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# Some Common Fixed point theorem for Weakly Compatible maps in Fuzzy Metric Spaces

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**Abstract**

In this paper, we shall prove some common fixed point theorems for a pair of weakly compatible maps in KM and GV fuzzy metric spaces. In addition to this fixed point results using. A property and CLR property are also proved in the mentioned spaces. Some suitable examples are also provided to prove the validity of our results.

**Keywords**: Fixed point, Fuzzy metric space, Weakly Compatible, E.A property, CLR Property.

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## 1. Introduction

The concept of fuzzy set was initiated by Zadeh [10] in 1965 as a mathematical way to represent vagueness in everyday life. To use this concept in analysis, several researchers have defined fuzzy metric spaces in several ways.

Fixed point theory in fuzzy metric spaces came into notice starting with the work of Helipern [17]. He introduced the concept of fuzzy mappings and proved some fixed point theorems for fuzzy contraction mappings in metric linear space, which is a fuzzy extension of Banach contraction principle.

George and Veeramani [6] have modified the concept of fuzzy metric space introduced by Kramosil and Michalek [8]. Most recently Gregori [7] have furnished several interesting examples of fuzzy metrics in the sense of George and Veeramani [6] and have also utilized such fuzzy metrics to color image processing.

**Definition 1.1** (Schweizer and Sklar [3]). A continuous t-norm is a binary operation $*$ on [0, 1] $a * 1 = a$ for all $a \in [0,1]$;

$a * b \leq c * d$ Satisfying the following conditions:

(i) $*$ is commutative and associative $a * 1 = a$ for all $a \in [0,1]$;

(ii) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ $(a, b, c, d \in [0,1]$;

(iii) The mapping $* : [0,1] \times [0,1] \rightarrow [0,1]$ is continuous.

In 1975, Kramosil and Michalek [8] gave a notion of fuzzy metric space which could be considered as a reformulation, in fuzzy context, of the notion of probabilistic metric space due to Menger [9].

**Definition 1.2** (Kramosil and Michalek [8]). A fuzzy metric space is a triple $(X, N,*)$ where $X$ is a non empty set, $*$ is a continuous $t$-norm and $N$ is fuzzy set on $X^2 \times [0,1]$ such that the following axioms hold:

(KM-1) $N(x, y, 0) = 0$ for all $x, y \in X$;

(KM-2) $N(x, y, t) = 1$ for all $x, y \in X$ where $t > 0 \Leftrightarrow x = y$;

(KM-3) $N(x, y, t) = N(y, x, t)$ for all $x, y \in X$;

(KM-4) $N(x, y, .) : [0, \infty) \rightarrow [0, 1]$ is left continuous for all $x, y \in X$;

(KM-5) $N(x, z, t + s) \geq N(x, y, t) * N(y, z, s)$ for all $x, y, z \in X$ and for all $s, t > 0$.

We will refer to these spaces as KM-fuzzy metric spaces.

**Lemma 1.3.** (Grabiec [11]). For every $x, y \in X$ the mapping $N(x, y, .)$ *is* nondecreasing on $[0, \infty]$.

George and Veeramani [6] introduced and studied a notion of fuzzy metric space which constitutes a modification of the one due to Kramosil and Michalek [8].

**Definition 1.4** (George and Veeramani [6]). A fuzzy matric space is a triple $(X, N,*)$ where $X$ is a non-empty set, $*$ is a continuous $t$-norm and $N$ is a fuzzy metric set on $X^2 \times [0,1]$ and the following conditions are satisfied for all $x, y \in X$ and $t, s > 0$:

(GV-1) $N(x, y, t) > 0$;

(GV-2) $N(x, y, t) = 1 \Leftrightarrow x = y$;

(GV-3) $N(x, y, t) = N(y, x, t)$;

(GV-4) $N(x, y, .) : (0, \infty) \rightarrow [0,1]$ is continuous;

(GV-5) $N(x, z, t + s) \geq N(x, y, t) * N(y, z, s)$.

From (GV-1) and (GV-2), it follows that if $x \neq y$, then $0 < N(x, y, t) < 1$ for all $t > 0$.

In what follows, fuzzy metric spaces in the sense of George and Veeramani [6] will be called GV-fuzzy metric spaces.

**Definition 1.5.** Let $(X, N,*)$ be a (KM- or GV-) fuzzy metric space. A sequence $\{x_n\}$ in $X$ is said to be convergent to $x \in X$ if

$$\lim_{n\to\infty} N(x_n, x, t) = 1$$

For all $t > 0$.

**Definition 1.6.** Let $N(X, N,*)$ be a (KM- or GV-) fuzzy metric space. A sequence $\{x_n\}$ in $X$ is said to be G-Cauchy Sequence if

$$\lim_{n\to\infty} N(x_n, x_{n+m}, t) = 1,$$

For all $t > 0$ and $m \in \mathbb{N}$.

**Definition 1.7.** A fuzzy metric space $(X, N,*)$ is called G-complete if every G-Cauchy sequence converges to a point in $X$.

**Definition 1.8**. Let $\varphi$ be class of all mappings $\varphi : [0,1] \to [0,1]$ satisfying the following properties:

$(\varphi 1)$ $\varphi$ is continuous and non-decreasing on $[0,1]$;

$(\varphi 2)$ $\varphi(x) > x$ For all $x \in (0,1)$.

**Lemma 1.9.** (Schweizer and Sklar [3]). If $(X, N,*)$ is a KM- fuzzy metric space and $\{x_n\}, \{y_n\}$ are sequence in $X$ such that

$$\lim_{n\to\infty} x_n = x, \qquad \lim_{n\to\infty} y_n = y,$$

then

$$\lim_{n\to\infty} N(x_n, y_n, t) = N(x, y, t)$$

For every continuity point of $N(x, y, .)$.

**Definition 1.10.** (Zadeh[10]). Let $(X, N,*)$ $f$ and $g$ Bea fuzzy metric space, $f and\ g$ be two self-maps of a metric space (X, d). A point u in $X$ is called a coincidence point of $f$ and $g$ iff $fz = gz$ in this case, $u = fz = gz$ is called point of coincidence of $f$ and $g$.

**Definition1.11.** (Jungck and Rhoades [5]). A pair of self- mappings $(f, g)$ of a fuzzy metric space $(X, N,*)$ is said to be weakly compatible if they commute at the coincidence points $i.e.$, if $fz = gz$ for some $z \in X$, then $fgz = gfz$.

**Example1.12**:- Define $f, g$: $\mathbb{R} \to \mathbb{R}$ by $fx = \frac{x}{4}$, for all $x \in \mathbb{R}$ and $= x^2$ , for all $x \in \mathbb{R}$.

Here, 0 and $\frac{1}{2}$ are two coincidence points for all maps $f$ and$g$. Note that $f$ and $g$ commute at 0, i.e., $fg(0) = gf(0) = 0$, but$fg\left(\frac{1}{2}\right) = f(\frac{1}{8})= \frac{1}{32}$ and $(\frac{1}{2}\ )= g(\frac{1}{8}\ )= \frac{1}{512}$ so $f$and $g$ are not weakly compatible on $\mathbb{R}$.

**Note:- (**i) Compatible maps are weakly compatible but converse is not true in general.

(ii) Weakly compatibility and (E.A) property are independent to each other**.**

In 2002, Aamri and Moutawakil [1] introduced the notion of E.A property, as follows:

**Definition 1.13.** Let $f$ and $g$ are two self–mappings of a metric space $(X, d)$.Then the pair $(f, g)$ is said to satisfy E.A. property if there exists a sequence $\{x_n\}$ in $X$ such that $\log_{n\to\infty} fx_n = \log_{n\to\infty} gx_n = t$ for some $t \in X$.

**Definition 1.14.** Let $f$ and $g$ are two self-mappings of a metric space $(X, d)$. Then the pair $(f, g)$ is said to satisfy $(CLR_g)$ property if there exists a sequence $\{x_n\}$ in $X$ such that $\lim_{n\to\infty} fx_n = \log_{n\to\infty} gx_n = gx$ for some $x \in X$.

**Lemma. 1.15.** If $(X, N,*)$ is a KM-fuzzy metric space and $\{x_n\}, \{y_n\}$ are sequences in $X$ such that $x_n \to x, y_n \to y$ , then $N(x_n, y_n, t) \to N(x, y, t)$ for every continuity point t of $N(x, y, .)$.

**Example1.16:-**Let $X = [0, \infty)$, $f, g : X \to X$, and $fx = \frac{x}{5}$ and $gx = \frac{3x}{5}$ for each $x \in X$. Consider the sequence$\{x_n\} = \frac{1}{n}$ so that $\log_{n\to\infty} fx_n = \log_{n\to\infty} gx_n = 0 = g(0)$ ,where $0 \in X$. Hence, the pair $(f, g)$ satisfies the $(CLR_g)$ property.

**Lemma 1.17.** (M. Grabiec 11). Let $(X, N,*)$ be a KM-(or GV-) fuzzy metric space. If there exists a constant $k \in (0,1)$ such that

$$N(x, y, kt) \geq N(x, y, t),$$

For all x, y $\in X$, $t > 0$, then $x = y$.

**2. Weakly Compatible Maps and E.A. Property**

Now we will prove our result for weakly compatible maps along with the E.A. Property as follows:-

**Theorem 2.1.** Let $(X, N *)$ be KM-fuzzy metric space and $f$ and $g$ be self-mappings of $X$ satisfying

$$N(fx, fy, kt) \geq \{N(gx, gy, t) * N(fx, gx, t) * N(fy, gy, t) * N(fx, gy, t) * N(fy, gx, t)\} \quad \textbf{(1)}$$

$\forall x, y \in X,\ t > 0,$ Suppose that the pair $(f, g)$ satisfy the E.A. property and $(f, g)$ is weakly compatible. Then $f$ and $g$ have a unique common fixed point in $X$.

**Proof:-**Since $f$ and $g$ satisfy E.A. property, there exists a sequence $\{x_n\}$ in $X$ such that

$\text{Limit}_{n\to\infty} fx_n = \text{Limit}_{n\to\infty} gx_n = z$ , for some $z \in X$.

Since $f(X)$ is closed subset of $X$, therefore $\text{Limit}_{n\to\infty} fx_n = z \in f(X)$. Hence, there exist a point $u \in X$, such that $fu = z$.

Now we assert that $N(gu, z, t) = 1$. if not, then using condition (1), we have

$$N(gu, fx_n, kt) \geq$$

$$\{N(gu, gx_n, t) * N(fu, gu, t) * N(fx_n, gx_n, t) * N(fu, gx_n, t) * N(fx_n, gu, t)\}$$

$\forall\, n \in \mathbb{N}$, Which on Letting $n \to \infty$

$$N(gu, z, kt) \geq \{N(gu, z, t) * N(z, gu, t) * N(z, z, t) * N(z, z, t) * N(z, gu, t)\}$$

$$N(gu, z, kt) \geq N(gu, z, t)$$

Since $z \neq gu$ , therefore $0 < N(z, gu, t_0) < 1$, for some $t_0 > 0$.As $N(z, gu, .)$ is left continuous and $N(z, gu, .)$ is non-decreasing, It can assume at the most countable points of discontinuity. If we assume that $t_0$ is a continuity point of $N(z, gu, .)$, then in view of lemma 1.17, one gets $z = gu$. So that $fu = z = gu$.

This shows that $u$ is coincidence point of the pair $(f, g)$.

Since $fu = gu$ and the pair $(f, g)$ is weakly compatible, therefore $fz = fgu = gfu = gz$.

Now we need to show that $z$ is a common fixed point of $(f, g)$. To accomplish this, we assert that $N(fz, z, t) = 1$, if not then using condition (1)

$$N(fz, z, kt) = N(fz, fu, kt)) \geq$$

$$\{N(gz, gu, t) * N(fz, gz, t) * N(fu, gu, t) * N(fz, gu, t) * N(fu, gz, t)\}$$

$$\geq \{N(fz, z, t) * N(fz, fz, t) * N(z, z, t) * N(fz, z, t)\}$$

$$N(fz, z, kt) \geq N(fz, z, t)$$

If $fz \neq z$, then$0 < N(fz, z, t_0) < 1$, for some $t_0 > 0$. As $N(fz, z, .)$ Is left continuous and $N(fz, z, .)$ is nondecreasing, it has only (at most) countable points of discontinuity. If we

Suppose that $t_0$ is continuity point of $N(fz, z, .)$, then (in view of condition $\phi_2$) and Lemma 1.17

so that $fz = z$.

Which shows that $z$ is common fixed point of the pair $(f, g)$.

**Uniqueness**:- Finally, we will prove that a common fixed point of $f$ and $g$ is unique. Let us suppose that $w$ is common fixed point of $f$ and $g$ in which $w \neq z$. It follows from condition (1)

$$N(z, w, kt) = N(fz, fw, kt)$$
$$\geq \{N(gz, gw, t) * N(fz, gz, t) * N(fw, gw, t) * N(fz, gw, t) * N(fw, gz, t)\}$$
$$N(z, w, kt) \geq (N(z, w, t)$$

From Lemma 1.17, we conclude that $w = z$

**Theorem 2.2.** Let$(X, N,*)$ be GV-fuzzy metric space and $f$ and $g$ be self-mappings of $X$ satisfying the condition

$$N(fx, fy, kt) \geq \{N(gx, gy, t) * N(fx, gx, t) * N(fy, gy, t) * N(fx, gy, t) * N(fy, gx, t)\} \qquad \textbf{(2)}$$

$\forall\, x, y \in X$ , $t > 0$, suppose that the pair $(f, g)$ satisfy the E.A property and $(f, g)$ is weakly compatible then $f$ and $g$ have a unique common fixed point in $X$.

**Proof.** Since $f$ and $g$ satisfy E.A. property, there exists a sequence $\{x_n\}$ in $X$ such that

$\text{Limit}_{n\to\infty} fx_n = \text{Limit}_{n\to\infty} gx_n = z$ , for some $z \in X$

Since $fX$ is closed subset of $X$, therefore $\lim_{n\to\infty} fx_n = z \in fX$. Hence $\exists$ a point $u \in X$, such that $fu = z$, Using condition (2), We have

$$N(gu, fx_n, kt) \geq \{N(gu, gx_n, t) * N(fu, gu, t) * N(fx_n, gx_n, t) * N(fu, gx_n, t) * N(fx_n, gu, t)\}$$

$\forall\, n \in \mathbb{N}$, Which on letting n$\to\infty$ , one gets

$$N(gu, z, kt) \geq \{N(gu, z, t) * N(z, z, t) * N(z, z, t) * N(z, z, t) * N(z, z, t)\}$$
$$N(gu, z, kt) \geq (N(gu, z, t))$$

From Lemma 1.17, we get$z = gu$. So that $fu = z = gu$.

Hence $u$ is coincidence point of the pair $(f, g)$.

Since $fu = gu$ and the pair $(f, g)$ is weakly compatible, therefore $fz = fgu = gfu = gz$.

Now we need to show that $z$ is a common fixed point of $(f, g)$. To accomplish this, We assert that $N(fz, z, t) = 1$, if not then using condition (2)

$$N(fz,z,kt) = N(fz,fu,kt))$$
$$\geq \{N(gz,gu,t) * N(fz,gz,t) * N(fu,gu,t) * N(fz,gu,t) * N(fu,gu,t)\}N(fz,z,kt)$$
$$\geq \{N(fz,z,t) * N(fz,fz,t) * N(z,z,t) * N(fz,z,t)\}$$
$$N(fz,z,kt) \geq \big(N(fz,z,t)\big)$$

From Lemma 1.17 we get $fz = z$.

Which shows that $z$ is common fixed point of the pair $(f,g)$.

**Uniqueness**:- Finally, we will prove that a common fixed point of $f$ and $g$ is unique. Let us suppose that $w$ is common fixed point of $f$ and $g$ in which $w \neq z$. It follows from condition

$$N(z,w,kt) = N(fz,fw,kt)$$
$$\geq \{N(gz,gw,t) * N(fz,gz,t) * N(fw,gw,t) * N(fz,gw,t) * N(fw,gz,t)\}$$
$$N(z,w,kt) \geq \{N(z,w,t) * 1 * 1 * N(z,w,t) * N(w,z,t)\}$$
$$N(z,w,kt) \geq (N(z,w,t))$$

In the view of Lemma 1.17, we conclude that $w = z$

## 3. CLR property and Weakly Compatible Maps

Now we prove our result for weakly compatible maps along with the $CLR_g$ property as follows.

**Theorem 3.1.** Let $(X,N,*)$ be a KM- fuzzy metric space and $f$ and $g$ be pair of self – mapping suppose that

(1) The pair $(f,g)$ enjoy the $(CLR_g)$ property;

(2) For all $x,y \in X$ , $t > 0$ and for some k $\in$ (0,1)

$$N(fx,fy,kt)$$
$$\geq \{N(gx,gy,t) * N(fx,gx,t) * N(fy,gy,t) * N(fx,gy,t) * N(fy,gx,t)\} \qquad \textbf{(3)}$$

Then the pair $(f,g)$ have a coincidence point each. Moreover, $f,g$ have a unique common fixed point provided the pair $(f,g)$ are weakly compatible.

$\forall x,y \in X$, Where $t > 0$. If $f$ and $g$ satisfy the $CLR_g$ property, then $f$ and $g$ have a unique common fixed point.

**Proof.** Since $f$ and $g$ satisfy the$(CLR_g)$ property, there exists a sequence $\{x_n\}$ in $X$ such that

$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = gx$, for some $x \in X$, now we assert that $fx = gx$.

By making use of (3) with $x = x_n$, $y = x$, we get

$$N(fx_n,fx,kt) \geq \{N(gx_n,gx,t) * N(fx_n,gx_n,t) * N(fx,gx,t) * N(fx_n,gx,t) * N(fx,gx_n,t)\}$$

Letting $n \to \infty$

$$N(gx, fx, kt) \geq \{N(gx, gx, t) * N(gx, gx, t) * N(fx, gx, t) * N(gx, gx, t) * N(fx, gx, t)\}$$

$$N(gx, fx, kt) \geq \{N(gx, gx, t) * N(gx, gx, t) * N(fx, gx, t) * N(gx, gx, t) * N(fx, gx, t)\}$$

$$N(fx, gx, kt) \geq \{1 * 1 * N(fx, gx, t) * 1 * N(fx, gx, t)\}$$

$$N(fx, gx, kt) \geq N(fx, gx, t)$$

In view of Lemma 1.17, we have $fx = gx$,

Now we let $z = fx = gx$, since the pair $(f, g)$ is waekly compatible we get $fz = fgx = gfx = gz$. Now we show that $z$ is common fixed point of the mappings $f$ and $g$ .

by making use of (3) with $x = z, y = x$

$$N(fz, fx, kt) \geq \{N(gz, gx, t) * N(fz, gz, t) * N(fx, gx, t) * N(fz, gx, t) * N(fx, gz, t)\}$$

$$\geq \{N(fz, z, t) * 1 * 1 * N(fz, z, t) * N(z, fz, t)\}$$

$$N(fz, z, kt) \geq N(fz, z, t),$$

On employing Lemma 1.17, we get

$z = fz = gz$.

Which shows that z is a common fixed point of the mappings $f$ and $g$.

**Uniqueness**: Let w be another common fixed point of the mappings $f$ and $g$ on using (3) with $x = z, y = w$, we get

$$N(fz, fw, kt) \geq \{N(gz, gw, t) * N(fz, gz, t) * N(fw, gw, t) * N(fz, gw, t) * N(fw, gz, t)\}$$

$$N(z, w, kt) \geq \{N(z, w, t) * N(z, z, t) * N(w, w, t) * N(z, w, t) * N(w, z, t)\}$$

$$N(z, w, kt) \geq N(z, w, t)$$

Appealing to Lemma 1.17, we have $z = w$ therefore the mappings $f$ and $g$ have a unique common fixed point in $X$.

Next, we let $z = fx = gx$ , since $f$ and $g$ are weakly compatible mappings, $fg(x) = gf(x)$

Which implies that $fz = fgx = gfx = gz$ .

Next we claim that $fz = z$ .

Using condition (3), we get

$$N(fz, z, kt) \geq \{N(gz, gx, t) * N(fz, gz, t) * N(fx, gx, t) * N(fz, gx, t) * N(fx, gz, t)\}$$

$$\geq \{N(fz, z, t) * 1 * 1 * N(fz, z, t) * N(z, fz, t)\}$$

$$N(fz, z, kt) \geq N(fz, z, t)$$

Appealing to Lemma 1.17, we have $fz = z$ and then $fz = gz = z$, $f$ and $g$ have a common fixed point that is z.

Now, we will prove that common fixed point of $f$ and $g$ is unique.

Let us suppose that $w$ is another common fixed point in which w$\neq z$.

$$N(z,w,kt) = N(fz,fw,kt)$$
$$\geq \{N(gz,gw,t) * N(fz,gz,t) * N(fw,gw,t) * N(fz,gw,t) * N(fw,gz,t)\}$$
$$N(z,w,kt) \geq \{N(z,w,t) * 1 * 1 * N(z,w,t) * N(w,z,t)\}$$
$$N(z,w,kt) \geq \big(N(z,w,t)\big)$$

Owing to Lemma 1.17, we conclude that $w = z$.

**Theorem 3.2.** Let $(X,N,*)$ be a GV- fuzzy metric space and $f$ and $g$ weakly compatible self –mappings of $X$ such that for some

$$N(fx,fy,kt)$$
$$\geq \{N(gx,gy,t) * N(fx,gx,t) * N(fy,gy,t) * N(fx,gy,t) * N(fy,gx,t)\} \qquad \textbf{(4)}$$

$\forall x,y \in X$, Where $t > 0$. If $f$ and $g$ satisfy the $CLR_g$ property. Then $f$ and $g$ have a unique common fixed point.

**Proof**. It follows from $f$ and $g$ satisfying the $CLR_g$ property that we can find a sequence $\{x_n\}$ in $X$ such that

$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = gx$, foe some $x \in X$

Let t be a continuity point of$(X,N,*)$.

$$N(fx_n,fx,kt) \geq \{N(gx_n,gx,t) * N(fx_n,gx_n,t) * N(fx,gx,t) * N(fx_n,gx,t) * N(fx,gx_n,t)\}$$

Letting n$\to \infty$

$$N(gx,fx,kt) \geq \{N(gx,gx,t) * N(gx,gx,t) * N(fx,gx,t) * N(gx,gx,t) * N(fx,gx,t)\}$$
$$N(fx,gx,kt) \geq \{1 * 1 * N(fx,gx,t) * 1 * N(fx,gx,t)\}$$
$$N(fx,gx,kt) \geq N(fx,gx,t)$$

In the view of Lemma 1.17, we have $gx = fx$

Since $f$ and $g$ are weakly compatible mappings $fgx = gfx$

Which implies $fz = gz$.

Next, we will show that $fz = z$. we will suppose that $fz \neq z$ and using condition (4)

$$N(fz,z,kt) = N(fz,fx,kt)$$
$$\geq \{N(gz,gx,t) * N(fz,gz,t) * N(fx,gx,t) * N(fz,gx,t) * N(fx,gz,t)$$
$$* N(fx,gz,t)\}$$
$$\geq \{N(z,fx,t) * 1 * 1 * N(fz,fx,t) * N(fx,fz,t) * N(fx,fz,t)\}$$
$$N(fz,z,kt) \geq N(fz,z,t)$$

Therefore $fz = z$ , $f$ and $g$ have a common fixed point that is $z$.

Finally, we will prove that a common fixed point of $f$ and $g$ is unique. Let us suppose that w is a common fixed point of $f$ and $g$ in which $w \neq z$ . It follows from condition (4)

$$N(z,w,kt) = N(fz,fw,t)$$
$$\geq \{N(gz,gw,t) * N(fz,gz,t) * N(fw,gw,t) * N(fz,gw,t) * N(fz,gw,t) * N(fw,gz,t)\}$$
$$\geq \{N(z,w,t) * 1 * 1 * N(z,w,t) * N(z,w,t) * N(w,z,t)\}$$
$$\geq \big(N(z,w,t)\big)$$

From Lemma 1.17, we conclude that $w = z$.

**Example 3.1.G** Let $(X, M, *)$ be a GV-fuzzy metric space wherein $X = \left[0, \frac{1}{2}\right]$, $a * b = ab$ with $M(x,y,t) = \frac{t/_2}{t/_2 + |x-y|}$ for all $t > 0$.

Define self-mappings $f, g, S \; and \; T on \; X$ by

$$f(x) = \begin{cases} \frac{1}{2} & if\ x \in \left[0,\frac{1}{2}\right] \cap Q \\ \frac{1}{4} & if\ x \notin [0,\frac{1}{2}] \cap Q \end{cases}, \qquad g(x) = \begin{cases} \frac{1}{2} & if\ x \in \left[0,\frac{1}{2}\right] \cap Q \\ \frac{1}{8} & if\ x \notin [0,\frac{1}{2}] \cap Q \end{cases}$$

$$S(x) = \begin{cases} \frac{1}{2} & if\ x \in \left[0,\frac{1}{2}\right] \cap Q \\ 0 & if\ x \notin \left[0,\frac{1}{2}\right] \cap Q \end{cases} \quad \text{and} \quad T(x) = \begin{cases} \frac{1}{2} & if\ x \in \left[0,\frac{1}{2}\right] \cap Q \\ 0 & if\ x \notin \left[0,\frac{1}{2}\right] \cap Q \end{cases}$$

Then $f(X) = \left\{\frac{1}{2}, \frac{1}{4}\right\} \not\subset \left\{0, \frac{1}{2}\right\} = S(X) \; and \; g(X) = \left\{\frac{1}{2}, \frac{1}{8}\right\} \not\subset \left\{0, \frac{1}{2}\right\} = T(X)$

Now, if we take $x \in [0,\frac{1}{2}] \cap Q \quad and \quad y \notin [0,\frac{1}{2}] \cap Q$

$$M(fx,gy,t) \geq \varphi(\min\{M(Sx,Ty,t), M(fx,Sx,t), M(gy,Ty,t), M(fx,Ty,t), M(Sx,gy,t)\})$$

$$M\left(\frac{1}{2},\frac{1}{8},t\right) \geq \varphi\left(\min\left\{M\left(\frac{1}{2},0,t\right), M\left(\frac{1}{2},\frac{1}{2},t\right), M\left(\frac{1}{8},0,t\right), M\left(\frac{1}{2},0,t\right), M\left(\frac{1}{2},\frac{1}{8},t\right)\right\}\right)$$

Or

$$\frac{t/_2}{t/_2 + |\frac{1}{2} - \frac{1}{8}|} \geq \varphi(\min\left\{\frac{\frac{t}{2}}{\frac{t}{2} + \left|\frac{1}{2} - 0\right|}, \frac{\frac{t}{2}}{\frac{t}{2} + \left|\frac{1}{2} - \frac{1}{2}\right|}, \frac{\frac{t}{2}}{\frac{t}{2} + \left|\frac{1}{8} - 0\right|}, \frac{\frac{t}{2}}{\frac{t}{2} + \left|\frac{1}{2} - 0\right|}, \frac{\frac{t}{2}}{\frac{t}{2} + \left|\frac{1}{2} - \frac{1}{8}\right|}\right\})$$

$$\frac{t/_2}{t/_2 + \frac{3}{4}} \geq \varphi\left(\min\left\{\frac{\frac{t}{2}}{\frac{t}{2} + \frac{1}{2}}, \frac{\frac{t}{2}}{\frac{t}{2}}, \frac{\frac{t}{2}}{\frac{t}{2} + \frac{1}{8}}, \frac{\frac{t}{2}}{\frac{t}{2} + \frac{1}{2}}, \frac{\frac{t}{2}}{\frac{t}{2} + \frac{3}{4}}\right\}\right)$$

$$\geq \varphi\left(\frac{\frac{t}{2}}{\frac{t}{2} + \frac{1}{2}}\right) > \frac{\frac{t}{2}}{\frac{t}{2} + \frac{1}{2}}$$

Which is true for all $t > 0$ $\big(here\ \varphi(s) = \sqrt{s}\big)$. Thus all the condition of Theorem 4 are satisfied. Notice that $\frac{1}{2}$ is the unique common fixed point of all the involved mappings.

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