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Coincidence Point and Fixed Point Theorems in Dualistic Partial Metric Space

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Abstract

In this paper, we define dualistic Λ_{Ξ}-contraction, generalized dualistic Λ_{Ξ}-contraction and Dass-Gupta dualistic rational Λ_{Ξ}-contraction. We establish some common fixed-point theorems for Λ_{Ξ}-contraction, generalized dualistic Λ_{Ξ}-contraction and Dass-Gupta dualistic rational Λ_{Ξ}-contraction in the setting of dualistic partial metric spaces. Our results extend and generalize some well-known results of [5] and [11]. We also provide an example that shows the usefulness of these contractions.

Keywords**:** fixed point theorem, dualistic partial metric space, contraction.

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1. INTRODUCTION

During the past twenty years, one of the most active areas of study has been fixed point theory. Novel and captivating outcomes are attained, primarily in two aspects: altering the framework (the composition of the abstract space, such as the b-metric, delta symmetric quasi-metric, or non-symmetric metric space, among others) or modifying the characteristics of the operators.

The common idea of a metric space has been numerous times generalized. A partial metric (PM) space, which Matthews developed and examined, is one such generalization [7]. He verified the exact correspondence between the so-called weightable quasi-metric spaces and PM spaces. PM space has certain generalizations. One major modification to Matthews' definition of the PM, for instance, was suggested by O'Neill [12] and involved moving their range from $[0,\infty)$ to $(-\infty,\infty)$. Dualistic partial metric is the term used to refer to the PM space in the O'Neill sense, and a pair $(\mathcal{U}, d^*)$ here $\mathcal{U}$ is a nonempty set and d^* is a dualistic partial metric on $\mathcal{U}$ is called a dualistic partial metric space, according to [12]. O'Neill established multiple links between PM space and the topological features of domain theory in this manner. Studying Banach's contraction principle is the first step in creating contractual requirements. Several fixed-point theorems for some generalized metric space have made use of these criteria.

A coincidence point in mathematics is the location at which two or more functions coincide, or intersect, indicating that they have the same value at that particular position. Numerous areas of mathematics, such as algebra, differential equations, and calculus, are interested in coincidence points. They can be used to solve equations, comprehend the behavior of mathematical models, and solve optimization problems. A key finding in the theory of fixed-point theorems is the Coincidence Point Theorem, which establishes the circumstances in which two mappings share a fixed point. Aydi *et al*. [3] proved some coincidence and common fixed-point results in partially ordered cone metric spaces. Since then, there have been many results related to coincidence and common fixed-point, we refer to ([5], [13], [14]) and references therein.

In this paper, we shall propose three types of contraction, namely, Λ_{Ξ}-contraction, generalized dualistic Λ_{Ξ}-contraction -contraction, and Dass-Gupta dualistic rational Λ_{Ξ}-contraction -contraction. Also, prove some common fixed-point results via dualistic Λ_{Ξ}-contraction, generalized dualistic Λ_{Ξ}-contraction and Dass-Gupta dualistic rational Λ_{Ξ}-contraction in a dualistic partial metric space. Our result extends and generalizes some well-known results of [4]. Also, we verify our results with an example.

2. PRELIMINARIES

We recall some mathematical basics and definitions to make this paper self-sufficient.

Definition 2.1 (see [7]) Let $\mathcal{U}$ be a non-empty set. A partial metric (PM) on $\mathcal{U}$ is a function $d: \mathcal{U} \times \mathcal{U} \to [0, \infty)$ complying with following axioms, for all $\xi, \zeta, \eta \in \mathcal{U}$

(d_1) $\xi = \zeta \Leftrightarrow d(\xi, \zeta) = d(\xi, \xi) = d(\zeta, \zeta)$;

(d_2) $d(\xi, \xi) \leq d(\xi, \zeta)$;

(d_3) $d(\xi, \zeta) = d(\zeta, \xi)$;

(d_4) $d(\xi, \zeta) \leq d(\xi, \eta) + d(\eta, \zeta) - d(\eta, \eta)$

The pair $(\mathcal{U}, d)$ is called a partial metric space (PM space).

Definition 2.2 (see [12]) Let $\mathcal{U}$ be a non-empty set. A dualistic partial metric on $\mathcal{U}$ is a function $d^*: \mathcal{U} \times \mathcal{U} \to (-\infty, \infty)$ satisfying the following axioms, for all $\xi, \zeta, \eta \in \mathfrak{D}$

(d_1^*) $\xi = \zeta \Leftrightarrow d^*(\xi, \zeta) = d^*(\xi, \xi) = d^*(\zeta, \zeta)$;

(d_2^*) $d^*(\xi,\xi) \leq d^*(\xi,\zeta)$;

(d_3^*) $d^*(\xi,\zeta) = d^*(\zeta,\xi)$;

(d_4^*) $d^*(\xi,\eta) + d^*(\zeta,\zeta) \leq d^*(\xi,\zeta) + d^*(\zeta,\eta)$

The pair $(\mathcal{U}, d^*)$ is called a dualistic partial metric space (dualistic partial metric space).

Remark 2.3 Noting that each PM is a dualistic partial metric, but the converse is false. Indeed, define d^* on $(-\infty,\infty)$ as $d^*(\xi,\zeta) = \max\{\xi,\zeta\}, \forall \xi,\zeta \in (-\infty,\infty)$. Obviously, d^* is a dualistic partial metric on $(-\infty,\infty)$. Since $d^*(\xi,\zeta) < 0 \notin [0,\infty), \forall\, \xi,\zeta \in (-\infty,0)$ and then d^* is not a PM on $(-\infty,\infty)$. This confirms our remark.

Example 2.4 (see [10], [12])

(1) Define $d_\rho^*\colon \mathcal{U} \times \mathcal{U} \to (-\infty,\infty)$ by $d_\rho^*(\xi,\zeta) = d(\xi,\zeta) + b$, where ρ is a metric on a nonempty set $\mathcal{U}$ and $b \in (-\infty,\infty)$ is arbitrary constant, then it is easy to check that d_ρ^* verifies axioms $(d_1^*) - (d_4^*)$ and hence $\left(\mathcal{U}, d_\rho^*\right)$ is a dualistic partial metric C space.

(2) Let d be a PM defined on a non-empty set $\mathcal{U}$. The function $d^*\colon \mathcal{U} \times \mathcal{U} \to (-\infty,\infty)$ defined by $d^*(\xi,\zeta) = d(\xi,\zeta) - d(\xi,\xi) - d(\zeta,\zeta)$ satisfies the axioms $(d_1^*) - (d_4^*)$ and so it defines a dualistic partial metric on $\mathcal{U}$. Note that $d^*(\xi,\zeta)$ may have negative values.

(3) Let $\mathcal{U} = (-\infty,\infty)$. Define $d^*\colon \mathcal{U} \times \mathcal{U} \to (-\infty,\infty)$ by $d^*(\xi,\zeta) = |\xi - \zeta|$ if $\xi \neq \zeta$ and $d^*(\xi,\zeta) = -\gamma$ if $\xi = \zeta$ and $\beta > 0$. We can easily see that d^* is a dualistic partial metric on $\mathcal{U}$.

O'Neill [12] established that each dualistic partial metric d^* on $\mathcal{U}$ generates a $\mathcal{T}_0$ topology $\tau(d^*)$ on $\mathcal{U}$ having a base, the family of d^*-balls $\left\{\mathcal{B}_{d^*}(\xi,\epsilon) \,\middle|\, \xi \in \mathfrak{D}, \epsilon > 0\right\}$, where

$$\mathcal{B}_{d^*}(\xi,\epsilon) = \left\{\zeta \in \mathcal{U} \,\middle|\, d^*(\xi,\zeta) < d^*(\xi,\xi) + \epsilon\right\}. \tag{2.1}$$

If $(\mathcal{U}, d^*)$ is a dualistic partial metric space, then the function $\rho_{d^*} : \mathcal{U} \times \mathcal{U} \to [0,\infty)$ defined by

$$\rho_{d^*}(\xi,\zeta) = d^*(\xi,\zeta) - d^*(\xi,\xi) \tag{2.2}$$

defines a quasi-metric on $\mathcal{A}$ such that $\tau(d^*) = \tau(\rho_{d^*})$ and

$$\rho_{d^*}^s(\xi,\zeta) = \max\{\rho_{d^*}(\xi,\zeta), \rho_{d^*}(\zeta,\xi)\} \tag{2.3}$$

defines a metric on $\mathcal{U}$.

Definition ***2.5*** (see [11]) Let $(\mathcal{U}, d^*)$ be a dualistic partial metric space.

1. A sequence $\{\xi_j\}$ in $\mathcal{U}$ is said to converge or to be convergent if there is a $\xi \in \mathcal{U}$ such that
$$\lim_{j\to\infty} d^*(\xi_j, \xi) = d^*(\xi, \xi).$$
ξ is called the *limit* of $\{\xi_j\}$ and we write $\xi_j \to \xi$.
2. A sequence $\{\xi_j\}$ in $\mathcal{U}$ is said to be Cauchy sequence if $\lim_{j,j\to\infty} d^*(\xi_n, \xi_m)$ exists and is finite.
3. A dualistic partial metric space $\mathcal{U} = (\mathcal{U}, d^*)$ is said to be complete if every Cauchy sequence $\{\xi_j\}$ in $\mathcal{U}$ converges, with respect to $d(d^*)$, to a point $\xi \in \mathcal{U}$ such that
$$d^*(\xi, \xi) = \lim_{j,j\to\infty} d^*(\xi_j, \xi_j).$$

Remark ***2.6*** For a sequence, convergence with respect to metric space may not imply convergence with respect to dualistic partial metric space. Indeed, if we take $\gamma = 1$ and $\left\{\xi_j = \frac{1-j}{j} : j \geq 1\right\}_{j\in\mathbb{N}} \subset \mathcal{U}$ as in Example 2.4 (3). Mention that $\lim_{j\to\infty} d(\xi_j, -1) = -1$ and therefore, $\xi_j \to -1$ with respect to d. On the other hand, we make a conclusion that $\xi_j \nrightarrow -1$ with respect to d^* because $\lim_{j\to\infty} d^*(\xi_j, -1) = \lim_{j\to\infty} d^*|\xi_j - (-1)| = \lim_{j\to\infty} \left|\frac{1-j}{j} + 1\right| = 0$ and $d^*(-1, -1) = -1$.

Lemma ***2.7*** (see [11]) Let $(\mathcal{U}, d^*)$ be a dualistic partial metric space.

(1) Every Cauchy sequence in $(\mathcal{U}, \rho^s_{d^*})$ is also a Cauchy sequence in $(\mathcal{U}, \rho^*)$.

(2) A dualistic partial metric$(\mathcal{U}, d^*)$ is complete if and only if the induced metric space $(\mathcal{U}, \rho^s_{d^*})$ is complete.

(3) A sequence $\{\xi_j\}$ in $\mathcal{U}$ converges to a point $\xi \in \mathcal{U}$ with respect to $d(\rho^s_{d^*})$ if and only if $d^*(\xi, \xi) = \lim_{j\to\infty} d^*(\xi_j, \xi) = \lim_{j,j\to\infty} d^*(\xi_j, \xi_j)$.

Definition 2.8 (see [4]) Let $(\mathcal{U}, \rho)$ be a metric space. A mapping and $\Delta: \mathcal{U} \to \mathcal{U}$ is said to be a Dass-Gupta Rational contraction if there exist real numbers $\mathfrak{r}_1, \mathfrak{r}_2 \in [0,1)$ with $\mathfrak{r}_1 + \mathfrak{r}_2 < 1$ such that
$$\rho(\Delta\xi, \Delta\zeta) \leq \mathfrak{r}_1 \frac{[1 + \rho(\xi, \Delta\xi)]\rho(\zeta, \Delta\zeta)}{1 + \rho(\xi, \zeta)} + \mathfrak{r}_2 \rho(\xi, \zeta)$$
for all $\xi, \zeta \in \mathcal{U}$.

In dualistic partial metric space, we define dualistic Λ_Ξ-contraction, generalized dualistic Λ_Ξ-contraction and Dass-Gupta dualistic rational Λ_Ξ-contraction in dualistic partial metric space. We establish some common fixed-point theorems for Λ_Ξ-

contraction, generalized dualistic Λ_Ξ-contraction and Dass-Gupta dualistic rational Λ_Ξ-contraction defined on a dualistic partial metric space. These theorems expand and generalize several intriguing findings from metric fixed-point theory to the dualistic partial metric setting. We also provide an example that shows the usefulness of these contractions.

3. COINCIDENCE POINT RESULTS

This section contains some coincidence point and common fixed-point theorems for dualistic Λ_Ξ-contraction, dualistic rational Λ_Ξ-contraction and generalized dualistic Λ_Ξ-contraction, and deductions. We begin with the following definitions.

Definition 3.1 Let $(\mathcal{U}, d^*)$ be a dualistic partial metric space and Λ, Ξ be two self-mappings on . We say the mapping Λ, a dualistic Λ_Ξ-contraction, if for every $\xi, \zeta \in \mathcal{U}$, there exist real numbers $\alpha, \beta, \gamma \geq 0$ with $\alpha + \beta + \gamma < 1$ such that

$$|d^*(\Lambda\xi, \Lambda\zeta)| \leq \alpha|d^*(\Xi\xi, \Xi\zeta)| + \beta|d^*(\Xi\xi, \Lambda\xi)| + \gamma|d^*(\Xi\zeta, \Lambda\zeta)|. \tag{3.1}$$

Definition 3.2 Let $(\mathcal{U}, d^*)$ be a dualistic partial metric space and Λ, Ξ be two self-mappings on . We say the mapping Λ, a dualistic rational Λ_Ξ-contraction, if for every $\xi, \zeta \in \mathcal{U}$there exist real numbers $\alpha, \beta, \gamma, \delta \geq 0$ with $\alpha + \beta + \gamma + \delta < 1$ such that

$$\begin{aligned}|d^*(\Lambda\xi, \Lambda\zeta)| \leq{}& \alpha|d^*(\Xi\xi, \Xi\zeta)| + \beta|d^*(\Xi\xi, \Lambda\xi)| + \gamma|d^*(\Xi\zeta, \Lambda\zeta)| \\ &+\delta\frac{[1+|d^*(\Xi\xi,\Lambda\xi)|]|d^*(\Xi\zeta,\Lambda\zeta)|}{1+|d^*(\Xi\xi,\Xi\zeta)|}.\end{aligned} \tag{3.2}$$

Definition 3.3 Let $(\mathcal{U}, d^*)$ be a dualistic partial metric space. and Λ, Ξ be two self-mappings on $\mathcal{U}$. We say the mapping Λ, a generalized dualistic Λ_Ξ-contraction, if for every $\xi, \zeta \in \mathcal{U}$, there exists a number λ with $0 \leq \lambda < 1$ such that

$$|d^*(\Lambda\xi, \Lambda\zeta)| \leq \lambda \max\left\{\begin{matrix}|d^*(\Xi\xi, \Xi\zeta)|, \big||d^*(\Xi\xi, \Lambda\xi)|, |d^*(\Xi\zeta, \Lambda\zeta)|\big|, \\ \frac{|d^*(\Xi\xi,\Lambda\xi)|+|d^*(\Xi\zeta,\Lambda\zeta)|}{2}\end{matrix}\right\}. \tag{3.3}$$

Theorem 3.4 Let $(\mathcal{U}, d^*)$ be a complete dualistic partial metric space and $\Lambda, \Xi: \mathcal{U} \to \mathcal{U}$ be two mappings such that $\Lambda(\mathcal{U}) \subset \Xi(\mathcal{U})$ and Λ is a dualistic Λ_Ξ-contraction. If $\Lambda(\mathcal{U})$ or $\Xi(\mathcal{U})$ is a complete subspace of $\mathcal{U}$, then Λ and Ξ have a coincidence point. Further, if Λ, Ξ are weakly compatible mappings, then Λ and Ξ have a unique common fixed point.

Proof. Let ξ_0 be an arbitrary point in $\mathcal{U}$. Since $\Lambda(\mathcal{U}) \subset \Xi(\mathcal{U})$, we can find $\xi_1 \in \mathcal{U}$ such that $\Lambda\xi_0 = \Xi\xi_1$. In general, ξ_j is chosen such that $\Lambda\xi_j = \Xi\xi_{j+1}$ for $j = 0,1,2, \ldots$. If $\Lambda\xi_j = \Lambda\xi_{j-1} = \Xi\xi_j$ for some $j \in \mathbb{N}$, then $\zeta = \Lambda\xi_j = \Lambda\xi_{j-1} = \Xi\xi_j$ is a point of coincidence of Λ and Ξ. Suppose that $\Lambda\xi_j \neq \Lambda\xi_{j-1}$ and thus $\Xi\xi_j \neq \Xi\xi_{j+1}$ for all $j \in \mathbb{N}$. By the dualistic E_Ξ-contraction condition (3.1), we obtain

$$\begin{aligned}\left|d^*(\Lambda\xi_j, \Lambda\xi_{j+1})\right| &\leq \alpha\left|d^*(\Xi\xi_j, \Xi\xi_{j+1})\right| + \beta\left|d^*(\Xi\xi_j, \Lambda\xi_j)\right| + \gamma\left|d^*(\Xi\xi_{j+1}, \Lambda\xi_{j+1})\right|. \\ &= \alpha\left|d^*(\Lambda\xi_{j-1}, \Lambda\xi_j)\right| + \beta\left|d^*(\Lambda\xi_{j-1}, \Lambda\xi_j)\right| + \gamma\left|d^*(\Lambda\xi_j, \Lambda\xi_{j+1})\right|.\end{aligned} \tag{3.4}$$

The last inequality gives

$$\left|d^*(\Lambda\xi_j, \Lambda\xi_{j+1})\right| \leq \frac{1-\gamma}{\alpha+\beta}\left|d^*(\Lambda\xi_{j-1}, \Lambda\xi_j)\right| = \lambda\left|d^*(\Lambda\xi_{j-1}, \Lambda\xi_j)\right|$$

where $\lambda = \frac{1-\gamma}{\alpha+\beta}$. From this, we can write,

$$\begin{aligned}\left|d^*(\Lambda\xi_j, \Lambda\xi_{j+1})\right| &\leq \lambda\left|d^*(\Lambda\xi_{j-1}, \Lambda\xi_j)\right| \\ &\leq \lambda^2\left|d^*(\Lambda\xi_{j-2}, \Lambda\xi_{j-1})\right| \\ &\leq \cdots \leq \lambda^j|d^*(\Lambda\xi_0, \Lambda\xi_1)|.\end{aligned} \tag{3.5}$$

Now, consider the self-distance

$$\begin{aligned}\left|d^*(\Lambda\xi_j, \Lambda\xi_j)\right| &\leq \alpha\left|d^*(\Xi\xi_j, \Xi\xi_j)\right| + \beta\left|d^*(\Xi\xi_j, \Lambda\xi_j)\right| + \gamma\left|d^*(\Xi\xi_j, \Lambda\xi_j)\right| \\ &= \alpha\left|d^*(\Lambda\xi_{j-1}, \Lambda\xi_{j-1})\right| + \beta\left|d^*(\Lambda\xi_{j-1}, \Lambda\xi_j)\right| + \gamma\left|d^*(\Lambda\xi_{j-1}, \Lambda\xi_j)\right| \\ &= \alpha\left|d^*(\Lambda\xi_{j-1}, \Lambda\xi_{j-1})\right| + (\beta+\gamma)\left|d^*(\Lambda\xi_{j-1}, \Lambda\xi_j)\right| \\ &\leq \alpha\left|d^*(\Lambda\xi_{j-1}, \Lambda\xi_{j-1})\right| + (\beta+\gamma)\lambda^{j-1}|d^*(\Lambda\xi_0, \Lambda\xi_1)|\end{aligned} \tag{3.6}$$

Similarly,

$$\left|d^*(\Lambda\xi_{j-1}, \Lambda\xi_{j-1})\right| \leq \alpha\left|d^*(\Lambda\xi_{j-2}, \Lambda\xi_{j-2})\right| + (\beta+\gamma)\lambda^{j-2}|d^*(\Lambda\xi_0, \Lambda\xi_1)|$$

The inequality (3.6) implies that

$$\begin{aligned}\left|d^*(\Lambda\xi_j, \Lambda\xi_j)\right| &\leq \alpha\left\{\alpha\left|d^*(\Lambda\xi_{j-2}, \Lambda\xi_{j-2})\right| + (\beta+\gamma)\lambda^{j-2}|d^*(\Lambda\xi_0, \Lambda\xi_1)|\right\} \\ &\quad +(\beta+\gamma)\lambda^{j-1}|d^*(\Lambda\xi_0, \Lambda\xi_1)| \\ &\leq \alpha^2\left|d^*(\Lambda\xi_{j-2}, \Lambda\xi_{j-2})\right| + (\beta+\gamma)\left(\alpha\lambda^{j-2} + \lambda^{j-1}\right)|d^*(\Lambda\xi_0, \Lambda\xi_1)|\end{aligned}$$

Proceeding further in a similar way, we get

$$\begin{aligned}\left|d^*(\Lambda\xi_j, \Lambda\xi_j)\right| &\leq \alpha^j|d^*(\Lambda\xi_0, \Lambda\xi_0)| \\ &\quad +(\beta+\gamma)\left(\alpha^{j-1} + \alpha^{j-2}\lambda + \cdots + \alpha\lambda^{j-2} + \lambda^{j-1}\right)|d^*(\Lambda\xi_0, \Lambda\xi_1)| \\ &\leq \alpha^j|d^*(\Lambda\xi_0, \Lambda\xi_0)| + \mu^j|d^*(\Lambda\xi_0, \Lambda\xi_1)|\end{aligned}$$

where $\mu^j = (\beta+\gamma)\left(\alpha^{j-1} + \alpha^{j-2}\lambda + \cdots + \alpha^2\lambda^{j-3} + \alpha\lambda^{j-2} + \lambda^{j-1}\right)$.

The equation implies (2.2) that

$$\rho_{d^*}(\Lambda\xi_j, \Lambda\xi_{j+1}) \leq \left|d^*(\Lambda\xi_j, \Lambda\xi_{j+1})\right| - d^*(\Lambda\xi_j, \Lambda\xi_j)$$

$$\leq \left|d^*\left(\Lambda\xi_j, \Lambda\xi_{j+1}\right)\right| + \left|d^*\left(\Lambda\xi_j, \Lambda\xi_j\right)\right|$$
$$\leq \lambda^j|d^*(\Lambda\xi_0, \Lambda\xi_1)| + \alpha^j|d^*(\Lambda\xi_0, \Lambda\xi_0)| + \mu^j|d^*(\Lambda\xi_0, \Lambda\xi_1)|$$
$$= \left(\lambda^j + \mu^j\right)|d^*(\Lambda\xi_0, \Lambda\xi_1)| + \alpha^j|d^*(\Lambda\xi_0, \Lambda\xi_0)|$$

Now, for $i > j$, we have

$$\begin{aligned}\rho_{d^*}\left(\Lambda\xi_j, \Lambda\xi_i\right) &\leq \rho_{d^*}\left(\Lambda\xi_j, \Lambda\xi_{j+1}\right) + \rho_{d^*}\left(\Lambda\xi_{j+1}, \Lambda\xi_{j+2}\right) + \cdots + \rho_{d^*}(\Lambda\xi_{i-1}, \Lambda\xi_i) \\ &\leq \left(\lambda^j + \mu^j\right)|d^*(\Lambda\xi_0, \Lambda\xi_1)| + \alpha^j|d^*(\Lambda\xi_0, \Lambda\xi_0)| \\ &+\left(\lambda^{j+1} + \mu^{j+1}\right)|d^*(\Lambda\xi_0, \Lambda\xi_1)| + \alpha^{j+1}|d^*(\Lambda\xi_0, \Lambda\xi_0)| + \cdots \\ &+\left(\lambda^{i-1} + \mu^{i-1}\right)|d^*(\Lambda\xi_0, \Lambda\xi_1)| + \alpha^{i-1}|d^*(\Lambda\xi_0, \Lambda\xi_0)| \\ &= \left(\lambda^j + \lambda^{j+1} + \cdots \lambda^{i-1}\right)|d^*(\Lambda\xi_0, \Lambda\xi_1)| \\ &+\left(\mu^j + \mu^{j+1} + \cdots + \mu^{i-1}\right)|d^*(\Lambda\xi_0, \Lambda\xi_1)| \\ &+\left(\alpha^j + +\alpha^{j+1} + \cdots + \alpha^{i-1}\right)|d^*(\Lambda\xi_0, \Lambda\xi_0)| \\ &\leq \frac{\lambda^j}{1-\lambda}|d^*(\Lambda\xi_0, \Lambda\xi_1)| + \frac{\mu^j}{1-\mu}|d^*(\Lambda\xi_0, \Lambda\xi_1)| + \frac{\alpha^j}{1-\alpha}|d^*(\Lambda\xi_0, \Lambda\xi_0)|\end{aligned}$$

The last inequality gives

$$\rho_{d^*}\left(\Lambda\xi_j, \Lambda\xi_j\right) \leq \frac{\lambda^j}{1-\lambda}|d^*(\Lambda\xi_0, \Lambda\xi_1)| + \frac{\mu^j}{1-\mu}|d^*(\Lambda\xi_0, \Lambda\xi_1)| + \frac{\alpha^j}{1-\alpha}|d^*(\Lambda\xi_0, \Lambda\xi_0)| \quad \textbf{(3.7)}$$

We conclude that

$$\lim_{j,j\to\infty} \rho^s_{d^*}\left(\Lambda\xi_j, \Lambda\xi_j\right) = \lim_{j,j\to\infty} \max\left\{\rho_{d^*}\left(\Lambda\xi_j, \Lambda\xi_j\right), \rho_{d^*}\left(\Lambda\xi_j, \Lambda\xi_j\right)\right\} = 0,$$

thus, $\left\{\Lambda\xi_j\right\}$ is a Cauchy sequence in $(\mathcal{U}, \rho^s_{d^*})$. Since $(\mathcal{U}, d^*)$ is a complete dualistic partial metric space, by Lemma 2.1(2), $(\mathcal{U}, \rho^s_{d^*})$ is a complete metric space. Consequently, there exists an element $\eta \in \Lambda(\mathcal{U}) \subset \mathcal{U}$ such that such that $\Lambda\xi_j \to \eta$ as $j \to \infty$, that is $\lim_{j\to\infty} \rho_{d^*}\left(\Lambda\xi_j, \eta\right) = 0$ and by Lemma 2.1 (3), we know that

$$d^*(\eta, \eta) = \lim_{j\to\infty} d^*\left(\Lambda\xi_j, \eta\right) = \lim_{j,j\to\infty} d^*\left(\Lambda\xi_j, \Lambda\xi_j\right). \quad \textbf{(3.8)}$$

Since, $\lim_{j\to\infty} d_{d^*}\left(\xi_j, \eta\right) = 0$, by (2.2) and (4.8), we have

$$d^*(\eta, \eta) = \lim_{j\to\infty} d^*\left(\Lambda\xi_j, \eta\right) = \lim_{j,j\to\infty} d^*\left(\Lambda\xi_j, \Lambda\xi_j\right) = 0. \quad \textbf{(3.9)}$$

This shows that $\left\{\Lambda\xi_j\right\}$ is a Cauchy sequence converging to $\eta \in (\mathcal{U}, d^*)$. As $\eta \in \Lambda(\mathcal{U}) \subset \Xi(\mathcal{U})$, there exists $\sigma \in \mathcal{U}$ such that $\eta = \Xi\sigma$ and by (3.9), we have $d^*(\Xi\sigma, \Xi\sigma) = 0$. By condition (3.1), we have

$$\begin{aligned}\left|d^*\left(\Xi\xi_{j+1}, \Lambda\sigma\right)\right| &= \left|d^*\left(\Lambda\xi_j, \Lambda\sigma\right)\right| \\ &\leq \alpha\left|d^*\left(\Xi\xi_j, \Xi\sigma\right)\right| + \beta\left|d^*\left(\Xi\xi_j, \Lambda\xi_j\right)\right| + \gamma|d^*(\Xi\sigma, \Lambda\sigma)|\end{aligned}$$

$$= \alpha\left|d^*\left(\Lambda\xi_{j-1}, \Xi\sigma\right)\right| + \beta\left|d^*\left(\Lambda\xi_{j-1}, \Lambda\xi_j\right)\right| + \gamma|d^*(\Xi\sigma, \Lambda\sigma)|.$$

Applying limit as $j \to \infty$ and using equation (3.9), we have

$$\begin{aligned} |d^*(\Xi\sigma, \Lambda\sigma)| &\le \alpha|d^*(\Xi\sigma, \Xi\sigma)| + \beta|d^*(\Xi\sigma, \Xi\sigma)| + \gamma|d^*(\Xi\sigma, \Lambda\sigma)| \\ &= \gamma|d^*(\Xi\sigma, \Lambda\sigma)|, \end{aligned}$$

which implies that $|d^*(\Xi\sigma, \Lambda\sigma)| = 0$, because $\gamma < 1$ and then $d^*(\Xi\sigma, \Lambda\sigma) = 0$. Again from (3.1), we have

$$|d^*(\Lambda\sigma, \Lambda\sigma)| \le \alpha|d^*(\Xi\sigma, \Xi\sigma)| + \beta|d^*(\Xi\sigma, \Lambda\sigma)| + \gamma|d^*(\Xi\sigma, \Lambda\sigma)| = 0.$$

Since $\gamma < 1$, and $d^*(\Xi\sigma, \Xi\sigma) = 0, d^*(\Xi\sigma, \Lambda\sigma) = 0$, we get $|d^*(\Lambda\sigma, \Lambda\sigma)| = 0$. Hence, $d^*(\Lambda\sigma, \Lambda\sigma) = 0$, and

$$d^*(\Xi\sigma, \Xi\sigma) = d^*(\Lambda\sigma, \Lambda\sigma) = d^*(\Xi\sigma, \Lambda\sigma)$$

By using axiom (d_1^*), we have $\Xi\sigma = \Lambda\sigma$. Thus, $\eta = \Xi\sigma = \Lambda\sigma$ is a point of coincidence of Λ and Ξ. Since Λ and Ξ are weakly compatible mappings, $\eta = \Xi\sigma = \Lambda\sigma$ implies $\Lambda\eta = \Lambda\Xi\sigma = \Xi\Lambda\sigma = \Xi\eta$. By (3.1), we get

$$\begin{aligned} |d^*(\Lambda\sigma, \Lambda\eta)| &\le \alpha|d^*(\Xi\sigma, \Xi\eta)| + \beta|d^*(\Xi\sigma, \Lambda\sigma)| + \gamma|d^*(\Xi\eta, \Lambda\eta)| \\ &= \alpha|d^*(\Lambda\sigma, \Lambda\eta)| \end{aligned}$$

Thus,

$$d^*(\Lambda\sigma, \Lambda\eta) = 0 = d^*(\Lambda\sigma, \Lambda\sigma) = d^*(\Lambda\eta, \Lambda\eta).$$

Hence, $\eta = \Xi\eta = \Lambda\eta$, that is η is common fixed point of Λ and Ξ. To prove the uniqueness of η, suppose that there exists another common fixed point η^* of Λ and Ξ; we prove that $\eta = \eta^*$. By (3.1), we obtain

$$\begin{aligned} |d^*(\eta, \eta^*)| &= |d^*(\Lambda\eta, \Lambda\eta^*)| \\ &\le \alpha|d^*(\Xi\eta, \Xi\eta^*)| + \beta|d^*(\Xi\eta, \Lambda\eta)| + \gamma|d^*(\Xi\eta^*, \Lambda\eta^*)| \\ &= \alpha|d^*(\eta, \eta^*)| + \beta|d^*(\eta, \eta)| + \gamma|d^*(\eta^*, \eta^*)| \\ &= \alpha|d^*(\eta, \eta^*)| \end{aligned}$$

which implies that

$$(1-\alpha)|d^*(\eta, \eta^*)| \le 0.$$

This is possible only when $|d^*(\eta, \eta^*)| = 0$, since $\alpha < 1$. Hence, $d^*(\eta, \eta^*) = 0$ and then,

$$d^*(\eta, \eta^*) = d^*(\eta, \eta) = d^*(\eta^*, \eta^*)$$

By (d_1^*), we have $\eta = \eta^*$. Consequently, Λ and Ξ have a unique commom fixed point η.

Theorem 3.5 Let $(\mathcal{U}, d^*)$ be a complete dualistic partial metric space and $\Lambda, \Xi: \mathcal{U} \to \mathcal{U}$ be two mappings such that $\Lambda(\mathcal{U}) \subset \Xi(\mathcal{U})$ and Λ is a dualistic rational Λ_Ξ-contraction. If $\Lambda(\mathcal{U})$ or $\Xi(\mathcal{U})$ is a complete subspace of $\mathcal{U}$, then Λ and Ξ have a coincidence point.

Further, if Λ, Ξ are weakly compatible mappings, then Λ and Ξ have a unique common fixed point.

Proof Following the steps of proof of Theorem 3.4, we construct the sequence $\{\xi_j\}$ by iterating

$\Lambda\xi_0 = \Xi\xi_1, \Lambda\xi_j = \Xi\xi_{j+1}$ for $j = 0,1,2, ...$

where $\xi_0 \in \mathcal{U}$ is arbitrary point. By the dualistic rational Λ_Ξ-contraction condition (3.2), we obtain

$$\begin{aligned}
\left|d^*(\Lambda\xi_j, \Lambda\xi_{j+1})\right| &\leq \alpha\left|d^*(\Xi\xi_j, \Xi\xi_{j+1})\right| + \beta\left|d^*(\Xi\xi_j, \Lambda\xi_j)\right| + \gamma\left|d^*(\Xi\xi_{j+1}, \Lambda\xi_{j+1})\right| \\
&\quad +\delta\frac{[1+|d^*(\Xi\xi_j,\Lambda\xi_j)|]|d^*(\Xi\xi_{j+1},\Lambda\xi_{j+1})|}{1+|d^*(\Xi\xi_j,\Xi\xi_{j+1})|} \\
&\leq \alpha\left|d^*(\Lambda\xi_{j-1}, \Lambda\xi_j)\right| + \beta\left|d^*(\Lambda\xi_{j-1}, \Lambda\xi_j)\right| + \gamma\left|d^*(\Lambda\xi_j, \Lambda\xi_{j+1})\right| \\
&\quad +\delta\frac{[1+|d^*(\Lambda\xi_{j-1},\Lambda\xi_j)|]|d^*(\Lambda\xi_j,\Lambda\xi_{j+1})|}{1+|d^*(\Lambda\xi_{j-1},\Lambda\xi_j)|} \\
&= \alpha\left|d^*(\Lambda\xi_{j-1}, \Lambda\xi_j)\right| + \beta\left|d^*(\Lambda\xi_{j-1}, \Lambda\xi_j)\right| + \gamma\left|d^*(\Lambda\xi_j, \Lambda\xi_{j+1})\right| \\
&\quad +\delta\left|d^*(\Lambda\xi_j, \Lambda\xi_{j+1})\right| \qquad \textbf{(3.10)}
\end{aligned}$$

The last inequality gives

$$\left|d^*(\Lambda\xi_j, \Lambda\xi_{j+1})\right| \leq \frac{\alpha+\beta}{1-(\gamma+\delta)}\left|d^*(\Lambda\xi_{j-1}, \Lambda\xi_j)\right| = \mu\left|d^*(\Lambda\xi_{j-1}, \Lambda\xi_j)\right|.$$

where $\mu = \frac{\alpha+\beta}{1-(\gamma+\delta)}$. From this, we can write,

$$\begin{aligned}
\left|d^*(\Lambda\xi_j, \Lambda\xi_{j+1})\right| &\leq \mu\left|d^*(\Lambda\xi_{j-1}, \Lambda\xi_j)\right| \\
&\leq \mu^2\left|d^*(\Lambda\xi_{j-2}, \Lambda\xi_{j-1})\right| \\
&\leq \cdots \leq \mu^j|d^*(\Lambda\xi_0, \Lambda\xi_1)|. \qquad \textbf{(3.11)}
\end{aligned}$$

Now, consider the self-distance

$$\begin{aligned}
\left|d^*(\Lambda\xi_j, \Lambda\xi_j)\right| &\leq \alpha\left|d^*(\Xi\xi_j, \Xi\xi_j)\right| + \beta\left|d^*(\Xi\xi_j, \Lambda\xi_j)\right| + \gamma\left|d^*(\Xi\xi_j, \Lambda\xi_j)\right| \\
&\quad +\delta\frac{[1+|d^*(\Xi\xi_j,\Lambda\xi_j)|]|d^*(\Xi\xi_j,\Lambda\xi_j)|}{1+|d^*(\Xi\xi_j,\Xi\xi_j)|} \\
&\leq \alpha\left|d^*(\Lambda\xi_{j-1}, \Lambda\xi_{j-1})\right| + \beta\left|d^*(\Lambda\xi_{j-1}, \Lambda\xi_j)\right| + \gamma\left|d^*(\Lambda\xi_{j-1}, \Lambda\xi_j)\right| \\
&\quad +\delta\left|d^*(\Lambda\xi_{j-1}, \Lambda\xi_j)\right| \\
&= \alpha\left|d^*(\Lambda\xi_{j-1}, \Lambda\xi_{j-1})\right| + (\beta+\gamma+\delta)\left|d^*(\Lambda\xi_{j-1}, \Lambda\xi_j)\right| \qquad \textbf{(3.12)}
\end{aligned}$$

The inequality (3.11) implies that

$$\left|d^*(\Lambda\xi_j, \Lambda\xi_j)\right| \leq \alpha\left|d^*(\Lambda\xi_{j-1}, \Lambda\xi_{j-1})\right| + (\beta+\gamma+\delta)\mu^{j-1}|d^*(\Lambda\xi_0, \Lambda\xi_1)|$$

Similarly,

$$\left|d^*\left(\Lambda\xi_{j-1},\Lambda\xi_{j-1}\right)\right| \le \alpha\left|d^*\left(\Lambda\xi_{j-2},\Lambda\xi_{j-2}\right)\right| + (\beta+\gamma+\delta)\mu^{j-2}|d^*(\Lambda\xi_0,\Lambda\xi_1)|$$

Consequently,

$$\begin{aligned}\left|d^*\left(\Lambda\xi_j,\Lambda\xi_j\right)\right| &\le \alpha\left\{\alpha\left|d^*\left(\Lambda\xi_{j-2},\Lambda\xi_{j-2}\right)\right| + (\beta+\gamma+\delta)\mu^{j-2}|d^*(\Lambda\xi_0,\Lambda\xi_1)|\right\}\\ &+(\beta+\gamma+\delta)\mu^{j-1}|d^*(\Lambda\xi_0,\Lambda\xi_1)|\\ &= \alpha^2\left|d^*\left(\Lambda\xi_{j-2},\Lambda\xi_{j-2}\right)\right| + (\beta+\gamma+\delta)\alpha\mu^{j-2}|d^*(\Lambda\xi_0,\Lambda\xi_1)|\\ &+(\beta+\gamma+\delta)\mu^{j-1}|d^*(\Lambda\xi_0,\Lambda\xi_1)|\\ &= \alpha^2\left|d^*\left(\Lambda\xi_{j-2},\Lambda\xi_{j-2}\right)\right|\\ &+(\beta+\gamma+\delta)\left(\alpha\mu^{j-2}+\mu^{j-1}\right)|d^*(\Lambda\xi_0,\Lambda\xi_1)|\end{aligned}$$

Proceeding further in a similar way, we get

$$\begin{aligned}\left|d^*\left(\Lambda\xi_j,\Lambda\xi_j\right)\right| &\le \alpha^j|d^*(\Lambda\xi_0,\Lambda\xi_0)|\\ &+(\beta+\gamma+\delta)\left(\alpha^{j-2}+\alpha^{j-1}\mu+\cdots+\alpha\mu^{j-2}+\mu^{j-1}\right)|d^*(\Lambda\xi_0,\Lambda\xi_1)|\\ &\le \alpha^j|d^*(\Lambda\xi_0,\Lambda\xi_0)| + c^j|d^*(\Lambda\xi_0,\Lambda\xi_1)|\end{aligned}$$

where $c^j = (\beta+\gamma+\delta)\left(\alpha^{j-2}+\alpha^{j-1}\mu+\cdots+\alpha\mu^{j-2}+\mu^{j-1}\right)$.

The equation implies (2.2) that

$$\begin{aligned}\rho_{d^*}\left(\Lambda\xi_j,\Lambda\xi_{j+1}\right) &\le \left|d^*\left(\Lambda\xi_j,\Lambda\xi_{j+1}\right)\right| - d^*\left(\Lambda\xi_j,\Lambda\xi_j\right)\\ &\le \left|d^*\left(\Lambda\xi_j,\Lambda\xi_{j+1}\right)\right| + \left|d^*\left(\Lambda\xi_j,\Lambda\xi_j\right)\right|\\ &\le \mu^j|d^*(\Lambda\xi_0,\Lambda\xi_1)| + \alpha^j|d^*(\Lambda\xi_0,\Lambda\xi_0)| + c^j|d^*(\Lambda\xi_0,\Lambda\xi_1)|\end{aligned}$$

Now, for $i > j$, we have

$$\begin{aligned}\rho_{d^*}\left(\Lambda\xi_j,\Lambda\xi_i\right) &\le \rho_{d^*}\left(\Lambda\xi_j,\Lambda\xi_{j+1}\right) + \rho_{d^*}\left(\Lambda\xi_{j+1},\Lambda\xi_{j+2}\right) + \cdots + \rho_{d^*}(\Lambda\xi_{i-1},\Lambda\xi_i)\\ &\le \mu^j|d^*(\Lambda\xi_0,\Lambda\xi_1)| + \alpha^j|d^*(\Lambda\xi_0,\Lambda\xi_0)| + c^j|d^*(\Lambda\xi_0,\Lambda\xi_1)|\\ &+\mu^{j+1}|d^*(\Lambda\xi_0,\Lambda\xi_1)| + \alpha^{j+1}|d^*(\Lambda\xi_0,\Lambda\xi_0)| + c^{j+1}|d^*(\Lambda\xi_0,\Lambda\xi_1)|\\ &+\cdots\\ &+\mu^{i-1}|d^*(\Lambda\xi_0,\Lambda\xi_1)| + \alpha^{i-1}|d^*(\Lambda\xi_0,\Lambda\xi_0)| + c^{i-1}|d^*(\Lambda\xi_0,\Lambda\xi_1)|\\ &= \left(\mu^j+\mu^{j+1}+\cdots+\mu^{j-1}\right)|d^*(\Lambda\xi_0,\Lambda\xi_1)|\\ &+\left(\alpha^j+\alpha^{j+1}+\cdots+\alpha^{i-1}\right)|d^*(\Lambda\xi_0,\Lambda\xi_0)|\\ &+\left(c^j+c^{j+1}+\cdots+c^{j-1}\right)|d^*(\Lambda\xi_0,\Lambda\xi_1)|\\ &\le \left(\frac{\mu^j}{1-\mu}+\frac{c^j}{1-c}\right)|d^*(\Lambda\xi_0,\Lambda\xi_1)| + \frac{\alpha^j}{1-\alpha}|d^*(\Lambda\xi_0,\Lambda\xi_0)|\end{aligned}$$

The last inequality gives

$$\rho_{d^*}\left(\Lambda\xi_j,\Lambda\xi_j\right) \leq \left(\frac{\mu^j}{1-\mu}+\frac{c^j}{1-c}\right)|d^*(\Lambda\xi_0,\Lambda\xi_1)| + \frac{\alpha^j}{1-\alpha}|d^*(\Lambda\xi_0,\Lambda\xi_0)| \qquad \textbf{(3.13)}$$

We conclude that

$$\lim_{j,j\to\infty}\rho^s_{d^*}\left(\Lambda\xi_j,\Lambda\xi_j\right) = \lim_{j,j\to\infty}\max\left\{\rho_{d^*}\left(\Lambda\xi_j,\Lambda\xi_j\right),\rho_{d^*}\left(\Lambda\xi_j,\Lambda\xi_j\right)\right\} = 0,$$

thus, $\left\{\Lambda\xi_j\right\}$ is a Cauchy sequence in $(\mathcal{U},\rho^s_{d^*})$. Since $(\mathcal{U},d^*)$ is a complete dualistic partial metric space, by Lemma 2.1(2), $(\mathcal{U},\rho^s_{d^*})$ is a complete metric space. Consequently, there exists an element $\eta\in\Lambda(\mathcal{U})\subset\mathcal{U}$ such that such that $\Lambda\xi_j\to\eta$ as $j\to\infty$, that is $\lim_{j\to\infty}\rho_{d^*}\left(\Lambda\xi_j,\eta\right)=0$ and by Lemma 2.1 (3), we know that

$$d^*(\eta,\eta) = \lim_{j\to\infty}d^*\left(\Lambda\xi_j,\eta\right) = \lim_{j,j\to\infty}d^*\left(\Lambda\xi_j,\Lambda\xi_j\right). \qquad \textbf{(3.14)}$$

Since, $\lim_{j\to\infty}\rho_{d^*}\left(\xi_j,\eta\right)=0$, by (2.2) and (4.8), we have

$$d^*(\eta,\eta) = \lim_{j\to\infty}d^*\left(\Lambda\xi_j,\eta\right) = \lim_{j,j\to\infty}d^*\left(\Lambda\xi_j,\Lambda\xi_j\right) = 0. \qquad \textbf{(3.15)}$$

This shows that $\left\{\Lambda\xi_j\right\}$ is a Cauchy sequence converging to $\eta\in(\mathcal{U},d^*)$. As $\eta\in\Lambda(\mathcal{U})\subset\Xi(\mathcal{U})$, there exists $\sigma\in\mathcal{U}$ such that $\eta=\Xi\sigma$ and by (3.15), we have $d^*(\Xi\sigma\,,\Xi\sigma\,)=0$. By condition (3.2), we have

$$\begin{aligned}
\left|d^*\left(\Xi\xi_{j+1},\Lambda\sigma\right)\right| &= \left|d^*\left(\Lambda\xi_j,\Lambda\sigma\right)\right| \\
&\leq \alpha\left|d^*\left(\Xi\xi_j,\Xi\sigma\right)\right| + \beta\left|d^*\left(\Xi\xi_j,\Lambda\xi_j\right)\right| + \gamma|d^*(\Xi\sigma,\Lambda\sigma)| \\
&\quad +\delta\frac{\left[1+\left|d^*\left(\Xi\xi_j,\Lambda\xi_j\right)\right|\right]|d^*(\Xi\sigma,\Lambda\sigma)|}{1+\left|d^*\left(\Xi\xi_j,\Xi\sigma\right)\right|} \\
&\leq \alpha\left|d^*\left(\Lambda\xi_{j-1},\Xi\sigma\right)\right| + \beta\left|d^*\left(\Lambda\xi_{j-1},\Lambda\xi_j\right)\right| + \gamma|d^*(\Xi\sigma,\Lambda\sigma)| \\
&\quad +\delta\frac{\left[1+\left|d^*\left(\Lambda\xi_{j-1},\Lambda\xi_j\right)\right|\right]|d^*(\Xi\sigma,\Lambda\sigma)|}{1+\left|d^*\left(\Lambda\xi_{j-1},\Xi\sigma\right)\right|}
\end{aligned}$$

Applying limit as $j\to\infty$ and using equation (3.15), we have

$$\begin{aligned}
|d^*(\Xi\sigma,\Lambda\sigma)| &\leq \alpha|d^*(\Xi\sigma,\Xi\sigma)| + \beta|d^*(\Xi\sigma,\Xi\sigma)| + \gamma|d^*(\Xi\sigma,\Lambda\sigma)| + \delta|d^*(\Xi\sigma,\Lambda\sigma)|, \\
&= (\gamma+\delta)|d^*(\Xi\sigma,\Lambda\sigma)|
\end{aligned}$$

which implies that $|d^*(\Xi\sigma,\Lambda\sigma)|=0$, because $\gamma+\delta<1$ and then $d^*(\Xi\sigma,\Lambda\sigma)=0$. Again from (3.2), we have

$$\begin{aligned}
|d^*(\Lambda\sigma,\Lambda\sigma)| &\leq \alpha|d^*(\Xi\sigma,\Xi\sigma)| + \beta|d^*(\Xi\sigma,\Lambda\sigma)| + \gamma|d^*(\Xi\sigma,\Lambda\sigma)| \\
&\quad +\delta\frac{[1+|d^*(\Xi\sigma,\Lambda\sigma)|]|d^*(\Xi\sigma,\Lambda\sigma)|}{1+|d^*(\Xi\sigma,\Xi\sigma)|} \\
&= \alpha|d^*(\Xi\sigma,\Xi\sigma)|
\end{aligned}$$

Since $\alpha<1$, we get $|d^*(\Lambda\sigma,\Lambda\sigma)|=0$. Hence, $d^*(\Lambda\sigma,\Lambda\sigma)=0$, and

$$d^*(\Xi\sigma,\Xi\sigma) = d^*(\Lambda\sigma,\Lambda\sigma) = d^*(\Xi\sigma,\Lambda\sigma)$$

By using axiom (d_1^*), we have $\Xi\sigma=\Lambda\sigma$. Thus, $\eta=\Xi\sigma=\Lambda\sigma$ is a point of coincidence of Λ and Ξ. Since Λ and Ξ are weakly compatible mappings, $\eta = \Xi\sigma = \Lambda\sigma$ implies $\Lambda\eta = \Lambda\Xi\sigma = \Xi\Lambda\sigma = \Xi\eta$. By (3.2), we get

$$|d^*(\Lambda\sigma, \Lambda\eta)| \leq \alpha|d^*(\Xi\sigma, \Xi\eta)| + \beta|d^*(\Xi\sigma, \Lambda\sigma)| + \gamma|d^*(\Xi\eta, \Lambda\eta)| + \delta\frac{[1+|d^*(\Xi\sigma,\Lambda\sigma)|]|d^*(\Xi\eta,\Lambda\eta)|}{1+|d^*(\Xi\sigma,\Xi\eta)|}$$

Thus, $d^*(\Lambda\sigma, \Lambda\eta) = 0 = d^*(\Lambda\sigma, \Lambda\sigma) = d^*(\Lambda\eta, \Lambda\eta)$. Hence, $\eta = \Xi\eta = \Lambda\eta$, that is η is common fixed point of Λ and Ξ. To prove the uniqueness of η, suppose that there exists another common fixed point η^* of Λ and Ξ; we prove that $\eta = \eta^*$. By (4.2), we obtain

$$\begin{aligned} |d^*(\eta, \eta^*)| &= |d^*(\Lambda\eta, \Lambda\eta^*)| \\ &\leq \alpha|d^*(\Xi\eta, \Xi\eta^*)| + \beta|d^*(\Xi\eta, \Lambda\eta)| + \gamma|d^*(\Xi\eta^*, \Lambda\eta^*)| \\ &\quad + \delta\frac{(1+|d^*(\Xi\eta,\Lambda\eta)|)|d^*(\Xi\eta^*,\Lambda\eta^*)|}{1+|d^*(\Xi\eta,\Xi\eta^*)|} \\ &= \alpha|d^*(\eta, \eta^*)| \end{aligned}$$

which implies that $(1-\alpha)|d^*(\eta, \eta^*)| \leq 0$. This is possible only when $|d^*(\eta, \eta^*)| = 0$, since $\lambda < 1$. Hence, $d^*(\eta, \eta^*) = 0$ and then,

$$d^*(\eta, \eta^*) = d^*(\eta, \eta) = d^*(\eta^*, \eta^*)$$

By (d_1^*), we have $\eta = \eta^*$. Consequently, Λ and Ξ have a unique common fixed point η.

Theorem 3.6 Let $(\mathcal{U}, d^*)$ be a complete dualistic partial metric space and $\Lambda, \Xi: \mathcal{U} \to \mathcal{U}$ be two mappings such that $\Lambda(\mathcal{U}) \subset \Xi(\mathcal{U})$ and Λ is a dualistic generalized Λ_Ξ-contraction. If $\Lambda(\mathcal{U})$ or $\Xi(\mathcal{U})$ is a complete subspace of $\mathcal{U}$, then Λ and Ξ have a coincidence point. Further, if Λ, Ξ are weakly compatible mappings, then Λ and Ξ have a unique common fixed point.

Proof Following the steps of proof of Theorem 3.3, we construct the sequence $\{\xi_j\}$ by iterating

$\Lambda\xi_0 = \Xi\xi_1, \Lambda\xi_j = \Xi\xi_{j+1}$ for $j = 0,1,2, ...$

where $\xi_0 \in \mathcal{U}$ is arbitrary point. By the dualistic rational Λ_Ξ-contraction condition (3.3), we obtain

$$\begin{aligned} \left|d^*(\Lambda\xi_j, \Lambda\xi_{j+1})\right| &\leq \lambda \max\left\{\begin{matrix}\left|d^*(\Xi\xi_j, \Xi\xi_{j+1})\right|, \left|d^*(\Xi\xi_j, \Lambda\xi_j)\right|, \left|d^*(\Xi\xi_{j+1}, \Lambda\xi_{j+1})\right|, \\ \frac{|d^*(\Xi\xi_j,\Lambda\xi_j)|+|d^*(\Xi\xi_{j+1},\Lambda\xi_{j+1})|}{2}\end{matrix}\right\} \\ &= \lambda \max\left\{\begin{matrix}\left|d^*(\Lambda\xi_{j-1}, \Lambda\xi_j)\right|, \left|d^*(\Lambda\xi_{j-1}, \Lambda\xi_j)\right|, \left|d^*(\Lambda\xi_j, \Lambda\xi_{j+1})\right|, \\ \frac{|d^*(\Lambda\xi_{j-1},\Lambda\xi_j)|+|d^*(\Lambda\xi_j,\Lambda\xi_{j+1})|}{2}\end{matrix}\right\} \end{aligned} \tag{3.16}$$

If $\left|d^*(\Lambda\xi_{j-1}, \Lambda\xi_j)\right| < \left|d^*(\Lambda\xi_j, \Lambda\xi_{j+1})\right|$ for some j, from (3.16), we have

$$\left|d^*\left(\Lambda\xi_j,\Lambda\xi_{j+1}\right)\right| \leq \lambda \max\left\{\begin{matrix}\left|d^*\left(\Lambda\xi_{j-1},\Lambda\xi_j\right)\right|, \left|d^*\left(\Lambda\xi_{j-1},\Lambda\xi_j\right)\right|, \left|d^*\left(\Lambda\xi_j,\Lambda\xi_{j+1}\right)\right|, \\ \left|d^*\left(\Lambda\xi_j,\Lambda\xi_{j+1}\right)\right|\end{matrix}\right\}$$
$$= \lambda\left|d^*\left(\Lambda\xi_j,\Lambda\xi_{j+1}\right)\right|$$

which is a contradiction. Hence, $\left|d^*\left(\Lambda\xi_{j-1},\Lambda\xi_j\right)\right| \geq \left|d^*\left(\Lambda\xi_j,\Lambda\xi_{j+1}\right)\right|$ and so from (3.16), we have

$$\left|d^*\left(\Lambda\xi_j,\Lambda\xi_{j+1}\right)\right| \leq \lambda\left|d^*\left(\Lambda\xi_{j-1},\Lambda\xi_j\right)\right|$$

From this, we can write,

$$\begin{aligned}\left|d^*\left(\Lambda\xi_j,\Lambda\xi_{j+1}\right)\right| &\leq \lambda\left|d^*\left(\Lambda\xi_{j-1},\Lambda\xi_j\right)\right| \\ &\leq \lambda^2\left|d^*\left(\Lambda\xi_{j-2},\Lambda\xi_{j-1}\right)\right| \\ &\leq \cdots \leq \lambda^j|d^*(\Lambda\xi_0,\Lambda\xi_1)|. \end{aligned} \qquad \textbf{(3.17)}$$

Now, consider the self-distance

$$\begin{aligned}\left|d^*\left(\Lambda\xi_j,\Lambda\xi_j\right)\right| &\leq \lambda \max\left\{\begin{matrix}\left|d^*\left(\Xi\xi_j,\Xi\xi_j\right)\right|, \left|d^*\left(\Xi\xi_j,\Lambda\xi_j\right)\right|, \left|d^*\left(\Xi\xi_j,\Lambda\xi_j\right)\right|, \\ \frac{\left|d^*\left(\Xi\xi_j,\Lambda\xi_j\right)\right|+\left|d^*\left(\Xi\xi_j,\Lambda\xi_j\right)\right|}{2}\end{matrix}\right\} \\ &= \lambda \max\left\{\left|d^*\left(\Xi\xi_j,\Xi\xi_j\right)\right|, \left|d^*\left(\Xi\xi_j,\Lambda\xi_j\right)\right|\right\} \\ &= \lambda \max\left\{\left|d^*\left(\Lambda\xi_{j-1},\Lambda\xi_{j-1}\right)\right|, \left|d^*\left(\Lambda\xi_{j-1},\Lambda\xi_j\right)\right|\right\} \\ &\leq \lambda\left|d^*\left(\Lambda\xi_{j-1},\Lambda\xi_j\right)\right|\end{aligned}$$

Above inequality and (3.17) implies that

$$\left|d^*\left(\Lambda\xi_j,\Lambda\xi_j\right)\right| \leq \lambda\lambda^{j-1}|d^*(\Lambda\xi_0,\Lambda\xi_1)| = \lambda^j|d^*(\Lambda\xi_0,\Lambda\xi_1)| \qquad \textbf{(3.18)}$$

The equation implies (2.2) that

$$\begin{aligned}\rho_{d^*}\left(\Lambda\xi_j,\Lambda\xi_{j+1}\right) &\leq \left|d^*\left(\Lambda\xi_j,\Lambda\xi_{j+1}\right)\right| - d^*\left(\Lambda\xi_j,\Lambda\xi_j\right) \\ &\leq \left|d^*\left(\Lambda\xi_j,\Lambda\xi_{j+1}\right)\right| + \left|d^*\left(\Lambda\xi_j,\Lambda\xi_j\right)\right| \\ &\leq \lambda^j|d^*(\Lambda\xi_0,\Lambda\xi_1)| + \lambda^j|d^*(\Lambda\xi_0,\Lambda\xi_1)| = 2\lambda^j|d^*(\Lambda\xi_0,\Lambda\xi_1)|\end{aligned}$$

Now, for $i > j$, we have

$$\begin{aligned}\rho_{d^*}\left(\Lambda\xi_j,\Lambda\xi_i\right) &\leq \rho_{d^*}\left(\Lambda\xi_j,\Lambda\xi_{j+1}\right) + \rho_{d^*}\left(\Lambda\xi_{j+1},\Lambda\xi_{j+2}\right) + \cdots + \rho_{d^*}(\Lambda\xi_{i-1},\Lambda\xi_i) \\ &\leq 2\lambda^j|d^*(\Lambda\xi_0,\Lambda\xi_1)| + 2\lambda^{j+1}|d^*(\Lambda\xi_0,\Lambda\xi_1)| + \cdots \\ &\quad +2\lambda^{i-1}|d^*(\Lambda\xi_0,\Lambda\xi_1)| \\ &= 2\left(\lambda^j + \lambda^{j+1} + \cdots + \lambda^{j-1}\right)|d^*(\Lambda\xi_0,\Lambda\xi_1)| \\ &= \frac{2\lambda^j}{1-\lambda}|d^*(\Lambda\xi_0,\Lambda\xi_1)|\end{aligned}$$

The last inequality gives

$$\rho_{d^*}(\Lambda\xi_j, \Lambda\xi_j) \leq \frac{2\lambda^j}{1-\lambda} |d^*(\Lambda\xi_0, \Lambda\xi_1)| \quad \textbf{(3.19)}$$

We conclude that

$$\lim_{j,j\to\infty} \rho^s_{d^*}(\Lambda\xi_j, \Lambda\xi_j) = \lim_{j,j\to\infty} \max\{\rho_{d^*}(\Lambda\xi_j, \Lambda\xi_j), \rho_{d^*}(\Lambda\xi_j, \Lambda\xi_j)\} = 0,$$

thus, $\{\Lambda\xi_j\}$ is a Cauchy sequence in $(\mathcal{U}, \rho^s_{d^*})$. Since $(\mathcal{U}, d^*)$ is a complete dualistic partial metric space, by Lemma 2.1(2), $(\mathcal{U}, \rho^s_{d^*})$ is a complete metric space. Consequently, there exists an element $\eta \in \Lambda(\mathcal{U}) \subset \mathcal{U}$ such that such that $\Lambda\xi_j \to \eta$ as $j \to \infty$, that is $\lim_{j\to\infty} \rho_{d^*}(\Lambda\xi_j, \eta) = 0$ and by Lemma 2.1 (3), we know that

$$d^*(\eta,\eta) = \lim_{j\to\infty} d^*(\Lambda\xi_j, \eta) = \lim_{j,j\to\infty} d^*(\Lambda\xi_j, \Lambda\xi_j). \quad \textbf{(3.20)}$$

Since, $\lim_{j\to\infty} \rho_{d^*}(\xi_j, \eta) = 0$, by (2.2) and (3.20), we have

$$d^*(\eta,\eta) = \lim_{j\to\infty} d^*(\Lambda\xi_j, \eta) = \lim_{j,j\to\infty} d^*(\Lambda\xi_j, \Lambda\xi_j) = 0. \quad \textbf{(3.21)}$$

This shows that $\{\Lambda\xi_j\}$ is a Cauchy sequence converging to $\eta \in (\mathcal{U}, d^*)$. As $\eta \in \Lambda(\mathcal{U}) \subset \Xi(\mathcal{U})$, there exists $\sigma \in \mathcal{U}$ such that $\eta = \Xi\sigma$ and by (3.21), we have $d^*(\Xi\sigma, \Xi\sigma) = 0$.

By condition (3.3 we have

$$\begin{aligned} |d^*(\Xi\xi_{j+1}, \Lambda\sigma)| &= |d^*(\Lambda\xi_j, \Lambda\sigma)| \\ &\leq \lambda \max\left\{ \begin{matrix} |d^*(\Xi\xi_j, \Xi\sigma)|, |d^*(\Xi\xi_j, \Lambda\xi_j)|, |d^*(\Xi\sigma, \Lambda\sigma)|, \\ \frac{|d^*(\Xi\xi_j, \Lambda\xi_j)| + |d^*(\Xi\sigma, \Lambda\sigma)|}{2} \end{matrix} \right\} \\ &= \lambda \max\left\{ \begin{matrix} |d^*(\Lambda\xi_{j-1}, \Xi\sigma)|, |d^*(\Lambda\xi_{j-1}, \Lambda\xi_j)|, |d^*(\Xi\sigma, \Lambda\sigma)|, \\ \frac{|d^*(\Lambda\xi_{j-1}, \Lambda\xi_j)| + |d^*(\Xi\sigma, \Lambda\sigma)|}{2} \end{matrix} \right\} \end{aligned}$$

Applying limit as $j \to \infty$ and using equation (3.21), we have

$$\begin{aligned} |d^*(\Xi\sigma, \Lambda\sigma)| &\leq \lambda \max\left\{ \begin{matrix} |d^*(\Xi\sigma, \Xi\sigma)|, |d^*(\Xi\sigma, \Xi\sigma)|, |d^*(\Xi\sigma, \Lambda\sigma)|, \\ \frac{|d^*(\Xi\sigma, \Xi\sigma)| + |d^*(\Xi\sigma, \Lambda\sigma)|}{2} \end{matrix} \right\} \\ &= \lambda |d^*(\Xi\sigma, \Lambda\sigma)| \end{aligned}$$

which implies that $|d^*(\Xi\sigma, \Lambda\sigma)| = 0$, because $\lambda < 1$ and then $d^*(\Xi\sigma, \Lambda\sigma) = 0$. Again from (3.3), we have

$$|d^*(\Lambda\sigma, \Lambda\sigma)| \leq \lambda \max\left\{ \begin{matrix} |d^*(\Xi\sigma, \Xi\sigma)|, |d^*(\Xi\sigma, \Lambda\sigma)|, |d^*(\Xi\sigma, \Lambda\sigma)|, \\ \frac{|d^*(\Xi\sigma, \Lambda\sigma)| + |d^*(\Xi\sigma, \Lambda\sigma)|}{2} \end{matrix} \right\}$$

We get $|d^*(\Lambda\sigma, \Lambda\sigma)| = 0$. Hence, $d^*(\Lambda\sigma, \Lambda\sigma) = 0$, and

$$d^*(\Xi\sigma, \Xi\sigma) = d^*(\Lambda\sigma, \Lambda\sigma) = d^*(\Xi\sigma, \Lambda\sigma)$$

By using axiom (d^*_1), we have $\Xi\sigma = \Lambda\sigma$. Thus, $\eta = \Xi\sigma = \Lambda\sigma$ is a point of coincidence of Λ and $\varXi$. Since Λ and $\varXi$ are weakly compatible mappings, $\eta = \varXi\sigma = \Lambda\sigma$ implies $\Lambda\eta = \Lambda\varXi\sigma = \varXi\Lambda\sigma = \varXi\eta$. By (3.3), we get

$$|d^*(\Lambda\sigma, \Lambda\eta)| \leq \lambda \max\left\{\begin{matrix} |d^*(\varXi\sigma, \varXi\eta)|, |d^*(\varXi\sigma, \Lambda\sigma)|, |d^*(\varXi\eta, \Lambda\eta)|, \\ \dfrac{|d^*(\varXi\sigma, \Lambda\sigma)| + |d^*(\varXi\eta, \Lambda\eta)|}{2} \end{matrix}\right\}$$

$$= \lambda|d^*(\varXi\sigma, \varXi\eta)| = \lambda|d^*(\Lambda\sigma, \Lambda\eta)|$$

Thus, $d^*(\Lambda\sigma, \Lambda\eta) = 0 = d^*(\Lambda\sigma, \Lambda\sigma) = d^*(\Lambda\eta, \Lambda\eta)$. Hence, $\eta = \varXi\eta = \Lambda\eta$, that is η is common fixed point of Λ and $\varXi$. To prove the uniqueness of η, suppose that there exists another common fixed point η^* of Λ and $\varXi$; we prove that $\eta = \eta^*$. By (3.3), we obtain

$$\begin{aligned} |d^*(\eta, \eta^*)| &= |d^*(\Lambda\eta, \Lambda\eta^*)| \\ &\leq \lambda \max\left\{\begin{matrix} |d^*(\varXi\eta, \varXi\eta^*)|, |d^*(\varXi\eta, \Lambda\eta)|, |d^*(\varXi\eta^*, \Lambda\eta^*)|, \\ \frac{|d^*(\varXi\eta, \Lambda\eta)| + |d^*(\varXi\eta^*, \Lambda\eta^*)|}{2} \end{matrix}\right\} \\ &= \lambda|d^*(\eta, \eta^*)| \end{aligned}$$

which implies that $(1-\lambda)|d^*(\eta, \eta^*)| \leq 0$. This is possible only when $|d^*(\eta, \eta^*)| = 0$, since $\lambda < 1$. Hence, $d^*(\eta, \eta^*) = 0$ and then,

$$d^*(\eta, \eta^*) = d^*(\eta, \eta) = d^*(\eta^*, \eta^*)$$

By (d_1^*), we have $\eta = \eta^*$. Consequently, Λ and Ξ have a unique common fixed point η.

Now, we give an example in support of our results.

Example 3.1 Define $d^*: (-\infty, 0] \times (-\infty, 0] \to (-\infty, \infty)$ by $d^*(\xi, \zeta) = \max\{\xi, \zeta\}$. It is easy to check that $\left((-\infty, 0], d^*\right)$ is a complete dualistic partial metric space. Define $\Lambda, \Xi: (-\infty, 0] \to (-\infty, 0]$ as

$$\Lambda\xi = \frac{\xi}{2}, \Xi\xi = \frac{\xi}{4} \forall\, \xi \in (-\infty, 0].$$

Further, for all $\xi, \zeta \in (-\infty, 0]$ with $\xi \geq \zeta$ and $\alpha = \frac{1}{2}, \beta = \frac{1}{8}, \gamma = \frac{1}{8}, \delta = \frac{1}{8}$, we have

$$\begin{aligned} |d^*(\Lambda\xi, \Lambda\zeta)| &= \left|\max\left\{\frac{\xi}{2}, \frac{\zeta}{2}\right\}\right| = \left|\frac{\xi}{2}\right| \\ &\leq \frac{1}{2}|\xi| + \frac{1}{8}\left|\frac{\xi}{4}\right| + \frac{1}{8}\left|\frac{\zeta}{4}\right| \\ &\leq \frac{1}{2}|\max\{\xi, \zeta\}| + \frac{1}{8}\left|\max\left\{\frac{\xi}{4}, \frac{\xi}{2}\right\}\right| + \frac{1}{8}\left|\max\left\{\frac{\zeta}{4}, \frac{\zeta}{2}\right\}\right| \\ &= \alpha|d^*(\Xi\xi, \Xi\zeta)| + \beta|d^*(\Xi\xi, \Lambda\xi)| + \gamma|d^*(\Xi\zeta, \Lambda\zeta)| \end{aligned}$$

Cleary, (3.1) is satisfied. Also

$$\begin{aligned} |d^*(\Lambda\xi, \Lambda\zeta)| &= \left|\max\left\{\frac{\xi}{2}, \frac{\zeta}{2}\right\}\right| = \left|\frac{\xi}{2}\right| \\ &\leq \frac{1}{2}|\xi| + \frac{1}{8}\left|\frac{\xi}{4}\right| + \frac{1}{8}\left|\frac{\zeta}{4}\right| + \frac{1}{8}\frac{\left|\frac{\xi}{4}\right| + \left|\frac{\zeta}{4}\right|}{2} \end{aligned}$$

$$\leq \frac{1}{2}|\max\{\xi,\zeta\}| + \frac{1}{8}\left|\max\left\{\frac{\xi}{4},\frac{\xi}{2}\right\}\right| + \frac{1}{8}\left|\max\left\{\frac{\zeta}{4},\frac{\zeta}{2}\right\}\right| + \frac{1}{8}\frac{\left|\max\left\{\frac{\xi}{4},\frac{\xi}{2}\right\}\right| + \left|\max\left\{\frac{\zeta}{4},\frac{\zeta}{2}\right\}\right|}{2}$$

$$= \alpha|d^*(\Xi\xi,\Xi\zeta)| + \beta|d^*(\Xi\xi,\Lambda\xi)| + \gamma|d^*(\Xi\zeta,\Lambda\zeta)| + \delta\frac{|d^*(\Xi\xi,\Lambda\xi)| + |d^*(\Xi\zeta,\Lambda\zeta)|}{2}$$

$$\leq \frac{1}{2}\left\{|d^*(\Xi\xi,\Xi\zeta)|, |d^*(\Xi\xi,\Lambda\xi)|, |d^*(\Xi\zeta,\Lambda\zeta)|, \frac{|d^*(\Xi\xi,\Lambda\xi)| + |d^*(\Xi\zeta,\Lambda\zeta)|}{2}\right\}$$

$$= \lambda\left\{|d^*(\Xi\xi,\Xi\zeta)|, |d^*(\Xi\xi,\Lambda\xi)|, |d^*(\Xi\zeta,\Lambda\zeta)|, \frac{|d^*(\Xi\xi,\Lambda\xi)| + |d^*(\Xi\zeta,\Lambda\zeta)|}{2}\right\}$$

for $\lambda = \frac{1}{2}$ and $\forall\, \xi,\zeta \in (-\infty, 0]$ with $\xi \geq \zeta$. Cleary, (3.2) and (3.3) is satisfied. In the view of Theorem 3.5 and Theorem 3.6, Λ and Ξ have a unique common fixed point in $(-\infty, 0]$, indeed $\Xi 0 = \Lambda 0 = 0$.

CONFLICTS OF INTEREST

The authors declare that they have no conflicts of interest.

AUTHORS' CONTRIBUTIONS

All authors contributed equally and significantly in writing this article. All authors read and approved the final manuscript.

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