left(= U^i_{;i}\right)$ and shear σ for the metric Equation (4) have been achievable as follows:

$$\theta = 3\frac{\dot{R}}{R} + \frac{\beta}{R^3\left(1+\beta\int\frac{dt}{R^3}\right)^{\sigma}} \qquad \dots\dots\dots \qquad (9)$$

and

$$\sigma^2 = \frac{\beta^2}{3R^6\left(1+\beta\int\frac{dt}{R^3}\right)^{2\sigma}} \qquad \dots\dots\dots \qquad (10)$$

Considering a characteristic length l, establishing the volume behavior of the cosmic fluid satisfying the Equation of State of the Ideal Gas, can be perceived by the Equation

$$\frac{\dot{l}}{l} = \frac{\theta}{3} \qquad \dots\dots\dots \qquad (11)$$

This expanses to

$$l = R.\ \left(1 + \beta \int \frac{dt}{R^3}\right)^{\frac{1}{3}} \qquad \dots\dots\dots \qquad (12)$$

The density ρ and the pressure p in terms of the Hubble Parameter H, the deceleration parameter q and the shear stress σ may now be written as follows:

$$\rho = 3H^2 - \sigma^2 \qquad \dots\dots\dots \qquad (13)$$

$$p = H^2(2q - 1) - \sigma^2 \qquad \dots\dots\dots \qquad (14)$$

where

$$H = \frac{\dot{L}}{L} \text{ and } q = -\frac{\ddot{H}}{l^2}$$

the shear stress σ in terms of the scale factor l can be interpreted by the following Equation

$$\sigma^2 = \frac{\beta^2}{3l^6} \qquad \dots\dots\dots \qquad (15)$$

The conservation Equation is obtained as a consequence of the Equations (13) and (14) as

$$\dot{\rho} + 3(\rho + p)\frac{\dot{l}}{l} = 0 \qquad \dots\dots\dots \qquad (16)$$

We assume that the cosmological matter contained in the universe has an Ideal Gas Equation of State as given in Equation (3) by

$$p = \rho(\gamma - 1)e - \gamma p^0$$

Equation (16) can be integrated in this case to yield

$$\dot{\rho} + 3(\rho + p)\frac{\dot{l}}{l} = 0$$

$$\Rightarrow \dot{\rho} + 3\left\{\rho + \rho(\gamma - 1)e - \gamma p^0\right\}\frac{\dot{l}}{l} = 0$$

$$\Rightarrow \dot{\rho} + 3[\{1 + (\gamma - 1)e\}\rho - \gamma p^0]\frac{\dot{l}}{l} = 0$$

$$\Rightarrow \dot{\rho} = -3[\{1 + (\gamma - 1)e\}\rho - \gamma p^0]\frac{\dot{l}}{l}$$

$$\Rightarrow \frac{\dot{\rho}}{\{1+(\gamma-1)e\}\rho-\gamma p^0} = -3\frac{\dot{l}}{l}$$

$$\Rightarrow \frac{\frac{d\rho}{dt}}{\{1+(\gamma-1)e\}\rho-\gamma p^0} = -3\frac{\frac{dl}{dt}}{l}$$

$$\Rightarrow \frac{d\rho}{\{1+(\gamma-1)e\}\rho-\gamma p^0} = -3\frac{dl}{l}$$

Here we have considered.

$$\begin{cases} \{1 + (\gamma - 1)e\}\rho - \gamma p^0 = z \\ \Rightarrow \{1 + (\gamma - 1)e\}\, d\rho = dz \\ \Rightarrow d\rho = \dfrac{1}{\{1 + (\gamma - 1)e\}}\, .dz \end{cases}$$

$$\Rightarrow \frac{\frac{dz}{\{1+(\gamma-1)e\}}}{z} = -3\frac{dl}{l}$$

$$\Rightarrow \int \frac{dz}{z}.\frac{1}{\{1+(\gamma-1)e\}} = -3\int \frac{dl}{l} + C_1$$

$$\Rightarrow \int \frac{dz}{z} = -3\,\{1+(\gamma-1)e\}\int \frac{dl}{l} + \{1+(\gamma-1)e\}C_1$$

where C_1 is a constant of integration.

$$\Rightarrow \log z = -3\{1+(\gamma-1)e\}\log l + \{1+(\gamma-1)e\}C_1$$

$$\Rightarrow \log z = \log e^{-3\,\{1+(\gamma-1)e\}} + \log(3C)^{(1+\alpha)}$$

where $\{1+(\gamma-1)e\}C_1 = \log(3C)^{(1+\alpha)}$

$$\Rightarrow \log z = \log\left[(3C)^{(1+\alpha)}.\,e^{-3\,\{1+(\gamma-1)e\}}\right]$$

$$\Rightarrow \{1+(\gamma-1)e\}\rho - \gamma p^0 = (3C)^{(1+\alpha)}.\,e^{-3\,\{1+(\gamma-1)e\}}$$

$$\Rightarrow \rho = \frac{\gamma p^0}{\{1+(\gamma-1)e\}} + \frac{1}{\{1+(\gamma-1)e\}}.\left[(3C)^{(1+\alpha)}.\,e^{-3\,\{1+(\gamma-1)e\}}\right] \quad \dots\dots\dots \quad (17)$$

where the integrating constant is specified as $\log(3C)^{(1+\alpha)}$, $C > 0$ for mathematical convenience.

From Equation (3), we have

$$p = \rho(\gamma-1)e - \gamma p^0$$

$$\therefore p = \left[\frac{\gamma p^0}{\{1+(\gamma-1)e\}} + + \frac{1}{\{1+(\gamma-1)e\}}.\left[(3C)^{(1+\alpha)}.\,e^{-3\,\{1+(\gamma-1)e\}}\right]\right].(\gamma-1)e - \gamma p^0$$

$$= \frac{\gamma p^0.(\gamma-1)e}{\{1+(\gamma-1)e\}} + \frac{(\gamma-1)e}{\{1+(\gamma-1)e\}}.\ \left[(3C)^{(1+\alpha)}.\,e^{-3\,\{1+(\gamma-1)e\}}\right] - \gamma p^0$$

$$= \frac{(\gamma^2-\gamma)p^0 e}{\{1+(\gamma-1)e\}} + \frac{(\gamma-1)e}{\{1+(\gamma-1)e\}}.\ \left[(3C)^{(1+\alpha)}.\,e^{-3\,\{1+(\gamma-1)e\}}\right] - \gamma p^0 \quad \dots\dots\dots \quad (18)$$

The period of evolution satisfying the condition,

$$H^2 + \sigma^2 + p > 0$$

which with the help of (13) and (3) can be expressed as

$$\rho = 3H^2 - \sigma^2 \quad \dots\dots\dots \quad (13)$$

$$p = \rho(\gamma-1)e - \gamma p^0 \quad \dots\dots\dots \quad (3)$$

Now, Equation (13)$\Rightarrow$ $H^2 = \frac{\rho+\sigma^2}{3}$

Substituting the value of H^2 and p from above and Equation (3), we get

$$\frac{\rho+\sigma^2}{3} + \sigma^2 + \rho(\gamma-1)e - \gamma p^0 > 0$$

$$\Rightarrow \frac{\rho + \sigma^2 + 3\sigma^2 + 3\rho(\gamma - 1)e - 3\gamma p^0}{3} > 0$$

$\Rightarrow \rho + 4\sigma^2 + 3\rho(\gamma - 1)e - 3\gamma p^0 > 0$
$\Rightarrow 4\sigma^2 + [1 + 3(\gamma - 1)e]\rho - 3\gamma p^0 > 0$
$\Rightarrow 4\sigma^2 > 3\gamma p^0 - [1 + 3(\gamma - 1)e]\rho$
$\Rightarrow \sigma^2 > \frac{3\gamma p^0 - [1+3(\gamma-1)e]\rho}{4}$ ……… (19)

In terms of the scale factor, the condition for the period of decelerated expansion can be expressed as follows:

We have the shear σ in terms of l comes out as
$\sigma^2 = \frac{\beta^2}{3l^6}$ ……… (15)

Now,

$$\sigma^2 > \frac{3\gamma p^0 - [1+3(\gamma-1)e]\rho}{4} \quad \text{[Utilizing Equation (19)]}$$

$$\therefore \frac{\beta^2}{3l^6} > \frac{3\gamma p^0 - [1 + 3(\gamma - 1)e]\rho}{4}$$

$\Rightarrow 4\beta^2 > 3l^6[3\gamma p^0 - [1 + 3(\gamma - 1)e]\rho]$
i.e., $\frac{3}{4}l^6[3\gamma p^0 - [1 + 3(\gamma - 1)e]\rho] < \beta^2$
……… (20)

In the absence of Anisotropy which emphasizes the condition ($\beta = 0$), the condition (20) reduces to one obtained by Debnath et al. (2004), in the case of a flat FRW model.

These correspond to a mixture of a cosmological constant equal to $\frac{(\gamma^2 - \gamma)p^0 e}{\{1+(\gamma-1)e\}}$
Now at this stage, again citing the Equation which is prior to Equation (18) is

$$p = \frac{\gamma p^0.(\gamma - 1)e}{\{1 + (\gamma - 1)e\}} + \frac{(\gamma - 1)e}{\{1 + (\gamma - 1)e\}} . \left[(3C)^{(1+\alpha)}.\, l^{-3\{1+(\gamma-1)e\}}\right] - \gamma p^0$$

Dividing both sides by the term $(\gamma - 1)e$, we have

$$\frac{p}{(\gamma - 1)e} = \frac{\gamma p^0}{(\gamma - 1)e} + \frac{1}{\{1 + (\gamma - 1)e\}} . \left[(3C)^{(1+\alpha)}.\, l^{-3\{1+(\gamma-1)e\}}\right] - \frac{\gamma p^0}{(\gamma - 1)e}$$

$$\Rightarrow \frac{p}{(\gamma-1)e} = \rho - \frac{\gamma p^0}{(\gamma-1)e}$$

$$\Rightarrow p = \{(\gamma - 1)e\}\rho - \gamma p^0$$

Taking $\gamma = 0$, we have

$$p = -e\rho$$

Let $p = \omega\rho$ where $\omega = -e$ which is analogous to a Generalized Chaplygin Gas Equation.
Now

$$\rho = \frac{\gamma p^0}{\{1+(\gamma-1)e\}} + \frac{1}{\{1+(\gamma-1)e\}} \cdot \left[(3C)^{(1+\alpha)} . l^{-3\{1+(\gamma-1)e\}}\right]$$

where e is the internal energy per unit mass, say $e = \frac{1}{3}$
Again, when the scale factor l is small, the energy density ρ varies as

$$\rho \propto l^{-3\left\{1+(6.1-1).\frac{1}{3}\right\}}$$
$$= l^{-8} = (l^2)^{-4}$$

i.e., $\rho \propto \left(\frac{{l_0}^2}{l^2}\right)^4$ which corresponds to a Radiation- dominated universe and for $l = 0$, the energy density $\rho \propto \left(\frac{l_0}{l}\right)^3$ which corresponds to a Dust-like dominated universe. Also, when the scale factor l becomes large, the energy density $\rho \simeq$ constant which corresponds to a cosmological constant like Dominated universe.

From the Equation of state, the parameter $\omega = \frac{p}{\rho}$, we get from

$$\omega = \frac{p}{\rho} = \{\rho(\gamma-1)e - \gamma p^0\}.\frac{1}{\rho} \qquad [\because p = \rho(\gamma-1)e - \gamma p^0]$$

$$\Rightarrow \omega = (\gamma-1)e - \gamma\frac{p^0}{\rho}$$

$$\Rightarrow \gamma p^0 = \{(\gamma-1)e - \omega_0\}.\rho_0 \qquad \text{[Rewriting in this form]}$$

Here and afterwards, a subscript '0' means the value of the variable at time t_0
We see that when $(\gamma-1)e = \omega_0$, we get $\gamma p^0 = 0$ and the Modified Generalized Chaplygin Gas (MGCG) Equation of State reduces to Barotropic Equation of State,

$$\omega = \frac{p}{\rho} = [\rho(\gamma-1)e - \gamma p^0].\frac{1}{\rho} = (\gamma-1)e - \frac{\gamma p^0}{\rho}$$

$$\Rightarrow p = [(\gamma-1)e]\rho$$

Now, substituting the value of $\gamma p^0 = \{(\gamma-1)e - \omega_0\}.\rho_0$ in (17), we get

i.e., $\rho = \frac{\gamma p^0}{\{1+(\gamma-1)e\}} + \frac{1}{\{1+(\gamma-1)e\}}.\left[(3C)^{(1+\alpha)}.l^{-3\{1+(\gamma-1)e\}}\right]$ ……… (17)

∴ We have,

$$\rho = \frac{\{(\gamma-1)e-\omega_0\}\rho_0}{\{1+(\gamma-1)e\}} + \frac{1}{\{1+(\gamma-1)e\}}\left[(3C)^{(1+\alpha)}.l^{-3\{1+(\gamma-1)e\}}\right]$$

$$\Rightarrow \left[1 - \frac{(\gamma-1)e-\omega_0}{\{1+(\gamma-1)e\}}\right]\rho_0 = \frac{1}{\{1+(\gamma-1)e\}}.\left[(3C)^{(1+\alpha)}.{l_0}^{-3\{1+(\gamma-1)e\}}\right]$$

$$\Rightarrow (3C)^{(1+\alpha)} = (1+\omega_0)\rho_0.{l_0}^{-3\{1+(\gamma-1)e\}}$$

$$\Rightarrow (3C)^{(1+\alpha)} = (1+\omega_0)\rho_0.{l_0}^{3\{1+(\gamma-1)e\}}$$

Now putting the value of $\gamma p^0 = \{(\gamma-1)e - \omega_0\}\rho_0$ and $(3C)^{(1+\alpha)} = (1+\omega_0).\rho_0.{l_0}^{3\{1+(\gamma-1)e\}}$ in Equation (17) where Equation (17) as pointed out above is

$$\rho = \frac{\gamma p^0}{\{1+(\gamma-1)e\}} + \frac{1}{\{1+(\gamma-1)e\}}.\left[(3C)^{(1+\alpha)}.l^{-3\{1+(\gamma-1)e\}}\right] \qquad \dots\dots\dots (17)$$

$$\therefore \rho = \frac{\{((\gamma-1)e - \omega_0)\}\rho_0}{\{1+(\gamma-1)e\}} + \frac{1}{\{1+(\gamma-1)e\}}\left[(3C)^{(1+\alpha)}.l^{-3\{1+(\gamma-1)e\}}\right]$$

$$\Rightarrow \rho = \frac{\{((\gamma-1)e - \omega_0)\}\rho_0}{\{1+(\gamma-1)e\}} + \frac{1}{\{1+(\gamma-1)e\}}\left[(1+\omega_0)\rho_0 . l_0^{3\{1+(\gamma-1)e\}} . \, l^{-3\,\{1+(\gamma-1)e\}}\right]$$

$$\Rightarrow \rho = \frac{\rho_0}{\{1+(\gamma-1)e\}}\left[\{((\gamma-1)e - \omega_0)\} + (1+\omega_0).\left(\frac{l_0}{l}\right)^{3\{1+(\gamma-1)e\}}\right] \quad \dots\dots\dots (21)$$

Thus, Equation (21) gives the Equation of energy density ρ of the cosmos.
Also, from (18), we have

$$p = \frac{(\gamma^2-\gamma)p^0 e}{\{1+(\gamma-1)e\}} + \frac{(\gamma-1)e}{\{1+(\gamma-1)e\}}. \left[(3C)^{(1+\alpha)} . e^{-3\,\{1+(\gamma-1)e\}}\right] - \gamma p^0$$

$$\Rightarrow p = \left[\frac{\{(\gamma-1)e-\omega_0\}\rho_0}{\{1+(\gamma-1)e\}} + \frac{1}{\{1+(\gamma-1)e\}}.\left\{(1+\omega_0).\rho_0 . l_0^{3\{1+(\gamma-1)e\}} . l^{-3\,\{1+(\gamma-1)e\}}\right\}\right] (\gamma-1)e - \gamma p^0$$

$$\Rightarrow p = \left[\frac{\{(\gamma-1)e-\omega_0\}\rho_0}{\{1+(\gamma-1)e\}} + \frac{1}{\{1+(\gamma-1)e\}}.\left\{(1+\omega_0).\rho_0 . \left(\frac{l_0}{l}\right)^{3\{1+(\gamma-1)e\}}\right\}\right](\gamma-1)e - \gamma p^0$$

$$\Rightarrow p = \frac{\rho_0}{\{1+(\gamma-1)e\}}\left[\{(\gamma-1)e-\omega_0\} + (1+\omega_0)\left(\frac{l_0}{l}\right)^{3\{1+(\gamma-1)e\}}\right](\gamma-1)e - \gamma p^0 \quad \dots\dots \quad (22)$$

Equation (17) is

$$\rho = \frac{\gamma p^0}{\{1+(\gamma-1)e\}} + \frac{1}{\{1+(\gamma-1)e\}}.\left[(3C)^{(1+\alpha)} . l^{-3\,\{1+(\gamma-1)e\}}\right] \quad \dots\dots\dots (17)$$

This Equation can be approximated to

$$\rho \simeq \frac{(3c)^{(1+\alpha)}}{\{1+(\gamma-1)e\}. l^{3\,\{1+(\gamma-1)e\}}} \quad \dots\dots\dots \quad (23)$$

which is very large and corresponds to the universe dominated by an Equation of State

$$p = \rho(\gamma-1)e$$

For ρ, given in Equation (22), Friedmann Equation (13) reduces to

$$\rho = 3H^2 - \sigma^2 \quad \dots\dots\dots \quad (13)$$

where $H = \frac{\dot{l}}{l}$ and shear stress σ in terms of l comes out as $\sigma^2 = \frac{\beta^2}{3l^6}$
Therefore, substituting the values in Equation (13), we have

$$\frac{(3c)^{(1+\alpha)}}{\{1+(\gamma-1)e\}.\,l^{3\{1+(\gamma-1)e\}}}=3\frac{\dot{l}^2}{l^2}-\frac{\beta^2}{3l^6}$$

$$\Rightarrow 3\frac{\dot{l}^2}{l^2}=\frac{(3c)^{(1+\alpha)}}{\{1+(\gamma-1)e\}.\,l^{3\{1+(\gamma-1)e\}}}+\frac{\beta^2}{3l^6}$$

$$\Rightarrow \frac{\dot{l}^2}{l^2}=\frac{(3c)^{(1+\alpha)}}{3\{1+(\gamma-1)e\}.\,l^{3\{1+(\gamma-1)e\}}}+\frac{\beta^2}{9l^6}$$

$$\Rightarrow \dot{l}^2=\frac{(3c)^{(1+\alpha)}}{3\{1+(\gamma-1)e\}.\,l^{3+3(\gamma-1)e-2}}+\frac{\beta^2}{9l^4} \quad \dots\dots\dots (23)$$

$$\Rightarrow \dot{l}^2=\frac{(3c)^{(1+\alpha)}}{3\{1+(\gamma-1)e\}.\,l^{3(\gamma-1)e+1}}+\frac{\beta^2}{(3l^2)^2} \quad \dots\dots\dots (24)$$

From this Equation (23) on integration and setting constraints on the integrating constants, we shall get the time evolution of the scale factor l

1.5 Analysis Carried out on the Hubble Constant H :

We know that the Hubble Constant H is given by $H=\frac{\dot{l}}{l}$

$$\therefore H=\frac{\sqrt{\frac{(3c)^{(1+\alpha)}}{3\{1+(\gamma-1)e\}.\,l^{3+3(\gamma-1)e-2}}+\frac{\beta^2}{9l^4}}}{l} \quad \dots\dots\dots (25)$$

[Substituting the value of $\dot{l}$ from Equation (23)]

where e is the internal energy per unit mass and $e = c_v T$ where T is the temperature in Kelvin Scale. In the context of Thermodynamics and the Ideal Gas Equation, c_v stands for the Specific Heat Capacity at Constant Volume (or Molar Heat Capacity at Constant Volume).

From Equation (25), it is obvious that as temperature increases and a situation arises that $T \gg 0$ then the value of the Hubble Constant H becomes negligibly small and $H \to 0$. In the current ΛCDM model of the universe, as time T becomes exceedingly large and hypothetically tends to ∞, the Hubble parameter asymptotically approaches a constant related to dark energy, not zero, while the acceleration continues, preventing a halt. The temperature of the universe was exceedingly high at the very initial outset. Planck Temperature is acknowledged as the peak conceivable temperature and epitomizes a point structure where long-established apprising of the established theories of Physics which is on the brink of ascertaining a value of 1.416808 x 1032 Kelvin. At such an extravagant temperature point, the heat is so powerful that particles are unable to grasp its customary physical possessions and characteristics like gravity and electromagnetism, may possibly amalgamate into a single, unified force. This hypothesized integration tactic may perhaps propose for intuitions into the genesis of the cosmos, just fractions of a second posterior to the Big Bang cosmological phenomenon which at this time endures as a precinct merely acknowledgeable all the way through Theoretical Physics. The concept of Planck's Temperature, can only be speculated since this temperature is outside the domain of up-to-the minute experimental proficiencies.

1.6 Analysis Carried Out on Pressure p:

From Equation (22), we shall have the Equation for the pressure p as follows:

$$p = \frac{\rho_0}{\{1+(\gamma-1)e\}}\left[\{(\gamma-1)e-\omega_0\} + (1+\omega_0)\left(\frac{l_0}{l}\right)^{3\{1+(\gamma-1)e\}}\right](\gamma-1)e - \gamma p^0$$

In this above Equation (22), l being the value of the scale factor and l_0 represents the value of this scale factor at current time $t = t_0$

When the value of the scale factor l increases incredibly and a situation arises where $l \to \infty$, pressure decreases and in due course of time $p \to 0$

In the subject arena of Astrophysics and Cosmology, pressure, explicitly referring to the pressure due to the non-relativistic matter, decreases to zero or it turns out to be insignificantly small since the universe inflates in every possible direction to homogeneity and anisotropic property of the cosmos and eventually cools down, transitioning into a state often described as dust. With the persistence of radiation pressure, the total effective pressure of the matter component of the cosmos transpires to be zero.

Central facets of this transition and the relevant phase include:

- **When Pressure Reduces to Zero:**

This situation occurs when the kinetic energy of particles due to thermal motion becomes insignificant as compared to their rest-mass energy. As the universe expanded and cooled, matter come to an end state of being hot and radiation-dominated, dropping down in temperature and finally reach a cold state where particles and in later course galaxies have negligible unconventional velocities proportionate to the expansion.

- **Setting Apart and Subsistence of Dark Energy:**

While pressure due to the cosmic matter declines to zero, the cosmos divulges the existence of dark energy that is hypothesized to exert a negative pressure, accelerating the exponential expansion of the cosmos.

- **Early Universe Distinction:**

In the extreme early epoch of the universe, the pressure was not zero, as the cosmic matter was ambiguously hot in nature and relativistic prior to its cooling stage.

Thus, pressure becomes effectually zero when the cosmic matter contained in the universe becomes cold enough so that its associated kinetic energy does not subsidize significantly to the total energy density measured up to its mass [32], [33].

Again, analyzing on considering the internal energy e per unit mass and $e = c_v T$ where T is the temperature in Kelvin Scale. When temperature increases enormously, a situation occurs when T is very high and $T \to \infty$. Undre such ambience, the pressure decreases colossally and in some point pressure $p \to 0$ and pressure becomes negligibly small. The subsistence of dark energy gets authorized significantly signifying an exponential expansion incurred in the cosmos.

1.7 Analysis Carried on Energy Density ρ:

From Equation (21), we shall get the Equation of the energy- density ρ as

$$\rho = \frac{\rho_0}{\{1+(\gamma-1)e\}}\left[\{((\gamma-1)e - \omega_0)\} + (1+\omega_0).\left(\frac{l_0}{l}\right)^{3\{1+(\gamma-1)e\}}\right]$$

From this above Equation, it is clear that the larger the value, the scale factor of the cosmos acquires, the lower decreased value, the energy density accomplishes.
Thus, Mathematically, as $l \to \infty \Rightarrow \rho \to 0$.
The incredibly underlying concept that the energy density becomes zero in Cosmology, generally refers to the Zero- Energy Universe Hypothesis which posits that the total positive energy due to cosmic matter is well-adjusted by negative gravitational energy, totaling to a net density of zero. As the universe expands, matter density dilutes to zero but dark energy density remains constant. According to the physicist Stephen Hawking, the Zero- Energy Universe Hypothesis states that the positive energy of matter and the negative energy of gravitational fields accurately strike down [34].
As the cosmos undergoes incessant intensification, the density of matter $\propto l^{-3}$ and radiation $\propto l^{-4}$ which decreases toward zero and dark energy remains constant [35].

2.0 Inferences Drawn:
Here in this present study, an exotic fluid has been considered and its vivid cosmological inferences on an isotropic LRS Bianchi Type-I space- time having plane symmetry in the cosmos has been investigated. The exotic fluid obeys the Ideal Gas Equation of State which carries some interesting properties. It has been shown that the Ideal Gas Equation of State describes the evolution of the universe from a phase dominated by the cosmic fluid having an Equation of State $p = (\gamma - 1)\rho$ for small values of the scale factor to a phase dominated by a cosmological constant $\frac{(3c)^{(1+\alpha)}}{\{1+(\gamma-1)e\}.l^{3\{1+(\gamma-1)e\}}}$. The cosmos undergoes a decelerating expansion in the early phase of evolution which is crucial for the successful nucleosynthesis as well as structure formation. In the later phase, the universe expands with acceleration for large values of the scale factor. Mathematically, as the scale factor becomes indefinitely large with the increment of time scales, the energy density ρ of the cosmos decreases with respect to time intervals and approaches zero value and at times reaches this value zero properly. This situation triggers a cosmological scenario where the cosmos undergoes an exponential expansion in every permissible direction as the universe is assumed to be homogeneous and isotropic. Analytically, when the scale factor of the universe gradually becomes large and as time increases, it tends to achieve an infinite value, the pressure tends to zero. Now this situation occurs in the cosmos when the pressure of non-relativistic matter decelerates to an insufficiently small value or the value of zero. In Cosmology, this situation transpires as the cosmos undergoes rapid expansion and then cools down, transitioning into a state which is more precisely referred to as dust. Although the pressure due to radiation exists, the total effective pressure of the matter component becomes zero.

Conflicts of Interest:
The author declares no conflicts of interest for this present study.

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