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Symmetric Generalized Bi-(σ,τ)-Reverse Derivations in Prime and Semiprime Rings

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Abstract: Let R be a 2-torision free prime and semiprime ring and G be a symmetric generalized bi-(σ,τ)- reverse derivation of R with associated symmetric bi-(σ,τ)- reverse derivation$B$. (i) If $G([x,y],[u,v])=0$ for all $x,y,u,v\in R$ (ii) If $G((xoy),[u,v])=0$ for all $x,y,u,v\in R$ (iii) If $G([x,y],(uov))=0$ for all $x,y,u,v\in R$ (iv) If $G\big((xoy),(uov)\big)=0$ for all $x,y,u,v\in R$, then either R is commutative or G acts as a bi-multiplier.

Keywords: Prime ring, Semiprime ring, Symmetric bi- (σ,τ) -reverse derivation, Symmetric generalized bi-(σ,τ)-reverse derivation.

1. INTRODUCTION

The notion of reverse derivation has been introduced by Bresar and Vukman in [4] and the reverse derivations on semi prime rings have been studied by Samman and Alyamani in [13]. The concept of a symmetric bi-derivation has been introduced Maksa in [10], [11]. Ozturk and Jun in [12] have introduced the concept of symmetric bi-derivation of a near ring and studied some properties. The notion of generalized symmetric biderivations introduced by Argac [2]. Ceven and Ozturk in [5] introduce the concepts of symmetric bi-(σ,τ)-derivation of a near ring and gave some properties. In 2011, the concepts of (θ, φ)-reverse derivation and generalized (θ, φ)-reverse derivation has been introduced by Anwar Khaleel Faraj in [1]. In 2016, Jaya Subba Reddy et.al in [6, 8] has studied symmetric reverse bi-derivations on prime rings and semiprime rings, Results of Symmetric Reverse Bi-Derivations on Prime Rings [9] and also proved important results on Symmetric Bi-(σ,τ)-Derivations in Prime Rings [7]. In 2018, Asma Ali et.al in [3] has proved some results on generalized biderivations on prime and semiprime rings. In this paper we proved some results on symmetric generalized bi-(σ,τ)-reverse derivations in prime and semiprime rings.

2. PRELIMINARIES

Throughout this paper R will represent an associative ring with center $Z(R)$. A ring R is said to be prime (resp. semiprime) if $aRb = 0$ implies that either $a = 0$ or $b = 0$ (resp. $aRa = 0$ implies that $a = 0$). A ring R to be 2-torision free if $2x = 0$ implies$x = 0$, for all $x \in R$. For any $x, y \in R$, the symbol $[x, y]$ stands for the commutator $xy - yx$ and the symbol (xoy) stands for the anti commutator $xy + yx$. A mapping $B(.,.): R \times R \to R$ is called a symmetric if $B(x,y) = B(y,x)$, for all $x, y, z \in N$. A mapping $b: R \to R$ is said to be trace of B if $b(x) = B(x,x)$, for all $x \in R$, where $B(.,.): R \times R \to R$ is a symmetric bi-additive mapping. The trace of B satisfies the relation $b(x + y) = b(x) + 2B(x,y) + b(y)$, for all $x, y \in R$. An additive mapping $b: R \to R$ is called a reverse derivation if $b(xy) = b(y)x + yb(x)$, for all $x, y \in R$. A symmetric bi-additive mapping $B(.,.): R \times R \to R$ is called a symmetric bi-reverse derivation if $B(xz,y) = B(z,y)x + zB(x,y)$, for all $x, y, z \in R$. An additive mapping $b: R \to R$ is called a (σ,τ)-reverse derivation if $b(xy) = b(y)\sigma(x) + \tau(y)b(x)$, for all $x, y \in R$, where σ and τ to be a automorphisms of R. A symmetric bi-additive mapping $B(.,.): R \times R \to R$ is called a symmetric bi-(σ,τ)-reverse derivation if there exists functions $\sigma, \tau: R \to R$ such that $B(xz,y) = B(z,y)\sigma(x) + \tau(y)B(x,y)$, for all $x, y, z \in R$. An additive mapping $F: R \to R$ is called a generalized reverse derivation, if there exists a reverse derivation $b: R \to R$ such that $F(xy) = F(y)x + yb(x)$, for all $x, y \in R$. A symmetric bi-additive mapping $G(.,.): R \times R \to R$ is called a symmetric generalized bi-reverse derivation if there exists a symmetric bi-reverse derivation $B(.,.): R \times R \to R$ such that $G(xz,y) = G(z,y)x + zB(x,y)$, for all $x, y, z \in R$. An additive mapping $F: R \to R$ is said to be a generalized (σ,τ)-reverse derivation of R, if there exists a (σ,τ)-reverse derivation $b: R \to R$ such that $F(xy) = F(y)\sigma(x) + \tau(y)b(x)$, for all $x, y \in R$, where σ and τ are automorphisms of R. A symmetric bi-additive mapping $G(.,.): R \times R \to R$ is called a symmetric generalized bi-(σ,τ)-reverse derivation if there exists a symmetric bi-(σ,τ)-reverse derivation $B(.,.): R \times R \to R$ such that $G(xz,y) = G(z,y)\sigma(x) + \tau(z)B(x,y)$, for all $x, y, z \in R$, where σ and τ to be a automorphisms of R. An additive mapping $h: R \to R$ is called left (resp. right) multiplier of R if $h(xy) = h(x)y$ (resp. $h(xy) = xh(y)$) for all $x, y \in R$. A biadditive mapping $H: R \times R \to R$ is said to be a left (resp. right) bi-multiplier of R if $H(x,yz) = H(x,y)z$ (resp. $H(xz,y) = xH(z,y)$) for all $x, y, z \in R$. Throughout the present paper, we will make extensive use of the following basic identities:

$[x, yz] = y[x,z] + [x,y]z,$

$[xy, z] = [x,z]y + x[y,z],$

$xo(yz) = (xoy)z - y[x,z] = y(xoz) + [x,y]z,$

$(xy)oz = x(yoz) - [x,z]y = (xoz)y + x[y,z].$

3. MAIN RESULTS

Theorem 3.1: Let R be a 2-torision free semiprime ring and G be a symmetric generalized bi- (σ,τ)-reverse derivation of R with associated symmetric bi-(σ,τ)-reverse derivation B. If $G([x,y],[u,v]) = 0$ for all $x,y,u,v \in R$, then either R is commutative or G acts as a bi-multiplier.

Proof: We have $G([x,y],[u,v]) = 0$ for all $x,y,u,v \in R$. (3.1)

Replacing y by xy in equation (3.1), we get $G(x[x,y],[u,v]) = 0$ for all $x,y,u,v \in R$.

$G([x,y],[u,v])\sigma(x) + \tau[x,y]B(x,[u,v]) = 0$ for all $x,y,u,v \in R$.

Using equation (3.1) in the above equation, we get

$\tau[x,y]B(x,[u,v]) = 0$ for all $x,y,u,v \in R$. (3.2)

Replacing u by vu in equation (3.2), we get

$\tau[x,y]B(x,v[u,v]) = 0$ for all $x,y,u,v \in R$.

$\tau[x,y]B(x,[u,v])\sigma(v) + \tau[x,y]\tau[u,v]B(x,v) = 0$ for all $x,y,u,v \in R$.

Using equation (3.2) in the above equation, we get

$\tau[x,y]\tau[u,v]B(x,v) = 0$ for all $x,y,u,v \in R$. (3.3)

Substitute yr for y in equation (3.3), we get

$\tau(y)\tau[x,r]\tau[u,v]B(x,v) + \tau[x,y]\tau(r)\tau[u,v]B(x,v) = 0$ for all $x,y,u,v,r \in R$.

Using equation (3.3) in the above equation, we get

$\tau[x,y]\tau(r)\tau[u,v]B(x,v) = 0$ for all $x,y,u,v,r \in R$. (3.4)

Replacing $\tau(r)$ by $B(x,v)\tau(r)$ in equation (3.4), we get

$\tau[x,y]B(x,v)\tau(r)\,\tau[u,v]B(x,v) = 0$ for all $x,y,u,v,r \in R$.

$\tau[x,y]B(x,v)R\tau[u,v]B(x,v) = 0$ for all $x,y,u,v \in R$. (3.5)

Using semi primeness of R, we have either $\tau[x,y] = 0$ or $B(x,v) = 0$ for all $x,y,v \in R$.

If $\tau[x,y] = 0$, then R is commutative. In the later case we get G acts as a bi-multiplier.

Corollary 3.1: Let R be a 2-torision free semiprime ring and G be a symmetric generalized bi- (σ,τ)-reverse derivation of R with associated symmetric bi-(σ,τ)-reverse derivation B. If $G([x,y],z) = 0$ for all $x,y,z \in R$, then either R is commutative or G acts as a bi-multiplier.

Theorem 3.2: Let R be a 2-torision free semiprime ring and G be a symmetric generalized bi- (σ,τ)-reverse derivation of R with associated symmetric bi-(σ,τ)-reverse derivation B. If $G((xoy),[u,v]) = 0$ for all $x,y,u,v \in R$, then either R is commutative or G acts as a bi-multiplier.

Proof: We have $G((xoy),[u,v])=0$ for all $x,y,u,v\in R$. (3.6)

Replacing y by xy in equation (3.6), we get $G(x(xoy),[u,v])=0$ for all $x,y,u,v\in R$.

$G((xoy),[u,v])\sigma(x)+\tau(xoy)B(x,[u,v])=0$ for all $x,y,u,v\in R$.

Using equation (3.6) in the above equation, we get

$\tau(xoy)B(x,[u,v])=0$ for all $x,y,u,v\in R$. (3.7)

Replacing u by vu in equation (3.7), we get

$\tau(xoy)B(x,v[u,v])=0$ for all $x,y,u,v\in R$.

$\tau(xoy)B(x,[u,v])\sigma(v)+\tau(xoy)\tau[u,v]B(x,v)=0$ for all $x,y,u,v\in R$.

Using equation (3.7) in the above equation, we get

$\tau(xoy)\tau[u,v]B(x,v)=0$ for all $x,y,u,v\in R$. (3.8)

Substitute yr for y in equation (3.8), we get

$\tau(y)\tau(xor)\tau[u,v]B(x,v)+\tau[x,y]\tau(r)\tau[u,v]B(x,v)=0$ for all $x,y,u,v,r\in R$.

Using equation (3.8) in the above equation, we get

$\tau[x,y]\tau(r)\tau[u,v]B(x,v)=0$ for all $x,y,u,v,r\in R$ (3.9)

The equation (3.9) is same as equation (3.4) in theorem 3.1. Hence the result is proved.

Theorem 3.3: Let R be a 2-torision free prime ring and G be a symmetric generalized bi- (σ,τ) -reverse derivation of R with associated symmetric bi- (σ,τ) -reverse derivation B . If $G([x,y],(uov))=0$ for all $x,y,u,v\in R$, then either R is commutative or G acts as a bi-multiplier.

Proof: We have $G([x,y],(uov))=0$ for all $x,y,u,v\in R$. (3.10)

Replacing y by xy in equation (3.10), we get $G(x[x,y],(uov))=0$ for all $x,y,u,v\in R$.

$G([x,y],[u,v])\sigma(x)+\tau[x,y]B(x,(uov))=0$ for all $x,y,u,v\in R$.

Using equation (3.10) in the above equation, we get

$\tau[x,y]B(x,(uov))=0$ for all $x,y,u,v\in R$. (3.11)

Replacing u by vu in equation (3.11), we get

$\tau[x,y]B(x,v(uov))=0$ for all $x,y,u,v\in R$.

$\tau[x,y]B(x,(uov))\sigma(v)+\tau[x,y]\tau(uov)B(x,v)=0$ for all $x,y,u,v\in R$.

Using equation (3.11) in the above equation, we get

$\tau[x,y]\tau(uov)B(x,v)=0$ for all $x,y,u,v\in R$. (3.12)

Substitute yr for y in equation (3.12), we get

$\tau(y)\tau[x,r]\tau(uov)B(x,v)+\tau[x,y]\tau(r)\tau(uov)B(x,v)=0$ for all $x,y,u,v,r\in R$.

Using equation (3.12) in the above equation, we get

$\tau[x,y]\tau(r)\tau(uov)B(x,v)=0$ for all $x,y,u,v,r\in R$. (3.13)

Replacing $\tau(r)$ by $B(w,z)\tau(r)$ in equation (3.13), we get

$\tau[x,y]B(w,z)\tau(r)\tau(uov)B(x,v)=0$ for all $x,y,u,v,r,w,z\in R$.

$\tau[x,y]B(w,z)R\tau(uov)B(x,v)=0$ for all $x,y,u,v,w,z\in R$. (3.14)

Using primeness of R, we have either $\tau[x,y]B(w,z)=0$ or $\tau(uov)B(x,v)=0$ for all $x,y,u,v\in R$.

Now first suppose that $\tau[x,y]B(w,z)=0$ for all $x,y,w,z\in R$. (3.15)

Substituting ys instead of y in equation (3.15), we get

$\tau(y)\tau[x,s]B(w,z)+\tau[x,y]\tau(s)B(w,z)=0$ for all $x,y,w,z,s\in R$.

Using equation (3.15) in the above equation, we get

$\tau[x,y]\tau(s)B(w,z)=0$ for all $x,y,w,z,s\in R$.

$\tau[x,y]RB(w,z)=0$ for all $x,y,w,z\in R$. (3.16)

Using primeness of R, then either $\tau[x,y]=0$ or $B(w,z)=0$ for all $x,y,w,z\in R$.

If $\tau[x,y]=0$, then R is commutative. In the later case G acts as a bi-multiplier.

In second case suppose that $\tau(uov)B(x,v)=0$ for all $x,u,v\in R$. (3.17)

Substituting $\tau(uov)\tau(t)$ instead of $\tau(uov)$ in equation (3.17), we get

$\tau(uov)\tau(t)B(x,v)=0$ for all $x,u,v,t\in R$.

$\tau(uov)RB(x,v)=0$ for all $x,u,v\in R$. (3.18)

Using primeness of R, then either $\tau(uov)=0$ or $B(x,v)=0$ for all $x,u,v\in R$.

If $\tau(uov)=0$, then R is commutative. In the later case we get G acts as a bi-multiplier.

Theorem 3.4: Let R be a 2-torision free prime ring and G be a symmetric generalized bi- (σ,τ) -reverse derivation of R with associated symmetric bi- (σ,τ) -reverse derivation B . If $G\big((xoy),(uov)\big)=0$ for all $x,y,u,v\in R$, then either R is commutative or G acts as a bi-multiplier.

Proof: We have $G((xoy),(uov))=0$ for all $x,y,u,v\in R$. (3.19)

Replacing y by xy in equation (3.19), we get $G(x(xoy),(uov))=0$ for all $x,y,u,v\in R$.

$G((xoy),(uov))\sigma(x)+\tau(xoy)B(x,(uov))=0$ for all $x,y,u,v\in R$.

Using equation (3.19) in the above equation, we get

$\tau(xoy)B(x,(uov))=0$ for all $x,y,u,v\in R$. (3.20)

Replacing u by vu in equation (3.20), we get

$\tau(xoy)B(x, v(uov)) = 0$ for all $x, y, u, v \in R$.

$\tau(xoy)B\big(x, (uov)\big)\sigma(v) + \tau(xoy)\tau(uov)B(x, v) = 0$ for all $x, y, u, v \in R$.

Using equation (3.20) in the above equation, we get

$\tau(xoy)\tau(uov)B(x, v) = 0$ for all $x, y, u, v \in R$. (3.21)

Substitute yr for y in equation (3.21), we get

$\tau(y)\tau(xor)\tau(uov)B(x, v) + \tau[x, y]\tau(r)\tau(uov)B(x, v) = 0$ for all $x, y, u, v, r \in R$.

Using equation (3.21) in the above equation, we get

$\tau[x, y]\tau(r)\tau(uov)B(x, v) = 0$ for all $x, y, u, v, r \in R$. (3.22)

The equation (3.22) is same as equation (3.13) in theorem 3.2. Hence the result is proved.

Corollary 3.2: Let R be a 2-torision free prime ring and G be a symmetric generalized bi- (σ, τ)-reverse derivation of R with associated symmetric bi-(σ, τ)-reverse derivation B. If $G\big((xoy), z\big) = 0$ for all $x, y, z \in R$, then either R is commutative or G acts as a bi-multiplier.

REFERENCES

[1] Anwar Khaleel Faraj.: On Generalizd (θ,φ)-Reverse Derivations of Prime rings, Iraqi Journal of Science, 52(2) (2011), 218-224.

[2] Argac, N.: Generalized derivation of prime rings, Algebra Colloq., 11(3) (2004), 399-400.

[3] Asma Ali, Faiza Shujat and Kapil Kumar.: Generalized biderivation on prime and semi prime rings, International Journal of Computational and Applied Mathematics, 13(1) (2018), 33-41.

[4] Bresar, M. and Vukman, J.: On some additive mappings in rings with involution, an equations math., 38 (1989), 178-185.

[5] Ceven,Y. and Ozturk, M.A.: Some Properties of Symmetric bi-(σ, τ)-derivations in near-rings, Commmun.Korean Math. Soc., 22(4) (2007), 487-491.

[6] Jaya Subba Reddy, C. and Rama Krishna Naik, M.: Symmetric Reverse Bi-Derivations on prime rings, Research J. Pharm. and Tech, 9(9) (2016), 1496-1500.

[7] Jaya Subba Reddy, C. and Mallikarjunarao, S.: Some Results on Symmetric Bi-(σ, τ) Derivations in Prime Rings**,** Open Journal of Applied & Theoretical Mathematics, 2(4) (2016), 973-981.

[8] Jaya Subba Reddy, C., Venkat Bhaskara Rao, G. and Ramoorthy Reddy, B.: Symmetric Reverse (θ, φ) - Bi-Derivations on Prime and Semiprime Rings-Global Journal of Pure and Applied Mathematics, 12(3) (2016), 386-391.

[9] Jaya Subba Reddy, C., Sivakameshwara Kumar, A. and Ramoorthy Reddy, B.: Results of Symmetric Reverse Bi-Derivations on Prime Rings, Annals of Pure and Applied Mathematics, 16(1) (2018), 1-6.

[10] Maksa, Gy. A remark on symmetric bi additive functions having non-negative diagonalization, Glasnik Mat. 15(35) (1980), 279-282.

[11] Maksa,Gy.: On the trace of symmetric bi-derivations., C.R. Math. Rep. Acad. Sci. Canada, 9 (1987), 302-307.

[12] Ozturk, M.A. and Jun,Y.B.: On the trace of symmetric bi-derivations in near-rings, Inter.J.Pure and Appl.Math, 17(1) (2004), 9-102.

[13] Samman, M. and Alyamani, N.: Derivations and reverse derivations in semi prime rings, International Mathematical Forum, 2 (39), (2007), 1895-1902.