Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 19, Number 2 (2023), pp. 849-856

http://www.ripublication.com

On Trigonometric Moments of Stereographic Reflected Log-Logistic Distribution

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Abstract

In this paper, derivation and evaluation of trigonometric moments of Stereographic Reflected Log-Logistic distribution are discussed. Also, population characteristics of Stereographic Reflected Log-Logistic distribution are computed using MATLAB.

INTRODUCTION

The characteristic function (ch. f.) of a circular model with the probability density function $g(\theta)$ is defined as $\varphi_p(\theta) = \int_0^{2\pi} e^{ip\theta} g(\theta)\, d\theta$, $p \in \mathbb{Z}$. Ramabhadra Sarma et al (2009; 2011) derived the characteristic functions for some wrapped models on the lines of the result [Jammalamadaka and Sen Gupta (2001), p.31]. This proposition is not applicable for Stereographic Circular model in establishing the Characteristic Function. The characteristic function of a Stereographic Circular model is derived in terms of corresponding linear distribution by employing the subsequent theorem associated to the characteristic function of linear distribution derived by Lukacs (1970).

Theorem 1.1

Let X be a random variable with distribution function $F(x)$ and suppose that $S(x)$ is a finite and single-valued function of *x*. The Characteristic function of $\varphi_Y(t)$ of the random variable $Y = S(x)$ is then given by

$$\varphi_Y(t) = E\left(e^{itY}\right) = E\left(e^{itS(X)}\right) = \int_{-\infty}^{\infty} e^{itS(X)} dF(x) \cdot$$

The above theorem is applied to propose the characteristic function of a Stereographic Circular model by Phani et al (2013) in Theorem 1.2.

Theorem 1.2

If $G(\theta)$ and $g(\theta)$ are the cdf and the pdf of the Stereographic Circular model and F(*x*) and *f*(*x*) are the cdf and pdf of the respective linear model, then the characteristic function of a Stereographic circular model is

$\varphi_{X_S}(p) = \varphi_{2\tan^{-1}\left(\frac{x}{v}\right)}(p), p \in$

Proof:

$$\varphi_{X_S}(p) = \int_{-\pi}^{\pi} e^{ip\theta} d\left(G(\theta)\right) \quad , p \in$$

$$= \int_{-\pi}^{\pi} e^{ip\theta} d\left(F\left(v \tan\frac{\theta}{2}\right)\right)$$

$$= \int_{-\infty}^{\infty} e^{ip\left(2\tan^{-1}\left(\frac{x}{v}\right)\right)} f(x)dx, \quad \text{taking } x = v\tan\left(\frac{\theta}{2}\right)$$

$$= \varphi_{2\tan^{-1}\left(\frac{x}{v}\right)}(p)$$

It is observed that the integral cannot be achieved analytically in general form, techniques from MATLAB tool box are used for the computation of the values of the characteristic function. The graph for the characteristic function is plotted here. Numerical integration of Weddle's rule or Gauss - Laguerre or Gauss – Hermite methods [Abramowitz and Stegun (1965)] whichever is applicable, is used for the computation of the needed values for the characteristic functions of the Stereographic circular models in plotting the characteristic functions.

STEREOGRAPHIC REFLECTED LOG-LOGISTIC DISTRIBUTION

A continuous random variable X is said to have a **Reflected Log-Logistic model** with scale parameter $\alpha > 0$ and shape parameter $\beta > 0$, then its probability density function and distribution function are given by

$$f(x)=\frac{\alpha\left(\frac{|x|}{\beta}\right)^{\alpha-1}}{2\beta\left[1+\left(\frac{|x|}{\beta}\right)^{\alpha}\right]^{2}}\ ,\ \alpha,\beta>0,\ -\infty<x<\infty\ (2.1)$$

$$F(x)=\frac{1}{2}\left[\frac{2+\left(\frac{|x|}{\beta}\right)^{-\alpha}}{1+\left(\frac{|x|}{\beta}\right)^{-\alpha}}\right],\alpha,\beta>0,\ -\infty<x<\infty\ (2.2)$$

Sreekanth et al (2018) derived pdf and cdf of Stereographic Reflected Log-Logistic distribution by applying inverse Stereographic projection defined by a one-one mapping from Phani et al (2012) $x=v\tan\left(\frac{\theta}{2}\right),-\pi<\theta<\pi,\ v>0$,we get a three parametric symmetric circular distribution on a unit circle.

Definition 2.1:

A random variable X_S on unit circle is said to have Stereographic Reflected Log-Logistic Distribution with shape parameter $\alpha>0$, scale parameter $\sigma>0$ and concentration parameter $v>0$, denoted by SRLLG(α,σ) ,then the probability density and cumulative distribution functions are given by

$$g(\theta)=\frac{\alpha\sigma\sec^{2}\left(\frac{\theta}{2}\right)\left(\sigma\left|\tan\left(\frac{\theta}{2}\right)\right|\right)^{\alpha-1}}{4\left(1+\left(\sigma\left|\tan\left(\frac{\theta}{2}\right)\right|\right)^{\alpha}\right)^{2}}\ ,\text{where }\alpha,\sigma>0\ ,-\pi<\theta<\pi\ \ (2.3)\text{ and}$$

$$G(\theta)=\begin{cases}1-\frac{1}{2}\left[\frac{2+\left(\sigma\left|\tan\left(\frac{\theta}{2}\right)\right|\right)^{-\alpha}}{1+\left(\sigma\left|\tan\left(\frac{\theta}{2}\right)\right|\right)^{-\alpha}}\right],\ \alpha,\sigma>0,\ -\pi<\theta<0\\ \frac{1}{2}\left[\frac{2+\left(\sigma\left|\tan\left(\frac{\theta}{2}\right)\right|\right)^{-\alpha}}{1+\left(\sigma\left|\tan\left(\frac{\theta}{2}\right)\right|\right)^{-\alpha}}\right],\ \alpha,\sigma>0,\ 0<\theta<\pi\end{cases}\qquad(2.4)$$

Respectively. Here $g(\theta)$ satisfies the conditions of circular model. The new model SRLLG(α,σ) is a circular model "**STEREOGRAPHIC REFLECTED LOG - LOGISTIC MODEL".** It can be viewed that the Stereographic Reflected Log - Logistic Model is symmetric model.

THE CHARACTERISTIC FUNCTION OF STEREOGRAPHIC REFLECTED LOG-LOGISTIC MODEL

$$\Phi_{X_S}(p)=\int_{-\pi}^{\pi} e^{ip\theta} g(\theta)\, d\theta$$

$$=\int_{-\pi}^{\pi} \cos p\theta \frac{\alpha\sigma \sec^2\left(\frac{\theta}{2}\right)\left[\sigma\left|\tan\left(\frac{\theta}{2}\right)\right|\right]^{\alpha-1}}{4\left(1+\left(\sigma\left|\tan\left(\frac{\theta}{2}\right)\right|\right)^{\alpha}\right)^2} d\theta + i\int_{-\pi}^{\pi} \sin p\theta \frac{\alpha\sigma \sec^2\left(\frac{\theta}{2}\right)\left[\sigma\left|\tan\left(\frac{\theta}{2}\right)\right|\right]^{\alpha-1}}{4\left(1+\left(\sigma\left|\tan\left(\frac{\theta}{2}\right)\right|\right)^{\alpha}\right)^2} d\theta$$

$$=\int_{0}^{\pi} \cos p\theta \frac{\alpha\sigma \sec^2\left(\frac{\theta}{2}\right)\left[\sigma\left|\tan\left(\frac{\theta}{2}\right)\right|\right]^{\alpha-1}}{2\left(1+\left(\sigma\left|\tan\left(\frac{\theta}{2}\right)\right|\right)^{\alpha}\right)^2} d\theta \text{ , since } \sin p\theta \text{ is odd}$$

3.1 Trigonometric moments of the Stereographic Reflected Log-Logistic Model

The trigonometric moments of the distribution are given by $\{\varphi_p : p=\pm1,\pm2,\pm3,\dots\}$, where $\varphi_p=\alpha_p+\beta_p$, with $\alpha_p=E(\cos p\theta)$ and $\beta_p=E(\sin p\theta)$ being the p^{th} order cosine and sine moments of the random angle θ, respectively. Because the **Stereographic Reflected Log-Logistic** distribution is symmetric, it follows that the sine moments are zero. Thus $\varphi_p=\alpha_p$.

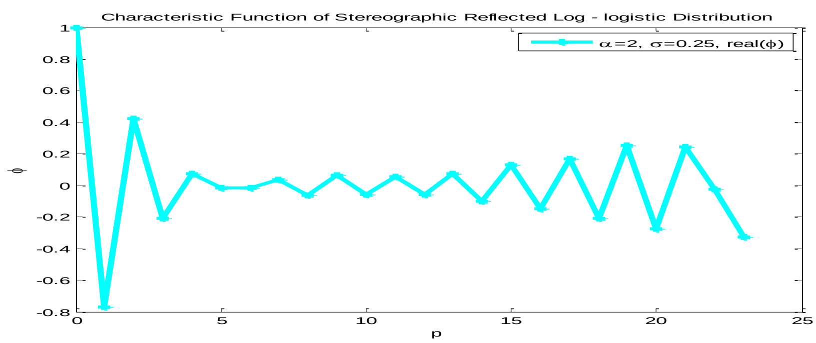

Figure:- 1 Graph of characteristic function of Stereographic Reflected Log – Logistic

Distribution for $\alpha = 2,\ \sigma = 0.25$

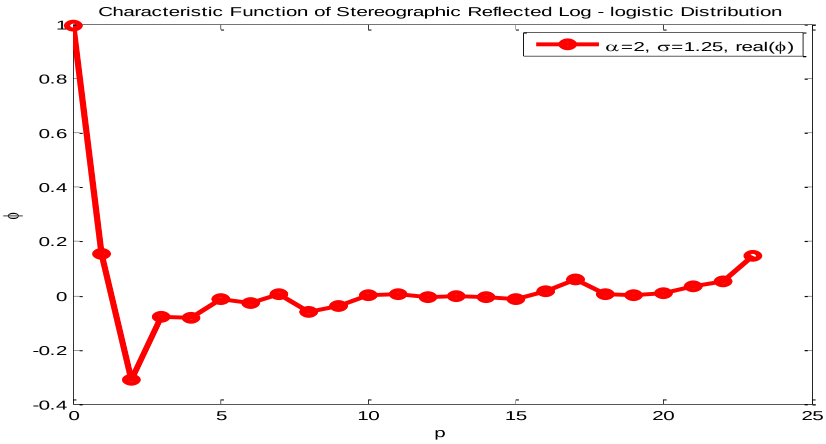

Figure 2 Graph of characteristic function of Stereographic Reflected Log – Logistic Distribution for $\alpha = 2,\ \sigma = 1.25$

Expressions for mean direction, resultant length, circular variance, circular standard deviation, central trigonometric moments, skewness and kurtosis for circular distributions are adopted from Mardia and Jupp (2000) and these characteristics for the Stereographic Reflected Log - Logistic model are also resulted on their respective trigonometric moments α_p and β_p . To compute the characteristic function for all integral values of p, we use numerical integration of Weddle's rule.

Table 1 Population Characteristics of SRLLG for $\alpha = 2$

	$\sigma = 0.25$	$\sigma = 0.5$	$\sigma = 0.75$	$\sigma = 1.25$
Trigonometric Moments				
α_1	-0.7457	-0.4382	-0.1685	0.1548
α_2	0.3666	-0.0979	-0.3210	-0.3102
Resultant Length				
ρ_1	0.7457	0.4382	0.1685	0.1548
ρ_2	0.3666	0.0979	0.3210	0.3102
Mean Direction				
μ_0	0	0	0	0
Circular Variance				
V_0	0.2543	0.5618	0.8315	0.8452
Circular Standard Deviation				
σ_0	0.7661	1.2846	1.8874	1.9317
	1.4167	2.1558	1.5076	1.5302
Central Trigonometric Moments				
α_1^*	0.7457	0.4382	0.1685	0.1548
α_2^*	0.3666	0.3479	0.3210	0.3102
Kurtosis				
γ_2^0	0.8872	0.5934	0.4630	0.4333

PROGRAM LISTINGS

The MATLAB program listings of population characteristics of the Stereographic Reflected Log - Logistic model are provided here.

4.1 Program for graph of characteristic function of Stereographic Reflected Log - Logistic Distribution and population characteristics of Stereographic Reflected Log - Logistic Distribution

```
a=0.1031;%input('enter the value of a =');
th=linspace(0+a,pi-a,24);
h=pi/24;
sig=0.25%input('enter the value of sigma =');% o.2;
alpha=2
f=(alpha.*sig).*(((sec(th/2))).^2.*(sig.*abs(tan(th/2))).
^(alpha-1))./(2*(1+(sig.*abs(tan(th/2))).^(alpha)).^2);
c=[1 5 1 6 1 5 1];
C1=zeros(1,24);
%S1=zeros(1,24);
for p=0:23
yc=cos(p.*th).*f;
%ys=sin(p.*th).*f;
s1=0;
%s2=0;
k=p+1;
for m=1:4
for i=1:6
s1=s1+c(i)*yc(6*(m-1)+i);
%s2=s2+c(i)*ys(6*(m-1)+i);
end
end
s1;%s2;
re=(3*h/10)*s1;
%im=(3*h/10)*s2;
C1(k)=re;
```

```
%S1(k)=im;
end
C1;%S1;
phi=C1;
phi(1)
p=0:23;
plot(p,real(phi),'-*r')
hold on
%plot(p,imag(phi),'-*m')
ylabel('\phi')
legend('\alpha=2, \sigma=1.25, real(\phi)')
alpha=[real(phi(2)),real(phi(3))]
beta=[0,0];%[imag(phi(2)),imag(phi(3))]
[mu,rho1,v0,sig0,gamma1,gamma2]=circpropnew(alpha,beta)
title('Characteristic Function of Stereographic Reflected
Log - Logistic Distribution')
xlabel('p')
```

Circpropnew

```
function[mu,rho1,v0,sig0,gamma1,gamma2]=circpropnew(alpha,beta)
rho=sqrt((beta(1)).^2+(alpha(1)).^2);
rho;
mu=atan(beta./alpha);
fprintf('Mean Direction =');
  mu
fprintf('Resultant Length =');
rho1=sqrt((beta).^2+(alpha).^2)
p=1:2;
fprintf('Central Trignometric Moments alpha1 & alpha2: \n')
p=1:2;
alphag(p)=rho1.*cos(mu-p*mu(1))
fprintf('circular variance =');
v0=1-rho1(1) %circular variance;
```

```
fprintf('circular standard deviation =');
sig0=sqrt(abs(log(1./((rho1).^2))))% circular standard deviation;
fprintf('kurtosis =');
gamma2=(alphag(2)-((1-v0).^4))./(v0.^2) %kurtosis;
```

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