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ADDOBT– A New Additional Obtainable Optimal Tour Plans Generation Method to Traveling Salesman Problems

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Abstract

A traveling salesman problem (TSP) is an assignment problem (AP) with added restrictions, in which the intention is to attain the best possible way of visiting all the cities only once and returning back to the starting point that minimize the overall travel distance or cost. It occupies the most important role in the real life situations. The prominent method used to solve the TSPs is the Hungarian method. In this research article, we have acknowledged and established a list of TSPs which are having additional available optimal tour plans (OTPs). For the recognized TSPs, the **add**itional **obt**ainable OTPs are generated by means of a new method named ADDOBT.

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Keywords: Assignment Problem, Traveling Salesman Problem, Hungarian method, MEASSI method, ADDOBT method.

INTRODUCTION

The Traveling Salesman Problem (TSP) is a problem in combinatorial optimization deliberated in operations research. A set of cities is given to a salesman and he has to start from a city, visit all the cities only once and return back to the start city to complete a round tour such that the length of the tour is the shortest among all possible tour plans. Because the TSP is a particular class of AP with extra conditions, a solution to the TSP, in general, is found using the methods existing for solving APs. In the recent years significant number of zeros assignment methods and ones assignment methods have been published by quite a few researchers for solving APs.

In this section, we briefly bring in the important zeros and ones assignment methods published so far recently.

During 2014, Hadi Basirzadeh [2] presented an approach namely, Ones Assignment Method for solving the TSP, by a little modification in the procedure given in [1] to obtain a tour of the TSP. In this method, priority rule plays a vital role in order to make a tour.

In 2016, Mohammed Ahmed Shihab Alkailany [5] presented a New Revised Ones Assignment Method to solve TSP and the author claims that the results of the tests show that this method is better than the Ones Assignment Method. But, the solutions generated by this method for the TSPs shown in Example 1 and Example 3 of this paper have cycles and hence not feasible to the given TSPs. This method has not provided any clear cut ruleto make a tour when the solution consisting cycles.

In 2019, Janusz Czopik [4] offered a new polynomial time algorithm which uses the Hungarian algorithm for AP to solve TSP. The running time of this algorithm is as the algorithm of AP. In this method, the given TSP considered as an AP is solved by the Hungarian method of assignment, thereby an optimal solution is obtained. If this optimal solution contains more cycles, then it is removed by constructing a Modifying Distance (Cost or Time) Matrix and then solving the same by Hungarian Algorithm in order to obtain an optimal solution with only one cycle. This algorithm removes the cycles generated by the Hungarian algorithm, but does not guaranteed to yield an optimal solution to a TSP. By this algorithm, Example 3 shown in this paper has got a tour plan with length Z = 34 units instead the optimal tour plan generated by our algorithm [6, 8] with minimum length Z = 26 units.

In the same year 2019, Susanta Kumar Mohanta [7] projected a direct approach to discover an optimal solution to the TSP in a single shot with a rational amount of time from the network of a complete graph, complete digraph or connected graph. In this approach, assignments are made based on choosing a suitable assignment preference table consisting of absolute favorable costs of from cities or to cities and their frequencies.

R. Murugesan and T. Esakkiammal [6], in 2021, presented a new two phase method named ESA-ESAN for generating the OTPs of TSPs based on ones assignment technique.

T. Esakkiammal and R. Murugesan [8], in 2021, introduced another new two phase method named ESAN-ESA for producing the OTPs of TSPs based on zeros assignment approach.

In this article, we have identified and confirmed a list of TSPs which are having more than one OTP. The generation of the obtainable OTPs is done by applying the new method known as ADDOBT.

This article is designed as follows: In Section 1, introduction is given. In Section 2, the algorithm of the new proposed ADDOBT method is presented. In Section 3, one benchmark TSP having more available OTPs has been illustrated. A set of 10 benchmark TSPs (having additional obtainable OTPs) identified from the literatures and textbooks has been tabulated in Section 4. The obtainable OTPs generated by the new proposed ADDOBT method on the 10 benchmark TSPs are tabulated in Section 5. To end with, in Section 6, conclusions are drawn.

Length of a Tour For a *tour* to an n city TSP, the salesman travels precisely n arcs (or n edges). Adding of the values (distances or costs) in every arc of a tour yields the *length of the tour*. The length of a tour may be in terms of time or distance or currency units.

Optimal Tour A tour with the shortest length is called a *least length tour* or an *optimal tour*. The length of an optimal tour is denoted by the symbol Z. It is clear that, $Z \leq$ Length of a tour.

Lower Bound (LB) Adding together the minimum values in every row of a TSP provides a lower bound (LB) for Z.

Tighter Lower Bound (TLB) It is obvious that the TSP is a restricted version (having added constraints) of the AP. Therefore, we loosen up temporarily by removing the added constraints and solve it as an AP. If the optimal assignment plan to the AP is feasible to the TSP also, then it would have been the OTP to the TSP. If it is infeasible to the TSP, then the overall total distance (or cost) related to the optimal assignment plan is a lower bound to the optimal value Z. This lower bound is, in general, tighter than the existing LB for Z and therefore, it is called as tighter lower bound (TLB). Note that, $LB \leq TLB \leq Z$.

Upper Bound (UB) Sum of the average values in every row of a TSP gives an upper bound (UB) for Z. Note down that, always $Z \leq$ Length of a tour $\leq UB$.

Relation between the Bounds and the Length of a Tour

The relation between the various bounds and the length of a tour is given by

$LB \leq TLB \leq Z \leq$ Length of a tour $\leq UB$.

But, the bounding condition $TLB \leq Z$ is not true for very few instances.

Algorithm of the new proposed ADDOBT Method

The term ADDOBT has been coined from the first three letters of the two words **Add**itional and **Obt**ainable. The method ADDOBT consists of two phases. In the first phase, a set of optimal subtours is generated using the zeros assignment technique named MEASSI [9] and in the second phase connecting of all the subtours together to form a tour plan is carried out based on the available 0-entry or 1-entry or the next available higher entry among the upcoming unassigned cells in the associated reduced cost matrix. The algorithm is as follows:

Phase-I (Generating subtours)

In this phase, a set of optimal subtours is generated by using the MEASSI method. The readers may refer [9] for detailed explanation of the MEASSI method. Also, see the algorithm in Appendix A.

Algorithm of the Existing MEASSI Method

The term MEASSI has been coined from the first letter of the names of the authors **M**urugesan and **E**sakkiammal followed by the first four letters of the word **Assi**gnment. The MEASSI method is a very simple and efficient method for finding the optimal solution of an assignment problem without making a direct comparison of every solution. This method also works on the principle of reducing the given cost matrix to a reduced cost matrix (RCM) and making assignments to the selected zero-entry cells of the RCM in a way entirely different from that of by the Hungarian method. Here, the algorithm for the MEASSI method (minimization case) for determining the optimal solution of APs has been proposed. The following are the sequence of steps involved in it:

1) **Check the Balanced Condition.** Check whether the given AP is balanced or not. If the AP is balanced, go to Step 3; otherwise, go to Step 2.
2) **Convertinto Balanced AP.** If the AP is not balanced, then anyone of the following two cases may arise:
    a) If the number of rows exceeds the number of columns, set up required number of extra dummy columns with the assignment table to equalize with the rows. The unit assignment cost for the cells in these dummy column(s) is set to 0. Go to Step 4(a).or
    b) If the number of columns exceeds the number of rows, bring in required number of additional dummy rows with the assignment table to equalize with the columns. The unit assignment cost for the cells in these dummy row(s) is set to 0. Go to Step 4(b).
3) **Build the Reduced Cost Matrix (RCM).**
    a) **Do the Row Minimum Subtraction (RMS) Operation**.

       Subtract the minimum cost from each of the costs of every individual row of the balanced AP. This will result in a resultant matrix.
    b) **Do the Column Minimum Subtraction (CMS) Operation**.

       Subtract the minimum cost from each of the costs of every respective column of the resultant matrix obtained in Step 3(a). Go to Step 5.
4) **Build the Reduced Cost Matrix (RCM).**
    a) **Do the Column Minimum Subtraction (CMS) Operation**.

       Subtract the minimum cost from each of the costs of every individual column of the obtained balanced AP. This will result in anRCM.Go to Step 5.

    b) **Do the Row Minimum Subtraction (RMS) Operation**.

       Subtract the minimum cost from each of the costs of every respective row of the obtained balanced AP. This will result in anRCM. Go to Step 5.

Note: The resultant matrix obtained via Step 3(b) or Step 4(a) or (b) is known as the*reduced cost matrix* (RCM). It is well-known that there will be at least one 0-entry in each row and in each column of an RCM. The cells having only 0-entries in anRCM are called *0-entry cells*.

5) **Make assignments one by one in the RCM by applying the Tie Breaking Techniques.**
   a) For each row, find the difference between the first minimum (0) and the second minimum (may be 0 or other). Let it be d1. Then find the difference between the first minimum (0) and the third minimum (0 or other). Let it be d2. Add d1 and d2. Write the resulting sum of two differences (STD) under the STD column by enclosing it in parentheses against the respective row. Similarly, do the same computation for each column.
   b) Mark by *, the maximum among the STD values computed for rows and columns, along the corresponding row(s) and/ column(s).
   c) Choose the row or column which is marked by * and assign to the cell having 0-entry in that row or column. If tie occurs among certain 0-enty cells in that chosen row or column, select the 0-entry cell which has the highest original cost figure for assignment. If tie occurs among the highest original cost figure, break the tie arbitrarily for assignment and such a situation may produce an alternative optimal solution to the given AP.
   d) Again, if tie occurs in case of (ii) amongst *, then select all these rows and columns (marked by *). For each such rows and columns identify all the 0-entry cells. For each such 0-entry cells compute the total of the row entries sum and the column entries sum of the corresponding row as well as column of the RCM. Select the 0-entry cell for assignment for which the computed total is the maximum. If tie occurs among certain 0-entry cells with the same computed total, then consider each such 0-entry cell for assignment as a separate case and continue further for getting complete assignment plan and finally select the best plan (solution). Such a situation may generate an alternative optimal solution to the given AP.
6) R**educe the RCM further.**After performing Step 5, delete the corresponding row as well as the column of the cell for which an assignment is made for further computation as they will not be taken into account for making any more assignments.
7) **Develop a revised RCM.**Check whether the resultant matrix obtained via Step 6 possesses at least one 0-entry in each row and in each column. If so, go to Step 5 for making the next assignment; otherwise, go to Step 3 for building a new revised RCM.
8) **Repeat Steps**. Repeat Steps 3 to 7 until and unless all the assignments have been made.
9) **Write the assignments.**Write the assignments one by one row-wise.
10) **Compute the Overall Cost.** Finally, compute the overall cost corresponding to the assignments [ignoring the assignments in the dummy row(s) or column(s)] obtained in Step 9 using the original cost matrix.

Phase-II (Connecting duly of all the subtours to form a tour plan)

At the moment we will connect all the subtours together correctly to form a tour plan. The connecting operation is carried out by following the underneath steps:

Step 1: Consider the subtour of shortest length. If more than one subtours of shortest length occurs, then consider anyone of them arbitrarily.

Step 2: In the shortest length subtour, start the tour plan from the first city of the subtour and travel ahead along the sequence given in the subtour and identify the finally visited city (not the start city) and let it be i. In the i^{th} row of the corresponding reduced cost matrix (RCM), look for the city (column), not visited already, with the existing next 0-entry (or 1-entry or next available higher entry). Let it be j.

Step 3: Identify the subtour in which the city j lies and continue the journey from city j in the sequence given in the subtour in which j lies or continue the journey from city j to another city with the existing next 0-entry (or 1-entry or next available higher entry) in the associated RCM.

Step 4: If all the cities are visited only once, then return back to the start city; otherwise, spot the finally visited city in the subtour in which j lies. Let it be city k. In the k^{th} row of the corresponding RCM, look for the city (column), not visited already, with the existing next 0-entry (or 1-entry or next available higher entry). Let it be l.

Step 5: Repeat Steps (3) and (4) until a tour plan with size n (a feasible solution) has been obtained to the given TSP. Compute the length of the tour.

Step 6: Next, in the shortest length subtour, start the tour plan from the second city (now it is the start city) and travel ahead and identify the finally visited city (not the start city) and let it be p. In the p^{th} row of the corresponding RCM, look for the city (column), not visited already, with the next existing 0-entry (or 1-entry or next available higher entry). Let it be q. Repeat Steps (4) and (5) until a tour plan with size n (a feasible solution) has been obtained to the given TSP. Compute the length of the tour.

Step 7: Repeat Steps (3) to (6) until a tour plan with size n (a feasible solution) has been obtained to the given TSP starting from the last but one city in the shortest length subtour. Among the generated tour plans, identify the tour plan(s) with the minimum length and is the optimal tour plan(s) to the given TSP.

ILLUSTRATIVE TSP

Precise illustrative explanation makes the readers to be well-known with the proposed ADDOBT method thoroughly. In that way, the objective of this article – finding more available OTPs (if it exists) – is achieved. Keeping in mind, two TSPs from the literature have been illustrated.

Example 1: Consider the following 5-city symmetric TSP, referred from J.K. Sharma [3], whose distance (in miles) matrix is shown in Table 1.

Table 1: The given 5-city TSP

	City				
City	**1**	**2**	**3**	**4**	**5**
1	--	17	16	18	14
2	17	--	18	15	16
3	16	18	--	19	17
4	18	15	19	--	18
5	14	16	17	18	--

SOLUTION BY THE PROPOSED METHOD

By applying the steps of Phase-I of the proposed method, one can get the following two sets of subtours as shown in Table 2 and Table 3 respectively.

Table 2: Set-I of subtours and their length

Subtour	Length of Subtour (in miles)
1 → 3 → 5→ 1	16 + 17 + 14 = 47
2 → 4 → 2	15 + 15 = **30**
Tighter Lower Bound (TLB)	77

Table 3: Set-II of subtours and their length

Subtour	Length of Subtour (in miles)
1 → 5 → 3→ 1	14 + 17 + 16 = 47
2 → 4 → 2	15 + 15 = **30**
Tighter Lower Bound (TLB)	77

The least length subtour in Set-I are 2 → 4 → 2, which is well thought-out first in Phase-II.

By applying the steps of Phase-II of the proposed method on Set-I, one can have the following tours along with their length, which are shown in Table 4:

Table 4: Generated tours and their length from Set-I

Subtour	**Start city**	**Generated Tour**	**Length of Tour** (in miles)
2 → 4 → 2	2	2 → 4 → 3 → 5 → 1 → 2	15 + 19 + 17 + 14 + 17 = 82
		2 → 4 → 3 → 1 → 5 → 2	15 + 19 + 16 + 14 + 16 = **80**
	4	4 → 2 → 3 → 5 → 1 → 4	15+ 18 + 17 + 14 + 18 = 82
		4 → 2 → 3 → 1 → 5 → 4	15+ 18 + 16 + 14 + 18 = 81
		4 → 2 → 5 → 1 → 3 → 4	15+ 16 + 14 + 16 + 19 = **80**

Writing the OTP: As of Table 3, we see that, the tour with least length is 80 miles. Consequently, the two distinct OTPsare 2 → 4 → 3 → 1 → 5 → 2 and 4 → 2 → 5 → 1 → 3 → 4 with Z= 80.

Moreover, the least length subtour from Set-II is also 2 → 4 → 2, which is considered first in Phase-II.By applying the steps of Phase-II, we can have the following tours along with their length, which are shown in Table 5:

Table 5: Generated tours and their length from Set-II

Subtour	Start city	Generated Tour	Length of Tour (in miles)
2 → 4 → 2	2	2 → 4 → 3 → 1 → 5 → 2	15 + 19 + 16 + 14 + 16 = **80**
	4	4 → 2 → 3 → 1 → 5 → 4	15 + 18 + 16 + 14 + 18 = 81
		4 → 2 → 5 → 3 → 1 → 4	15 + 16 + 17 + 16 + 18 = 82
		4 → 2 → 5 → 1 → 3 → 4	15 + 16 + 14 + 16 + 19 = **80**

Writing the OTP: From Table 5, we see that, the tour with least length is 80. Accordingly, the two distinct OTPsare 2 → 4 → 3 → 1 → 5 → 2 and 4 → 2 → 5 → 1 → 3 → 4 with Z= 80.

Hence, the given TSP has two separate OTPs: 1 → 3 → 4 → 2 → 5 → 1 and 1 → 5 → 2 → 4 → 3 → 1) with Z= 80. In other words, the given TSP has additional available OTPs.

Example 2: Consider the following 7-city symmetric TSP, referred from J.K. Sharma [3], whose distance (in miles) matrix is shown in Table 6.

Table 6: The given 5-city TSP

	City						
City	**1**	**2**	**3**	**4**	**5**	**6**	**7**
1	--	86	49	57	31	69	50
2	86	--	68	79	93	24	5
3	49	68	--	16	7	72	67
4	57	79	16	--	90	69	1
5	31	93	7	90	--	86	59
6	69	24	72	69	86	--	81
7	50	5	67	1	59	81	--

SOLUTION BY THE PROPOSEDADDOBT METHOD

By applying the steps of Phase-I of the proposed method, one can get the following subtours as shown in Table 7.

Table 7: Subtours and their length

Subtour	Length of Subtour (in miles)
1 → 5 → 3→ 1	31 + 7 + 49 = 87
2 → 6 → 2	24+ 24 = 48
4 → 7 → 4	1 + 1 = **2**
Tighter Lower Bound (TLB)	137

Note that, the least length subtour is 4 → 7 → 4, which is well thought-out first in Phase-II.

By applying the steps of Phase-II of the proposed method, one can have the following tour plans along with their length, which are shown in Table 8.

Table 8: Generated tours and their length

Subtour	**Start city**	**Generated Tour**	**Length of Tour** (in miles)
4 → 7 → 4	4	4 → 7 → 2 → 6 → 1 → 5 → 3 → 4	1 + 5 + 24 + 69 + 31 + 7 + 16 = 153
	7	7 → 4 → 3 → 1 → 5 → 6 → 2 → 7	1 + 16 + 49 + 31 + 86 + 24 + 5 = 212
		7 → 4 → 3 → 5 → 1 → 6 → 2 → 7	1 + 16 + 7 + 31 + 69 + 24 + 5 = 153

Writing the OTP

As of Table 8, we see that, the tour with least length is 153 miles. Consequently, the OTPS are:

4 → 7 → 2 → 6 → 1 → 5 → 3 → 4 (equivalently, 1 → 5 → 3 → 4 → 7 → 2 → 6 → 1) and 7 → 4 → 3 → 5 → 1 → 6 → 2 → 7 (equivalently, 1 → 6 → 2 → 7 → 4 → 3 → 5 → 1) with Z= 153.Hence, the given TSP has two separate OTPs:

1 → 5 → 3 → 4 → 7 → 2 → 6 → 1 and 1 → 6 → 2 → 7 → 4 → 3 → 5 → 1 with Z= 153. In other words, the given TSP has additional obtainable OTPs.

Benchmark TSPs

In order to validate the objective of this article, we have identified and solved ten benchmark symmetric TSPs, in different relatively small sizes, each having more available OTPs, from a range of literatures and textbooks, which are shown in Table 9.

Table 9: List of benchmark symmetric TSPs having additional obtainable OTPs

TSP	**TSP**
Problem 1 $[C_{ij}]5\times5$= [-- 16 4 12 --; 16 -- 6 -- 8; 4 6 – 5 6; 12 -- 5 – 20; -- 8 6 20 --]	**Problem 6** $[C_{ij}]5\times5$= [-- 17 16 18 14; 17 – 18 15 16; 16 18 – 19 17; 18 15 19 – 18; 14 16 17 18 --]
Problem 2 $[C_{ij}]5\times5$= [-- 3 6 2 3; 3 -- 5 2 3; 6 5 – 6 4; 2 2 6 – 6; 3 3 4 6 --]	**Problem 7** $[C_{ij}]6\times6$= [-- 13 2 15 15 15; 13 -- 14 1 12 12; 2 14 – 16 14 14; 15 1 16 – 10 10; 15 12 14 10 – 4; 15 12 14 10 4 --]
Problem 3 $[C_{ij}]5\times5$= [-- 10 8 9 7; 10 -- 10 5 6; 8 10 – 8 9; 9 5 8 – 6; 7 6 9 6 --]	**Problem 8** $[C_{ij}]6\times6$= [-- 1 5 4 -- --; 1 -- 2 2 -- 2; -- 2 – 1 4 1; 4 2 1 – 2 2; -- -- 4 2 – 3; -- 2 1 2 3 --]
Problem 4 $[C_{ij}]5\times5$= [-- 10 12 14 8; 10 -- 13 8 9; 12 13 – 12 8; 14 8 12 – 11; 8 9 8 11 --]	**Problem 9** $[C_{ij}]6\times6$= [-- 12 29 22 13 24; 12 -- 19 3 25 6; 29 19 – 21 23 28; 22 3 21 – 4 5; 13 25 23 4 – 16; 24 6 28 5 16 --]
Problem 5 $[C_{ij}]5\times5$= [-- 8 4 9 9; 8 -- 6 7 10; 4 6 – 5 6; 9 7 5 – 4; 9 10 6 4 --]	**Problem 10** $[C_{ij}]7\times7$= [-- 86 49 57 31 69 50; 86 -- 68 79 93 24 5; 49 68 – 16 7 72 67; 57 79 16 – 90 69 1; 31 93 7 90 – 86 59; 69 24 72 69 86 – 81; 50 5 67 1 59 81 --]

Outcomes Analysis

On behalf of every one of the ten numbers of TSPs, listed in Table 4, the subtours generated by Phase-I along with their lengths and the OTPs generated by Phase-II along with their least lengths as a result of ADDOBT method are shown in Table 10.

Table 10: Subtours and the resultant OTPs generated by the ADDOBT method

Prob. No.	Subtours generated by Phase-I	Length of Subtours	OTPs generated by Phase-II	Length of Tour (Z)
1.	1 → 3 → 4 → 1	21		
	2 → 5 → 2	**16**	2 → 5 → 3 → 4 → 1 → 2	47
			5 → 2 → 1 → 3 → 4 → 5	47
			5 → 2 → 1 → 4 → 3 → 5	47
2.	1 → 4 → 1	**04**	4 → 1 → 5 → 3 → 2 → 4	16
	2 → 5 → 3 → 2	12		
	(Alternative subtours)			
	1 → 4 → 2 → 1	**07**	1 → 4 → 2 → 3 → 5 → 1	16
	3 → 5 → 3	08		
3.	1 → 3 → 1	**16**	1 → 3 → 4 → 2 → 5 → 1	34
	2 → 4 → 5 → 2	17	3 → 1 → 5 → 2 → 4 → 3	34
4.	1 → 3 → 5 → 1	28		
	2 → 4 → 2	**16**	2 → 4 → 3 → 5 → 1 → 2	46
	(Alternative subtours)		4 → 2 → 1 → 5 → 3 → 4	46
	1 → 5 → 3 → 1	28		
	2 → 4 → 2	**16**	4 → 2 → 1 → 5 → 3 → 4	46
5.	1 → 3 → 2 → 1	17		
	4 → 5 → 4	**08**	5 → 4 → 2 → 1 → 3 → 5	29
	(Alternative subtours)			
	1 → 2 → 3 → 1	18		
	4 → 5 → 4	**08**	4 → 5 → 3 → 1 → 2 → 4	29
6.	1 → 3 → 5 → 1	47		
	2 → 4 → 2	**30**	2 → 4 → 3 → 1 → 5 → 2	80
			4 → 2 → 5 → 1 → 3 → 4	80
7.	1 → 3 → 1	04		
	2 → 4 → 2	**02**	2 → 4 → 5 → 6 → 3 → 1 → 2	44
	5 → 6 → 5	08	2 → 4 → 6 → 5 → 3 → 1 → 2	44
8.	1 → 2 → 1	**02**	1 → 2 → 3 → 6 → 5 → 4 → 1	13
	3 → 6 → 3	**02**	2 → 1 → 4 → 5 → 6 → 3 → 2	13
	4 → 5 → 4	04		
9.	1 → 5 → 1	26		
	2 → 3 → 2	38	4 → 6 → 2 → 3 → 1 → 5 → 4	76
	4 → 6 → 4	**10**	6 → 4 → 5 → 1 → 3 → 2 → 6	76
10.	1 → 3 → 5 → 1	87		
	2 → 6 → 2	48	4 → 7 → 2 → 6 → 1 → 5 → 3 → 4	153
	4 → 7 → 4	**02**	7 → 4 → 3 → 5 → 1 → 6 → 2 → 7	153

As of Table 10,we see that the first TSP has three distinct OTPs and each of the remaining nine TSPs have two distinct OTPs.

Why the MEASSI method is chosen for generating subtours for a TSP?

In our research article [9] in AP, we have proved through 60 classical benchmark instances that the MEASSI method is the most efficient one which produces optimal solution directly to 57 instances. Therefore, the MEASSI method is chosen for generating subtours for a TSP.

Novelty in terms of methodology in the proposed ADDOBT method

The generated set of subtours by Phase-I of the proposed method are connected together exactly in Phase-II to form OTPs by considering the subtour of shortest length first. By starting the tour plan from the first (or second) city of this subtour and travel ahead along the sequence given in this subtour and connecting of all the remaining subtours together to form a tour plan is carried out based on the available 0-entry or 1-entry or the next available higher entry among the upcoming unassigned cells in the associated reduced cost matrix.

How is the proposed ADDOBT method different from the existing methods in TSP?

1. *In Hungarian Method:* The given TSP considered as an AP is solved by the Hungarian method of assignment [3], thereby an optimal solution is obtained. If this optimal solution violates the condition that salesman can visit each city only once, then one looks for the ‘next best’ solution by bringing the next (non-zero) cost element 1 along with the zero elements into the solution. If more than one cost element 1 occurs, then each such 1 is considered separately until a feasible solution to the TSP is obtained.
2. *In Ones Assignment Method:* In Ones Assignment Method [1], priority rule plays an important role to make a tour.
3. *In the New Revised Ones Assignment Method:* In the New Revised Ones Assignment Method [5], no clear cut rule is given to make a tour from the solution consisting cycles.
4. *In the Direct Approach:* In the Direct Approach [7], assignments are made based on choosing a suitable assignment preference table consisting of absolute favorable costs of from cities or to cities and their frequencies.
5. *In the Application of Hungarian Algorithm to solve TSP* [4]*:* The given TSP considered as an AP is solved by the Hungarian method of assignment, thereby an optimal solution is obtained. If this optimal solution contains more cycles, then it is removed by constructing a Modifying Distance (Cost or Time) Matrix and then solving the same by Hungarian Algorithm in order to obtain an optimal solution with only one cycle.

CONCLUSION

In this research article, we have branded a set of ten benchmark symmetric TSPs and established that each of the TSPs is having additional available optimal tour plans. The more available optimal tour plans have been generated all the way through the new method named ADDOBT. Therefore, it is explicit that by making use of the ADDOBT method one can generate an optimal tour plan and as well as further available optimal tour plans to a given TSP, provided it exists.

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