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Coupled Fixed Point Theorems in S_b Metric Space

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Abstract

In this research paper some coupled fixed point theorems for occasionally weakly compatible mappings in S_b metric space are proved. An example is given to support results. As an application, the existence of a solution of integral equation is also investigated.

Keywords: Occasionally weakly compatible mappings, coupled fixed point, S_b metric space**.**2000 Mathematics Subject Classification: 47H10; 54H25.

1. INTRODUCTION

Bakhtin introduced the concept of b-metric spaces as a generalization of metric spaces [2]. In these spaces, the triangular inequality of the usual metric function was replaced by a more general inequality consisting a constant b≥1 such that for b=1 we obtain the usual metric as a special case. Later several authors proved so many results on b-metric spaces. Mustafa and Sims defined the concept of a generalized metric space which is called a G metric space [9]. Sedghi, Shobe and Aliouche gave the notion of an S metric space and proved some fixed point theorems for a self-mapping on a complete S metric space [12]. Aghajani, Abbas and Roshan presented a new type of metric which is called $G_b$metric and studied some properties of this metric [1]. Recently Sedghi et al. [14] defined $S_b$metric spaces using the concept of S-metric spaces [12]. The concepts of coupled fixed points and mixed monotone property was recently introduced by Bhaskar and Lakshmikantham [3]. Jungck and Rhoades [7] proved some common fixed point theorems for occasionally weakly compatible maps. Shukla and Nigam [15] introduced occasionally weakly compatible mappings for coupled fixed point.In this paper some coupled fixed point theorems are proved for occasionally weakly compatible mappings in S_b metric space. The examples and application in support of results are also given.

2. Preliminary Notes

Definition 2.1[4] Let X be a nonempty set, $b \geq 1$ be a given real number and $d : X \times X \to [0,\infty)$ be a function satisfying the following conditions for all $x, y, z \in X$

(i) $d(x,y) = 0$ if and only if $x = y$.
(ii) $d(x,y) = d(y,x)$.
(iii) $d(x,z) \leq b[d(x,y) + d(y,z)]$.

Then the function d is called a $b - metric$ on X and the pair (X,d) is called a $b - metric\ space$.

Definition 2.2[9] Let X be a nonempty set and $G : X \times X \times X \to [0,\infty)$ be a function satisfying the following conditions.

(i) $G(x,y,z) = 0\ if\ x = y = z$.
(ii) $0 < G(x,x,y)$ for all $x, y \in X$ with $x \neq y$.
(iii) $G(x,x,y) \leq G(x,y,z)$ for all $x, y, z \in X$ with $y \neq z$.
(iv) $G(x,y,z) = G(x,z,y) = G(y,z,x) = \cdots$.
(v) $G(x,y,z) \leq G(x,a,a) + G(a,y,z)$ for all $x, y, z, a \in X$.

Then the function G is called a generalized metric or a $G - metric$ on X and the pair (X,G) is called a $G - metric\ space$.

Definition 2.3[1] Let X be a nonempty set, $b \geq 1$ be a given real number and $G_b: X \times X \times X \to [0,\infty)$ be a function satisfying the following conditions.

(i) $G_b(x,y,z) = 0$ if $x = y = z$.
(ii) $0 < G_b(x,x,y)$ for all $x, y \in X$ with $x \neq y$.
(iii) $G_b(x,x,y) \leq G_b(x,y,z)$ for all $x, y, z \in X$ with $y \neq z$.
(iv) $G_b(x,y,z) = G_b(x,z,y) = G_b(y,z,x) = \cdots$.
(v) $G_b(x,y,z) \leq b[G_b(x,a,a) + G_b(a,y,z)]$ for all $x, y, z, a \in X$.

Then the function G_b is called a generalized $b - metric$ or a G_b metric on X and the pair (X, G_b) is called a G_b-$metric\ space$.

Definition 2.4[12] Let X be a nonempty set and $S : X \times X \times X \to [0,\infty)$ be a function satisfying the following conditions for all $x, y, z, a \in X$.

(i) $S(x,y,z) = 0$ if and only if $x = y = z$.
(ii) $S(x,y,z) \leq S(x,x,a) + S(y,y,a) + S(z,z,a)$.

Then the function S is called an $S -$metric on X and the pair (X,S) is called an $S - metric\ space$.

Lemma 2.5[12] Let (X,S) be an $S - metric\ space$. Then we have

$$S(x,x,y) = S(y,y,x).$$

Definition 2.6[14] Let X be a nonempty set and $b \geq 1$ be a given real number. A function S_b: $X \times X \times X \rightarrow [0,\infty)$ is said to be S_b-metric if and only if for all $x,y,z,a \in X$ the following conditions are satisfied:

(i) $S_b(x,y,z) = 0$ if and only if $x = y = z$,
(ii) $S_b(x,y,z) \leq b[S_b(x,x,a) + S_b(y,y,a) + S_b(z,z,a)]$.

The pair (X,S_b) is called an S_b-metric space.

S_b -metric spaces are the generalizations of S-metric spaces since every S-metric is an S_b-metric with $b = 1$.

Example 2.7 Let $X = R$ and the function S_b be defined as

$$S_b(x,y,z) = \frac{1}{16}((|x - y| + |y - z| + |x - z|)^2)$$

Then the function is an S_b-metric with $b = 4$, but it is not an S-metric. Indeed, for $x = 4, y = 6, z = 8$ and $a = 5$ we get

$$S_b(4,6,8) = 4, S_b(4,4,5) = \frac{1}{4}, S_b(6,6,5) = \frac{1}{4}, S_b(8,8,5) = \frac{9}{4}$$

Hence we have

$$S_b(4,6,8) = 4 \leq S_b(4,4,5) + S_b(6,6,5) + S_b(8,8,5) = \frac{11}{4}$$

Which is a contradiction with (ii) of Definition 2.4.

Definition 2.8[14] Let (X,S_b) be an S_b-metric space and $b > 1$. An S_b-metric S_b is called symmetric if

(i) $S_b(x,x,y) = S_b(y,y,x)$, for all $x,y \in X$.

Lemma 2.9 Let (X,S_b) be an S_b-metric space, S_b be a symmetric S_b −metric with $b \geq 1$ and the function $d : X \times X \rightarrow [0,\infty)$ be defined by

$$d(x,y) = S_b(x,x,y),$$

For all $x,y \in X$. Then d is a b −metric on X.

Lemma 2.10 Let (X,d) be a b −metric space with $b \geq 1$ and the function $S_b : X \times X \times X \rightarrow [0,\infty)$ be defined by

$$S_b(x,y,z) = d(x,z) + d(y,z),$$

For all $x,y,z \in X$. Then S_b is an S_b −metric on X.

Definition 2.11[3] An element $(x,y) \in X \times X$ is called a

(i) Coupled fixed point of the mapping $f: X \times X \rightarrow X$ if
(ii) $f(x,y) = x,\ f(y,x) = y$.
(iii) Coupled coincidence point of the mapping $f: X \times X \rightarrow X$ and $g: X \rightarrow X$ if
(iv) $f(x,y) = g(x),\ f(y,x) = g(y)$.
(v) Common Coupled coincidence point of the mapping $f: X \times X \rightarrow X$ and $g: X \rightarrow$

X if

$$x = f(x,y) = g(x),\ y = f(y,x) = g(y)\ .$$

Definition 2.12[3] An element $x \in X$ is called a common coupled fixed point of the mappings $f: X \times X \to X$ and $g: X \to X$ if

$$x = f(x,x) = g(x).$$

Definition 2.13[3] Let $A, B: X \times X \to X$ and $S, T: X \to X$ be four mappings. Then, the pair of maps (B, S) and (A, T) are said to have Common Coupled coincidence point if there exist a, b in X such that

$$B(a,b) = S(a) = T(a) = A(a,b) \text{ and } B(b,a) = S(b) = T(b) = A(b,a).$$

Definition 2.14[15] The mappings $f: X \times X \to X$ and $g: X \to X$of a set X are occasionally weakly compatible (owc) iff there is a point $(x,y) \in X \times X$ which is a coincidence point of f and g at which f and g commute i.e. (f, g) are occasionally weakly compatible maps iff $f(x,y) = g(x),\ f(y,x) = g(y)$

implies $gf(x,y) = f(gx, gy),\ gf(y,x) = f(gy, gx)$ for $(x,y) \in X \times X$.

Example 2.15 Let $X = R$ and the function $S_b$be defined as

$$S_b(x,y,z) = \frac{1}{16}((|x - y| + |y - z| + |x - z|)^2)$$

Then the function is an S_b-metric with $b = 4$.

Let $f: X \times X \to X$ & $g: X \to X$ be defined by

$$f(x,y) = \frac{2x + y}{2}$$

$$g(x) = \begin{cases} x, & \text{if } 0 \le x < 1; \\ \frac{3}{2}, & \text{if } x \ge 1. \end{cases}$$

Here, (0, 0) and (1, 1) are two coincidence points of f and g. That is $f(0,0) = 0 = g(0), f(1,1) = 1 = g(1)$ but $gf(0,0) = 0 = f(g0, g0),\ gf(1,1) \neq f(g1, g1)$. Thus f and g are owc but not weakly compatible.

Example 2.16 Let $X = R$ and the function $S_b$be defined as

$$S_b(x,y,z) = \frac{1}{16}((|x - y| + |y - z| + |x - z|)^2)$$

Then the function is an S_b-metric with $b = 4$.

Let $f: X \times X \to X$ & $g: X \to X$ be defined by

$$f(x,y) = \frac{x+y}{2}$$

$$g(x) = \begin{cases} \frac{1}{2}, & \text{if } 0 \le x < 1; \\ \frac{x}{2}, & \text{if } x = 1; \\ 3, & \text{if } 1 < x \le 2; \\ x-1, & \text{if } x > 2. \end{cases}$$

Here, (1, 0), (0,1), (2,4) and (4,2) are coincidence points of f and g. That is

$f(1,0) = \frac{1}{2} = g(1), f(0,1) = \frac{1}{2} = g(0)$ and $f(2,4) = 3 = g(2), f(4,2) = 3 = g(4)$ but

$gf(1,0) = \frac{1}{2} = f(g1, g0)$, $gf(0,1) = \frac{1}{2} = f(g0, g1)$, $gf(2,4) \ne f(g2, g4)$, $gf(4,2) \ne f(g4, g2)$.

Thus f and g are *owc* but not weakly compatible.

3. Main Results

Theorem: 3.1 Let (X, S_b) be a $S_b$metric space with $b \ge 1$ and $A, B: X \times X \to X$ and $S, T: X \to X$ be four self-mappings satisfying the following conditions:

(i) $S_b(A(x,y), A(x,y), B(u,v) \le \frac{q}{b^4} max\{S_b(Sx, Sx, Tu), S_b(A(x,y), A(x,y), Sx), S_b(B(u,v), B(u,v), Tu)\}$
for all $x, y, u, v \in X$, $0 < q < 1$ and $b \ge \frac{3}{2}$

(ii) $y = B(x,y)$

Moreover if the pairs (A, S) and (B, T) are owc, then there exists a unique point x in X such that $A(x,x) = T(x) = B(x,x) = S(x) = x$.

Proof: Since the pairs (A, S) and (B, T) are owc so there are points a, b, a', b' in X such that

$A(a,b) = Sa$, $A(b,a) = Sb$ and

$B(a', b') = Ta'$, $B(b', a') = Tb'$

We claim that $Sa = Ta'$. If not, by inequality (i) we get

$S_b(A(a,b), A(a,b), B(a',b') \le \frac{q}{b^4} max\{S_b(Sa, Sa, Ta'), S_b(A(a,b), A(a,b), Sa), S_b(B(a',b'), B(a',b'), Ta')\}$

or $S_b(Sa, Sa, Ta') \le \frac{q}{b^4} max\{S_b(Sa, Sa, Ta'), S_b(Sa, Sa, Sa), S_b(Ta', Ta', Ta')\}$

$$\le \frac{q}{b^4} max\{S_b(Sa, Sa, Ta'), 0, 0\}$$

$$\le \frac{q}{b^4} S_b(Sa, Sa, Ta') \le q\, S_b(Sa, Sa, Ta')$$

$$\Rightarrow Sa = Ta'$$

Therefore $A(a,b) = Ta' = Sa = B(a',b')$

Similarly $A(b,a) = Tb' = Sb = B(b',a')$

Thus the pairs (A,S) and (B,T) have common coincidence points.

Let $A(a,b) = Ta' = Sa = B(a',b') = x$

and $A(b,a) = Tb' = Sb = B(b',a') = y$

Since (A,S) and (B,T) are owc

So $Sx = SA(a,b) = A(Sa,Sb) = A(x,y)$

and $Sy = SA(b,a) = A(Sb,Sa) = A(y,x)$

Also $Tx = TB(a',b') = B(Ta',Tb') = B(x,y)$

and $Ty = TB(b',a') = B(Tb',Ta') = B(y,x)$

Next we show that $x = y$, for this

putting $x = a$, $y = b$, $u = b'$, $v = a'$ in (ii),

$$S_b(x,x,y) = S_b(A(a,b),A(a,b),B(b',a'))$$

$$\leq \frac{q}{b^4} max\{S_b(Sa,Sa,Tb'),S_b(Sa,Sa,Sa),S_b(Tb',Tb',Tb')\}$$

$$\leq S_b(Sa,Sa,Tb') \leq S_b(x,x,y)$$

$$\Longrightarrow x = y$$

Now we prove that $Sx = Tx$

$$S_b(Sx,Sx,Tx) = S_b(Sx,Sx,Ty) = S_b(A(x,y),A(x,y),B(y,x))$$

$$\leq \frac{q}{b^4} max\{S_b(Sx,Sx,Ty),S_b(A(x,y),A(x,y),Sx),S_b(B(y,x),B(y.x),Ty)\}$$

$$\leq \frac{q}{b^4} max\{S_b(Sx,Sx,Tx),S_b(Sx,Sx,Sx),S_b(Ty,Ty,Ty)\}$$

$$\leq S_b(Sx,Sx,Tx)$$

$$\leq S_b(x,x,y)$$

$$\Longrightarrow Sx = Tx$$

Also by condition (ii) we have,

$$x = B(x,x)$$

Thus $A(x,x) = T(x) = B(x,x) = S(x) = x.$

Example 3.1.1 Let $X = [0,1]$ and the function S_b be defined as

$$S_b(x,y,z) = \frac{1}{16}((|x - y| + |y - z| + |x - z|)^2)$$

Then the function is an S_b-metric with $b = 4$.

Let $S, T: X \to X$ and $A, B: X \times X \to X$ defined by

$$A(x,y) = \frac{x+y}{16} \qquad S(x) = \begin{cases} x, & \text{if } 0 \le x < 1; \\ \frac{1}{8}, & \text{if } x \ge 1. \end{cases}$$

$$B(x,y) = y \qquad T(x) = \begin{cases} 15x, & \text{if } 0 \le x < 1; \\ 1, & \text{if } x \ge 1. \end{cases}$$

$$S_b(A(x,y), A(x,y), B(x,y) = \frac{1}{16}(|A(x,y) - B(x,y)| + |A(x,y) - B(x,y)|)^2$$

$$= \frac{1}{16}(2|A(x,y) - B(x,y)|)^2$$

$$= \frac{1}{16}\left(2\left(\frac{x+y}{16} - y\right)\ \right)^2$$

$$= \frac{1}{(16)^2}\frac{1}{16}\big(2(x - 15y)\big)^2$$

$$\le \frac{1}{4^4} S_b(Sx, Sx, Ty) \le \frac{q}{b^4} max\{S_b(Sx, Sx, Ty), S_b(A(x,y), A(x,y), Sx), S_b(B(x,y), B(x,y), Tx)\}$$

For all $x, y, u, v \in X$, $0 < q \le 1$ and $b = 4$

Clearly all the conditions of the above Theorem 3.1 are satisfied. Also

$$SA(0,0) = A(S0, S0) \text{ and } BT(0,0) = B(T0, T0)$$

So, (A, S) and (B, T) are *owc* maps and (0,0) is the common coupled fixed point of A, B, S and T.

Theorem: 3.2 Let (X, S_b) be a $S_b$metric space with $b \ge 1$ and $A, B: X \times X \to X$ and $S, T: X \to X$ be four self-mappings satisfying the following conditions:

(i) $S_b(A(x,y), A(x,y), B(u,v) \le \frac{h}{3}\ [S_b(Sx, Sx, Tu) + S_b(A(x,y), A(x,y)Tu) + S_b(Sx, Sx, B(u,v))$

For all $x, y, u, v \in X$, $0 \le h < \frac{1}{b^2}$ and $b \ge \frac{3}{2}$

(ii) $y = B(x,y)$

Moreover if the pairs (A, S) and (B, T) are owc, then there exists a unique point x in X such that $A(x,x) = T(x) = B(x,x) = S(x) = x$.

Proof: Since the pairs (A,S) and (B,T) are owc so there are points a, b, a', b' in X such that

$$A(a,b) = Sa,\ \ A(b,a) = Sb \ \text{ and}$$

$$B(a', b'\ ) = Ta',\ \ \ B(b', a'\ ) = Tb'$$

We claim that $Sa = Ta'$. If not, by inequality (i) we get

$$S_b(A(a,b),A(a,b),B(a',b'))$$
$$\leq \frac{h}{3}\left\{S_b(Sa,Sa,Ta') + S_b(A(a,b),A(a,b),Ta') + S_b\big(Sa,Sa,B(a',b')\big)\right\}$$

Or $\quad S_b(Sa,Sa,Ta') \leq \frac{h}{3}\{S_b(Sa,Sa,Ta') + S_b(Sa,Sa,Ta') + S_b(Sa,Sa,Ta')\}$

$\leq \; h\,S_b(Sa,Sa,Ta')$ Since $0 \leq h < \frac{1}{b^2}$ where $b \geq 1, \leq S_b(Sa,Sa,Ta')$

$$\Rightarrow Sa = Ta'$$

Therefore $A(a,b) = Ta' = Sa = B(a',b')$

Similarly $A(b,a) = Tb' = Sb = B(b',a')$

Thus the pairs (A,S) and (B,T) have common coincidence points.

Let $A(a,b) = Ta' = Sa = B(a',b') = x$

and $A(b,a) = Tb' = Sb = B(b',a') = y$

Since (A,S) and (B,T) are owc

So $Sx = SA(a,b) = A(Sa,Sb) = A(x,y)$

and $Sy = SA(b,a) = A(Sb,Sa) = A(y,x)$

Also $Tx = TB(a',b') = B(Ta',Tb') = B(x,y)$

and $Ty = TB(b',a') = B(Tb',Ta') = B(y,x)$

Next we show that $x = y$, for this

putting $x = a$, $y = b$, $u = b'$, $v = a'$ in (ii),

$$S_b(x,x,y) = S_b(A(a,b),A(a,b),B(b',a'))$$
$$\leq \frac{h}{3}\left\{S_b(Sa,Sa,Ta') + S_b(A(a,b),A(a,b),Tb') + S_b\big(Sa,Sa,B(b',a')\big)\right\}$$
$$\leq \frac{h}{3}\{S_b(Sa,Sa,Tb') + S_b(Sa,Sa,Tb') + S_b(Sa,Sa,Tb')\}$$

$\leq \; h\,S_b(Sa,Sa,Tb')$ Since $0 \leq h < \frac{1}{b^2}$ where $b \geq 1$,

$$= S_b(x,x,y)$$
$$\Longrightarrow x = y$$

Now we prove that $Sx = Tx$

$$S_b(Sx,Sx,Tx) = S_b(Sx,Sx,Ty) = S_b(A(x,y),A(x,y),B(y,x))$$
$$\leq \frac{h}{3}\left\{S_b(Sx,Sx,Ty) + S_b(A(x,y),A(x,y),Ty) + S_b\big(Sx,Sx,B(y,x)\big)\right\}$$
$$\leq \frac{h}{3}\{S_b(Sx,Sx,Ty) + S_b(Sx,Sx,Ty) + S_b(Sx,Sx,Ty)\}$$

$$\leq h\, S_b(Sx, Sx, Ty) \quad \text{Since } 0 \leq h < \frac{1}{b^2} \text{ where } b \geq 1,$$

$$= S_b(Sx, Sx, Ty)$$

$$\Longrightarrow Sx = Tx$$

Also by condition (ii) we have,

$$x = B(x, x)$$

Thus $A(x,x) = T(x) = B(x,x) = S(x) = x$

Corollary: 3.3 Let (X, S_b) be a $S_b$metric space with $b \geq 1$ and $A, B: X \times X \to X$ and $S, T: X \to X$ be four self-mappings satisfying the following conditions:

(i) $S_b(A(x,y), A(x,y), B(u,v) \leq$

$$\frac{q}{b^4}\, max\left\{\begin{array}{c} S_b(Sx,Sx,Tu), S_b(A(x,y),A(x,y),Sx), S_b(A(x,y),A(x,y),Tu), S_b\big(Sx,Sx,B(u,v)\big), \\ S_b\big(B(u,v),B(u,v),Tu\big) \end{array}\right\}$$

For all $x, y, u, v \in X,\ 0 < q < 1\ and\ b \geq \frac{3}{2}$

(ii) $y = B(x, y)$

Moreover if the pairs (A, S) and (B, T) are owc, then there exists a unique point x in X such that $A(x,x) = T(x) = B(x,x) = S(x) = x$.

3. Application to Integral Equations

In this section, we study the existence of a unique solution to an initial value problem as an application to Theorem 3.1.

Theorem 4.1 Consider the initial value problem

$$x^1(t) = T\big(t, x(t)\big),\ t \in I = [0,1],\ x(0) = x_0 \tag{4.1}$$

where $T: I \times R \times R \to R$ and $x_0 \in R$. Then there exists a unique solution in $C(I, R)$ for initial value problem (4.1).

Proof: The integral equation corresponding to initial value problem (4.1) is

$$x(t) = x_0 + \int_0^t T(s, x(s), x(s))ds, \qquad t \in I. \tag{4.2}$$

Let $S_b(x,y,z) = \frac{1}{16}((|x - y| + |y - z| + |x - z|)^2)$ for $x, y, z \in C(I, R)$. Then the function is an S_b-metric with $b = 4$. Define $A, B: X \times X \to X$ and $S, T: X \to X$ such that $x, y \in X,\ t \in I$

$$S(x)(t) = x(t), T(y)(t) = y(t)$$

$$A(x,y)(t) = \frac{x_0}{4^3} + \int_0^t T(s, x(s), y(s))ds$$

$$B(x,y)(t) = \frac{y_0}{4^3} + \int_0^t T(s, x(s), y(s))ds$$

$$S_b(A(x,y),A(x,y),B(x,y)) = \frac{1}{16}(|A(x,y)-B(x,y)| + |A(x,y)-B(x,y)|)^2$$

$$= \frac{1}{16}(2|A(x,y)-B(x,y)|)^2$$

$$= \frac{1}{4}\left|\frac{x_0}{4^3} + \int_0^t T(s,x(s),y(s))ds - \frac{y_0}{4^3} - \int_0^t T(s,x(s),y(s))ds\right|^2$$

$$= \frac{1}{4^4}|x(t)-y(t)|^2$$

$$\leq \frac{1}{4^4} S_b(Sx,Sx,Ty) \leq \frac{q}{b^4} max\{S_b(Sx,Sx,Ty), S_b(A(x,y),A(x,y),Sx), S_b(B(x,y),B(x,y),Tx)\}$$

For all $x, y, u, v \in X$, $0 < q \leq 1$ and $b = 4$.
Clearly all the conditions of the above Theorem 3.1 are satisfied. Then $x(t)$ is the unique solution of the integral equation (4.2).

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