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Transportation Problems with Twin Optimal Solutions

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Abstract

In this paper, we have identified and listed a set of balanced and unbalanced transportation Problems (TPs) having alternative optimal solution. For each of the identified TPs, we have applied the Modified Allocation (MODA) method to generate the optimal solution as well as the alternative optimal solution. The twin optimal solutions can used to perceive the substitution between objectives or objectives that are disagreement. Further, throughout the world the twin optimal solutions offer management with better flexibility in selecting and using resources.

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INTRODUCTION

The readers of this paper know the fundamentals of Transportation problems (TPs) and the key methods such as NWCM, LCM, VAM, MODI and Stepping Stone [2, 7, 8] available to solve them. Now, we shall see the very recent developments in TP area.

In 2021, Esakkiammal T. and Murugesan R. [1] proposed an innovative zero allocations approach named SOFTMIN which produces optimal solutions to most of the TPs. In 2022, Murugesan R. [6] established that the SOFTMIN method performs much better than the IASM method, but not a direct method to produce optimal solution to any given TP.

We further analyzed the process of allocation due to the SOFTMIN method on the near optimal solutions obtained for some ‘More Challenging’ TPs, and identified that very few changes made in the allocation process have improved the solution. This resulted in the ‘Improved SOFTMIN” (or briefly I-SOFT) method [3]. As for as our knowledge/search is concerned so far, no competing methods for generating best initial basic feasible solution (IBFS) on the identified some ‘More Challenging’ TPs are not available in the literature and thereby, the I-SOFT method may be the best one to produce the best IBFS to a given TP.

In 2022, Murugesan R. [4] proposed an innovative method named MODA (Modified Allocation) which tests the optimality of a solution and also optimizes the solution, if it is not optimal. By our further research we have identified the extra efforts made in the MODA method to trace and consider all possible loops starting and ending at an identified basic cell and passing through a non-basic cell. Consequently, we have simplified this difficulty by introducing the new idea of *Solution Improvement Loops* only to consider. This resulted in a revised version of the existing MODA method [5].

The paper is organized as follows: Section 1 – Briefs the introduction. Section 2 – Presents some basic definitions for the development of this paper. Section 3 – Presents the algorithm of the revised version of the MODA method. Section 4 – Illustrates one BTP having alternative optimal solution. Section 5 – Lists a set of 10 TPs having alternative optimal solutions. Section 6 – Displays the alternative optimal solutions of each of the problems. Section 7 – Draws the conclusion.

SOME BASIC DEFINITIONS

In this section, we present some basic definitions and concepts, which are necessary for the development of this paper.

Basic cell and Non-basic cell

A cell in a transportation table (TT) is said to be a *basic cell* if it is having some allocated quantity. The allocated quantity may be positive or zero. The other cells are called *non-basic cells*. A TT with size m × n will have at most (m+n-1) basic cells and the remaining cells are non-basic.

Non-degenerate BFS and Degenerate BFS

A basic feasible solution (BFS) to a TP is said to be *non-degenerate* if it contains exactly (m+n-1) numbers of basic cells, which are in independent positions. A BFS is said to be *degenerate* if it contains less than (m+n-1) numbers of basic cells.

Loop in a TT

A *loop* in a transportation table is an ordered set of even numbers (≥ 4) of cells having only one non-basic cell (NBC) and the remaining basic cells. The cells in a loop are called *corner cells*. The one NBC of a loop may occur at an add position or at an even position in the order of cells of the loop.

Net Cost Change (NCC) value of a loop

Consider one traced loop. Mark with a + sign and a – sign alternatively at each of the corner cells of the considered loop, starting from the non-basic cell (i, j) in it. Compute the *effect on cost* of the considered loop by adding together the original UTC found in each corner cell containing a + sign and then subtracting the original UTC found in each corner cell containing a – sign. This effect on cost is called the *net cost change* (NCC) value for the considered loop.

Solution improvement (SI) loop

A loop with negative NCC value only will improve the objective function value. Such a loop is called a *Solution Improvement (SI) loop*. If the identified basic cell (h, k) has two or more than two SI loops, then select the one having the minimum NCC value for implementation. If tie occurs among the SI loops with the same minimum NCC value, then implement any one loop arbitrarily. Such a situation will generate an alternative solution to the given TP. If the SI loop has NCC value as zero, this situation also generates an alternative solution to the given TP. In general, a SI loop will either eliminate the IBC from the basis or reduce the allocation at the IBC. However, if the SI loop has the NBC which occurs at the odd position in the order of cells of the loop it may increase the allocation value at the IBC. But, the IBC will be eliminated during the successive iterations.

Algorithm of the Existing 'Revised Version of the Moda' Method

The term MODA has been coined from the first three letters of the word 'Modified' and the first one letter of the word 'Allocation'. MODA is an iterative method which can be used for testing the optimality of an initial basic feasible solution (IBFS) and also optimize the IBFS, if it is not optimal, for transportation problems. The innovative way of improving a non-optimal solution to an optimal solution by the revised version of the MODA method is based on redistributing the allocation available at a currently allocated cell (basic cell) with largest 'unit transportation cost' (UTC) to another un-allocated cell (non-basic cell) and its subsequent induced reallocations. The algorithm of the revised version of the MODA method consists of two stages. In Stage #1, an IBFS is obtained to the given TP. In Stage #2, optimality testing of the obtained IBFS and also optimizing it, if it is not optimal, is carried out. Hereafter, we use the phrase 'MODA method' in the place of 'revised version of the MODA method'.

We use the following notations and abbreviations in the development of the algorithm of the MODA method:

$m \times n$	– Size of the unit cost matrix of the given TP
TT	– Transportation table
BTP	– Balanced transportation problem
UTP	– Unbalanced transportation problem
UTC	– Unit transportation cost
C_{ij}	– UTC available at the cell (i, j)
IBFS	– Initial basic feasible solution

$X = [x_{ij}]$	– A solution
X^*	– An optimal solution
TTC	– Total transportation cost
Z(X)	– TTC
$Z(X^*)$	– Minimum TTC
NBC	– Non-Basic Cell
IBC	– Identified Basic Cell
NCC	– Net Cost Change
SI	– Solution Improvement

Stage #1: Obtain an Ibfs to the Given Tp

For the given TP, first obtain an IBFS say $X^{(0)}$ with its associated total transportation cost $Z(X^{(0)})$ using any available method in TPs. We use the I-SOFT method [3] to obtain an IBFS because at present day it has been identified and established as the best method to find the best IBFS to TPs.

Stage #2: Test the Optimality of the Obtained Ibfs

Step 1: Construct the current solution table

Consider the transportation table (TT) en-squared with the obtained allocations (IBFS) $X^{(0)} = [x_{ij}]$ as the current solution table. Also, compute the corresponding TTC $Z(X^{(0)})$.

Step 2: Ensure the Non-degeneracy condition

Ensure the numbers of basic cells in the TT exactly equal to (m+n-1).

Step 3: Perform the Optimality Test on the IBFS $X^{(0)}$

(a) Determine $C(X^{(0)}) = \text{Max}\{c_{ij} : x_{ij} > 0\}$ and the corresponding basic cell as the identified basic cell (IBC). Let it be (h, k).

(i) If the IBC is unique, then go to Step (b) directly.

(ii) If there is two or more basic cells having the same largest UTC $C(X^{(0)})$, then select the basic cell having the maximum quantity of allocation as the IBC. Let it be (h, k) and go to Step (b).

(iii) If there is two or more basic cells having the same largest UTC $C(X^{(0)})$ and with the same maximum allocation quantity, then select any one such basic cell as the IBC. Let it be (h, k) and go to Step (b).

(b) Trace a SI loop starting and ending at the IBC (h, k) and passing through a non-basic cell. As it is a SI loop, it will have the Net Cost Change (NCC) value as non-positive (≤ 0). If there is a tie between two or more than two SI loops with the same NCC value, then select any one loop. Such a situation may generate alternative solutions to the given TP. If the NCC value of the SI loop is zero, then this will also indicate the existence of an alternative solution to the given TP.

(c) Implement this loop and obtain the better BFS, say $X^{(1)}$ with its associated TTC $Z(X^{(1)})$.

(d) If it is not possible to trace a SI loop starting and ending at the current IBC, then consider the next basic cell having UTC next to $C(X^{(0)})$ as the new IBC (h, k) and go to Step (a(i)).

Step 4: Repeat Steps 3(a) to (d) until no SI loop can be traced starting and ending at the new IBC with the current largest UTC. At this level, the solution under optimality test is the optimal one. Write the optimal solution X^* with its minimum TTC as $Z(X^*)$.

Alternative Optimal Solution

At the 'optimal level', if the NCC value of the SI loop is zero, then this indicates that the given TP has an alternative optimal solution. By implementing this loop we can get the alternative optimal solution to the given TP.

Important Note

1. One cannot restrict a SI loop with corner cells having UTCs less than or equal to the UTC of the identified basic cell.
2. One cannot restrict the place (even position or odd position) of the non-basic cell in a SI loop.

Numerical Illustration

Suitable illustrative explanation helps the readers to understand the algorithm of the MODA method in a better way, which produces an optimal solution and an alternative optimal solution, if it exists to a problem. Keeping in mind, one BTP from the literature is illustrated.

Example: Consider the following cost minimization type BTP with four sources and five destinations, as given in Table 1.

Table 1: The given BTP

Sources	Destinations					Supply
	D1	**D2**	**D3**	**D4**	**D5**	
S1	2	1	3	2	2	**20**
S2	3	2	1	1	1	**70**
S3	5	4	2	1	3	**30**
S4	7	5	5	2	3	**60**
Demand	**50**	**30**	**30**	**50**	**20**	**180**

Solution By The 'Moda Method'

Stage #1: Obtain an IBFS

In Stage #1, we solve the given BTP by using the I-SOFT method and obtain the IBFS ($X^{(0)}$) table. This is shown in Table 2

Stage #2: Optimizing the obtained solution by the MODA method

Construct the current solution table

Consider the transportation table (TT) en-squared with the obtained allocations (solution) as the current solution table. This is the IBFS and is shown in Table 2.

Table 2: The IBFS $X^{(0)}$ table obtained by the I-SOFT method

Sources	Destinations					Supply
	D1	D2	D3	D4	D5	
S1	2	**20** 1	3	2	2	**20**
S2	**50** 3	2	**20** 1	1	1	**70**
S3	5	4	**10** 2	**20** 1	3	**30**
S4	7	**10** 5	5	**30** 2	**20** 3	**60**
Demand	**50**	**30**	**30**	**50**	**20**	**180**

Writing the IBFS $X^{(0)}$

As of Table 2, the IBFS is $X^{(0)} = \{x_{12} = 20, x_{21} = 50, x_{23} = 20, x_{33} = 10, x_{34} = 20, x_{42} = 10, x_{44} = 30, x_{45} = 20\}$and the associated TTC is $Z(X^{(0)}) = \$400$.

Optimality Testing for the IBFS $X^{(0)}$ [First Iteration]

a) Determine $C(X^{(0)})$ = Max$\{c_{ij}\ ;\ x_{ij} > 0\}$ = Max {1, 3, 1, 2, 1, **5**, 2, 3} = 5 at the unique cell (h, k) = (S4, D2). Therefore, the IBC is (S4, D2).

b) Trace a Solution Improvement (SI) loop starting and ending at the IBC (S4, D2) and passing through one opt NBC. The possible such a loop is:

   Loop = {(S4, D2), (S2, D2), (S2, D3), (S3, D3), (S3, D4), (S4, D4)} passing through the NBC (S2, D2) with NCC value as 2 -1+2-1+2-5= -1.

c) By implementing this loop we obtain the following better BFS, say $X^{(1)}$, as shown in Table 3.

Table 3: A better BFS $X^{(1)}$ (Optimal Solution)

Sources	Destinations					Supply
	D1	D2	D3	D4	D5	
S1	2	**20** 1	3	2	2	**20**
S2	**50** 3	**10** 2	**10** 1	1	1	**70**
S3	5	4	**20** 2	**10** 1	3	**30**
S4	7	5	5	**40** 2	**20** 3	**60**
Demand	**50**	**30**	**30**	**50**	**20**	**180**

Writing the better BFS $X^{(1)}$

From Table 3, we see that the better BFS is $X^{(1)}$ = {x_{12} = 20, x_{21} = 50, x_{22} = 10, x_{23} = 10, x_{33} = 20, x_{34} = 10, x_{44} = 40, x_{45} = 20}and the associated TTC is Z($X^{(0}$) = $390. Note that $X^{(1)}$ is a better BFS than $X^{(0)}$ as Z($X^{(1}$) < Z($X^{(0}$).

Optimality Testing for the better BFS $X^{(1)}$ [Second Iteration]

(a) Determine C($X^{(1)}$**)** = Max{c_{ij} ; x_{ij} > 0} = Max {1, **3**, 2, 1, 2, 1, 2, **3**} = 3 at the cells (S2, D1) and (S4, D5) with quantity of allocations 50 and 20 units respectively. As the cell (S2, D1) is with the maximum allocation, we select the IBC as (S2, D1). Draw a Solution Improvement (SI) Loop starting and ending at (S2, D1). The possible such a loop is:

Loop = {(S2, D1), (S2, D2), (S1, D2), (S1, D1)} passing through the NBC (S1, D1) with NCC value as 2 -1+2-3=0.

(d) Further, it is not possible to trace a SI loop starting and ending at the new IBC (S4, D5). Similarly, it is not possible to trace any SI loop from other subsequent new IBCs (S2, D2), (S3, D3) and so on. This indicates that the current solution $X^{(1)}$ under optimality test is the optimal solution and we denote it by X^*.

Writing the Optimal Solution

The optimal solution (X^*) to the given TP is X^* = {x_{12} = 20, x_{21} = 50, x_{22} = 10, x_{23} = 10, x_{33} = 20, x_{34} = 10, x_{44} = 40, x_{45} = 20} with the minimum TTC of Z(X^*) = $390.

Alternative Optimal Solution

At the optimal level, as the NCC value of the SI loop starting and ending at the IBC (S2, D1) and passing through one NBC is zero, the given BTP has an alternative optimal solution. By implementing this loop we get an alternative optimal solution (X^{**}) as shown in Table 4.

Table 4: The Alternative Optimal Solution (X^{**}) table by the MODA method

	Destinations					
Sources	D1	D2	D3	D4	D5	Supply
S1	**20** 2	1	3	2	2	**20**
S2	**30** 3	**30** 2	**10** 1	1	1	**70**
S3	5	4	**20** 2	**10** 1	3	**30**
S4	7	5	5	**40** 2	**20** 3	**60**
Demand	**50**	**30**	**30**	**50**	**20**	**180**

Writing the Alternative Optimal Solution

The alternative optimal solution (X^{**}) to the given TP is X^{**} = {x_{11} = 20, x_{21} = 30, x_{22} = 30, x_{23} = 10, x_{33} = 20, x_{34} = 10, x_{44} = 40, x_{45} = 20} with the minimum TTC of Z(X^{**}) = $390.

Numerical Examples

By testing a large numbers of TPs from the literature and various textbooks, we have identified that 5 TPs of balanced category and 5 TPs of unbalanced category have alternative optimal solutions. The identified 10 TPs are listed in Table 5.

Table 5: A set of some TPs having twin optimal solutions

BTP Problem No.	UTP Problem No.
Problem 1 [C_{ij}] 3×3= [4 8 8; 16 24 16; 8 16 24] [S_i] 3×1= [61 82, 72] [D_j] 1×3= [72, 100, 43]	**Problem 1** [C_{ij}] 3×3= [4 8 8; 16 24 16; 8 16 24] [S_i] 3×1= [76, 82, 77] [D_j] 1×3= [72, 102, 41]
Problem 2 video [C_{ij}] 3×3= [6 3 5; 5 2 2; 12 7 8] [S_i] 3×1= [60, 80, 85] [D_j] 1×3= [75, 110, 40]	**Problem 2** [C_{ij}] 3×4= [3 48 14 2; 4 230 10; 36 8 12 12] [S_i] 3×1= [24, 24, 2] [D_j] 1×4= [6, 12, 3, 44]
Problem 3 [C_{ij}] 3×4= [6 1 9 3; 11 5 2 8; 10 12 4 7] [S_i] 3×1= [70, 55, 90] [D_j] 1×4= [85, 35, 50, 45]	**Problem 3** [C_{ij}] 3×4= [20 21 16 18; 17 28 14 16; 29 23 19 20] [S_i] 3×1= [10, 9, 7] [D_j] 1×4= [6, 10, 4, 5]
Problem 4 [C_{ij}] 4×5= [2 1 3 2 2; 3 2 1 1 1; 5 4 2 1 3; 7 5 5 2 3] [S_i] 4×1= [20, 70, 30, 60] [D_j] 1×5= [50, 30, 30, 50, 20]	**Problem 4** [C_{ij}] 3×4= [3 6 5 2; 4 --- 5 4; 3 4 4 4] [S_i] 3×1= [40, 50, 30] [D_j] 1×4= [50, 30, 40, 20]
Problem 5 [C_{ij}] 4×6= [9 12 9 6 9 10; 7 3 7 7 5 5; 6 5 9 11 3 11; 6 8 11 2 2 10] [S_i] 4×1= [5, 6, 2, 9] [D_j] 1×6= [4, 4, 6, 2, 4, 2]	**Problem 5** [C_{ij}] 4×6= [9 12 9 6 9 10; 7 3 7 7 5 5; 6 9 11 3 11 2; 6 11 2 2 10 2] [S_i] 4×1= [5, 6, 2, 2] [D_j] 1×6= [4, 4, 6, 2, 4, 2]

Twin Optimal Solutions of the Tps

For the identified 10 TPs (5 BTPs and 5 UTPs), as listed in Table 5, the IBFS ($X^{(0)}$) generated by the I-SOFT method, and the optimal solution (X^*) derived through the existing MODA method are shown in Table 6 and Table 7 respectively for the BTPs and UTPs . Also, the derived alternative optimal solutions (X^{**}) for the BTPs and the UTPs are shown in Table 8 and Table 9 respectively.

Table 6: Derivation of Optimal Solutions to the BTPs by the MODA method

BTP #	I-SOFT ($X^{(0)}$)	Optimal Solution (X^*) by MODA method	Minimum No. of iterations required to reach the optimal
1.	2688	2688	0
2.	1160	1160	0
3.	1165	1160	1
4.	400	390	1
5.	112	112	0

Table 7: Derivation of Optimal Solutions to the UTPs by the MODA method

UTP #	I-SOFT ($X^{(0)}$)	Optimal Solution (X^*) by MODA method	Minimum No. of iterations required to reach the optimal
1.	2752	2424	1
2.	188	180	1
3.	404	404	0
4.	440	440	0
5.	75	71	1

Table 8: Twin Optimal Solutions of the BTPs

BTP #	Optimal and Alternative Optimal Solutions
1.	$X^* = \{x_{12} = 61, x_{21} = 0, x_{22} = 39, x_{23} = 43, x_{31} = 72\}$ with $Z(X_1) = 2688$
	$X^{**} = \{x_{12} = 61, x_{21} = 39, x_{23} = 43, x_{31} = 33, x_{32} = 39\}$ with $Z(X_2) = 2688$
2.	$X^* = \{x_{11} = 60, x_{21} = 15, x_{22} = 25, x_{23} = 40, x_{32} = 85\}$ with $Z(X_1) = 1160$
	$X^{**} = \{x_{11} = 35, x_{12} = 25, x_{21} = 40, x_{23} = 40, x_{32} = 85\}$ with $Z(X_2) = 1160$
3.	$X^* = \{x_{12} = 30, x_{14} = 40, x_{22} = 5, x_{23} = 50, x_{31} = 85, x_{34} = 5\}$ with $Z(X_1) = 1160$
	$X^{**} = \{x_{11} = 40, x_{12} = 30, x_{22} = 5, x_{23} = 50, x_{31} = 45, x_{34} = 45\}$ with $Z(X_2) = 1160$
4.	$X^* = \{x_{12} = 20, x_{21} = 50, x_{22} = 10, x_{23} = 10, x_{33} = 20, x_{34} = 10, x_{44} = 40, x_{45} = 20\}$ with $Z(X_1) = 390$
	$X^{**} = \{x_{11} = 20, x_{21} = 30, x_{22} = 30, x_{23} = 10, x_{33} = 20, x_{34} = 10, x_{44} = 40, x_{45} = 20\}$ with $Z(X_2) = 390$
5.	$X^* = \{x_{13} = 5, x_{22} = 3, x_{23} = 1, x_{26} = 2, x_{31} = 1, x_{32} = 1, x_{41} = 3, x_{44} = 2, x_{45} = 4\}$ with $Z(X_1) = 112$
	$X^{**} = \{x_{13} = 5, x_{22} = 4, x_{23} = 0, x_{26} = 2, x_{31} = 1, x_{33} = 1, x_{41} = 3, x_{44} = 2, x_{45} = 4\}$ with $Z(X_1) = 112$

Table 9: Twin Optimal Solutions of the UTPs

UTP #	Optimal and Alternative Optimal Solutions
1.	$X^* = \{x_{12} = 76, x_{22} = 21, x_{23} = 41, x_{24} = 20, x_{31} = 72, x_{32} = 5\}$ with $Z(X_1) = 2424$
	$X^{**} = \{x_{12} = 76, x_{22} = 21, x_{23} = 41, x_{24} = 20, x_{31} = 51, x_{32} = 26\}$ with $Z(X_2) = 2424$
2.	$X^* = \{x_{14} = 24, x_{21} = 6, x_{22} = 12, x_{24} = 6, x_{33} = 2, x_{43} = 1, x_{44} = 14\}$ with $Z(X_1) = 180$
	$X^{**} = \{x_{14} = 24, x_{21} = 12, x_{22} = 12, x_{24} = 6, x_{34} = 2, x_{43} = 3, x_{44} = 12\}$ with $Z(X_2) = 180$
3.	$X^* = \{x_{12} = 3, x_{13} = 1, x_{14} = 5, x_{15} = 1, x_{21} = 6, x_{23} = 3, x_{32} = 7\}$ with $Z(X_1) = 404$
	$X^{**} = \{x_{12} = 3, x_{13} = 4, x_{14} = 2, x_{15} = 1, x_{21} = 6, x_{24} = 3, x_{32} = 7\}$ with $Z(X_2) = 404$
4.	$X^* = \{x_{11} = 20, x_{14} = 20, x_{21} = 10, x_{23} = 40, x_{31} = 20, x_{32} = 10, x_{42} = 20\}$ with $Z(X_1) = 440$
	$X^{**} = \{x_{11} = 20, x_{14} = 20, x_{21} = 30, x_{23} = 20, x_{32} = 10, x_{33} = 20, x_{42} = 20\}$ with $Z(X_2) = 440$
5.	$X^* = \{x_{11} = 3, x_{14} = 2, x_{22} = 4, x_{25} = 0, x_{26} = 2, x_{31} = 0, x_{35} = 2, x_{45} = 2, x_{51} = 1, x_{53} = 6\}$ with $Z(X_1) = 71$
	$X^{**} = \{x_{13} = 3, x_{14} = 2, x_{22} = 4, x_{25} = 0, x_{26} = 2, x_{31} = 0, x_{35} = 2, x_{45} = 2, x_{51} = 4, x_{53} = 3\}$ with $Z(X_1) = 71$

CONCLUSION

In this article, by testing a large numbers of balanced and unbalanced Transportation Problems (TPs), we have identified a set of ten TPs which are having alternative optimal solutions. For the identified TPs, we have applied the existing Modified Allocation (MODA) method to generate the optimal solution as well as the alternative optimal solution. The alternative optimal solutions can used to make out the changeover between objectives or objectives that are discrepancy. Also, throughout the globe the alternative optimal solutions suggest management with better flexibility in selecting and using resources.

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