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The C-Transformation of a Variant of the Riemann Zeta Function

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Abstract

Variants of Caceres' $X(z,n)$, $Y(z,n)$, and $C_n\{\frac{1}{n^z}\}-\zeta(z,n)$ functions are devised that are equivalent in the sense that there is a linear relationship between them. The variant of the Riemann zeta function has a similar relationship with these functions as the usual Riemann zeta function has with $X(z,n)$ and $Y(z,n)$. These functions are relevant to the Riemann hypothesis.
Keywords: Riemann zeta function, C-transformation of Riemann zeta function, Riemann hypothesis

1. INTRODUCTION

Caceres [1] defined the C-transformation as

$$C_n\{f\} = \sum_{k=1}^{n} f(k) - \int f(n)dn \tag{1}$$

and derived the following function

$$X(z,n) = \sum_{k=1}^{n} k^{-\alpha}(\cos(\beta \cdot \ln(k)) + \frac{1}{2}n^{-\alpha}\cos(\beta\ln(n)) + \tag{2}$$

$$i(\sum_{k=1}^{n} k^{-\alpha}(\sin(\beta \cdot \ln(k)) + \frac{1}{2}n^{-\alpha}\sin(\beta\ln(n))). \tag{3}$$

The associated function is

$$Y(z,n) = n^{(1-\alpha)}\frac{1}{[(1-\alpha)^2+\beta^2]}[((1-\alpha)\cdot\cos(\beta\ln(n)) + \beta\cdot\sin(\beta\ln)))+ \tag{4}$$

$$i(\beta\cdot\cos(\beta\ln(n)) - (1-\alpha)\cdot\sin(\beta\ln(n)))]. \tag{5}$$

Caceres' equation [24] is

$$Re(C_n\{f\}) = \sum_{k=1}^{n} k^{-\alpha}(\cos(\beta \cdot \ln(k)) + \tag{6}$$

$$\frac{1}{[(1-\alpha)^2+\beta^2]}(n^{(1-\alpha)}[(1-\alpha)\cdot\cos(\beta\cdot\ln(n)) + \beta\cdot\sin(\beta\cdot\ln(n))]) \tag{7}$$

where $f = \frac{1}{n^z}$. Caceres' equation [25] is

$$Im(C_n\{f\}) = -\sum_{k=1}^{n} k^{-\alpha}(\sin(\beta \cdot \ln(k)) + \tag{8}$$

$$\frac{1}{[(1-\alpha)^2+\beta^2]}(n^{(1-\alpha)}[\beta\cdot\cos(\beta\cdot\ln(n)) - (1-\alpha)\cdot\sin(\beta\cdot\ln(n))]) \tag{9}$$

Let $\zeta'(z,n)$ denote $\sum_{k=1}^{n}\frac{1}{(n-k+1)^z}$. $C_n\{\frac{1}{n-k+1}\}$ is derived from these formulas by substituting $\sum_{k=1}^{n}(n-k+1)^{-\alpha}$ for $\sum_{k=1}^{n}k^{-\alpha}$.

A plot of $C_n\{\frac{1}{(n-k+1)^z}\}$ for the first non-trivial Riemann zeta function zero and $n \leq 100000$ is

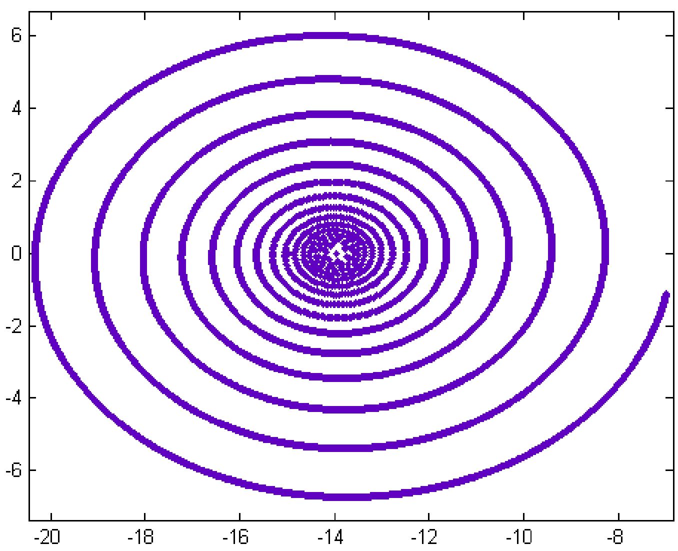

Figure 1

A plot of the inflection points (where the y-component of the curve approaches zero from above and then becomes negative) is

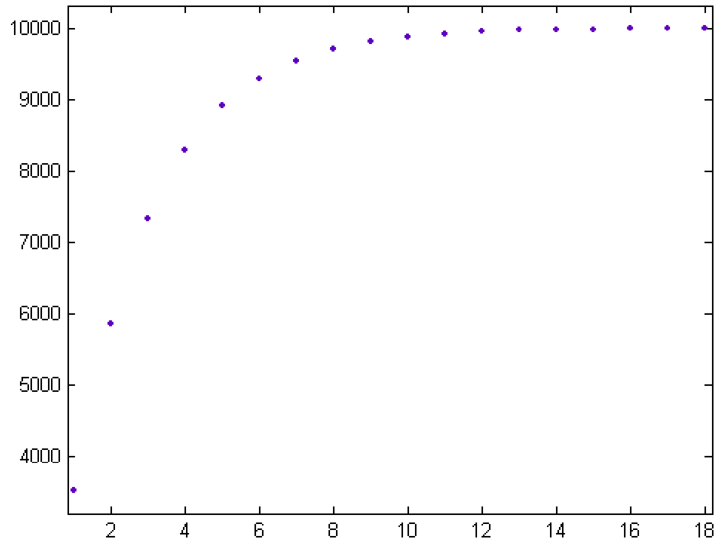

Figure 2

The real components of $C_n\{\frac{1}{(n-k+1)^z}\} - \zeta'(z,n)$ for the first zeta function zero and $n \leq 10000$ equal 6.9855. A plot of the imaginary components is

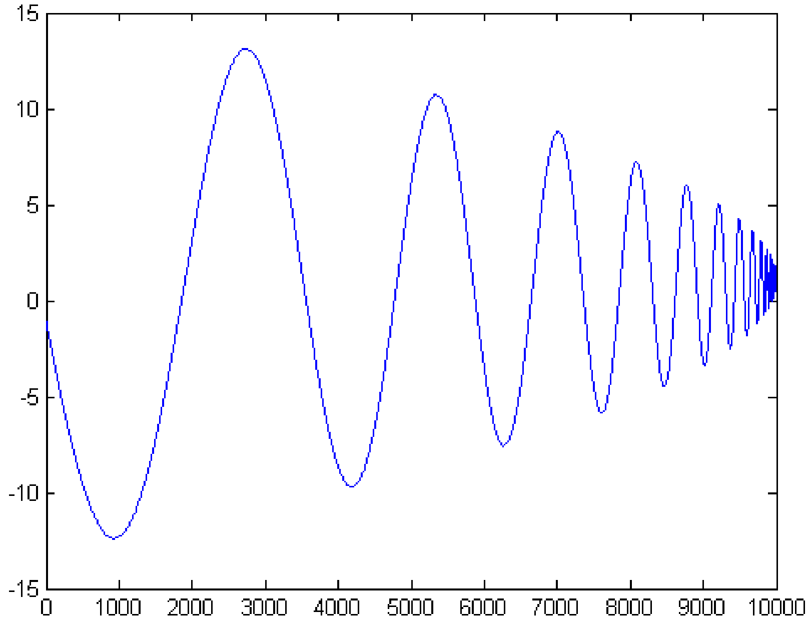

Figure 3

A plot of the n values of the inflection points is

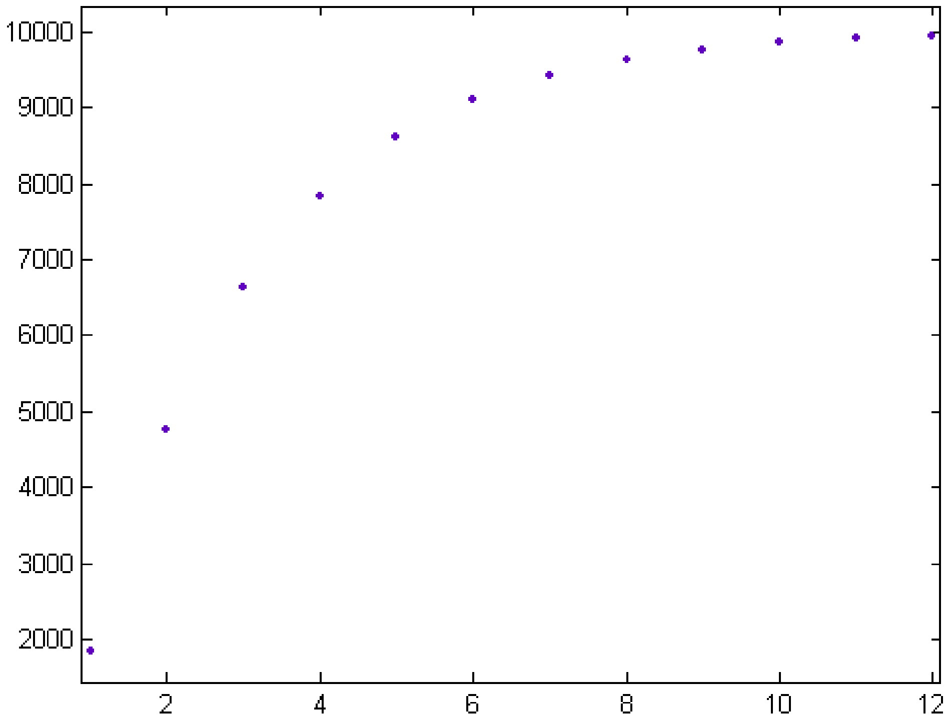

Figure 4

The values are 1857, 4772, 6642, 7842, 8613, 9108, 9426, 9630, 9761, 9846, 9900, and 9935.

Let $X'(z,n)$ denote

$$\sum_{k=1}^{n}(n-k+1)^{-\alpha}(\cos(\beta\cdot\ln(k))+\frac{1}{2}n^{-\alpha}\cos(\beta\ln(n))+ \tag{10}$$

$$i(\sum_{k=1}^{n}(n-k+1)^{-\alpha}(\sin(\beta\cdot\ln(k))+\frac{1}{2}n^{-\alpha}\sin(\beta\ln(n))). \tag{11}$$

for $k = 1$ to n. A plot of $X'(z, n)$ for the first zeta function zero and $n \leq 10000$ is

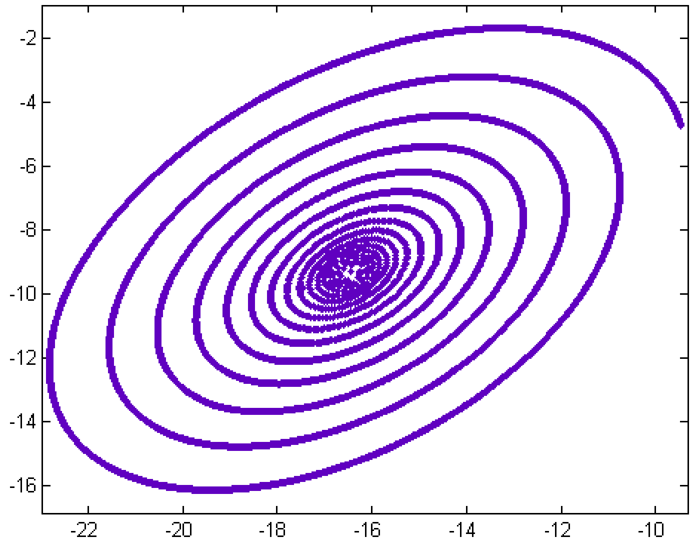

Figure 5

A plot of the n values of the inflection points is

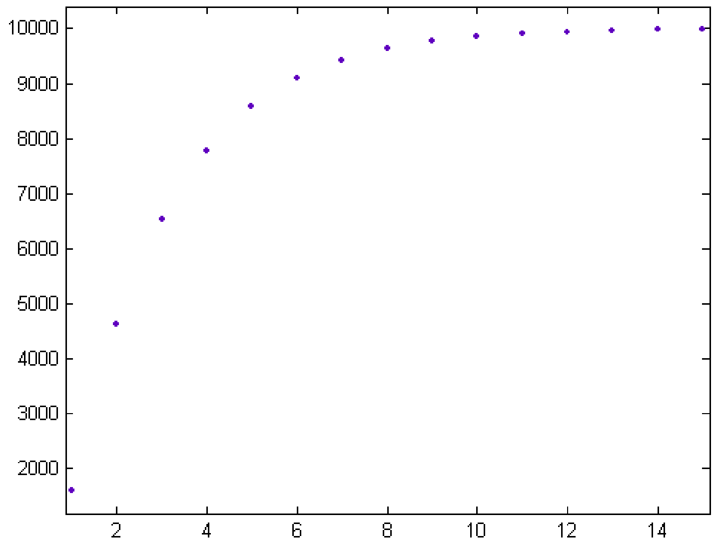

Figure 6

The n values of the inflection points are 1610, 4617, 6546, 7783, 8577, 9086, 9413, 9623, 9758, 9844, 9900, 9936, 9959, 9974, and 9983. The first twelve of these values have a linear relationship with the n values of the inflection points of

$C_n\{\frac{1}{(n-k+1)^z}\} - \zeta'(z,n)$ above. A plot of the n values of the inflection points of $C_n\{\frac{1}{(n-k+1)^z}\} - \zeta'(z,n)$ versus the first twelve n values of the inflection points of $X'(z,n)$ is

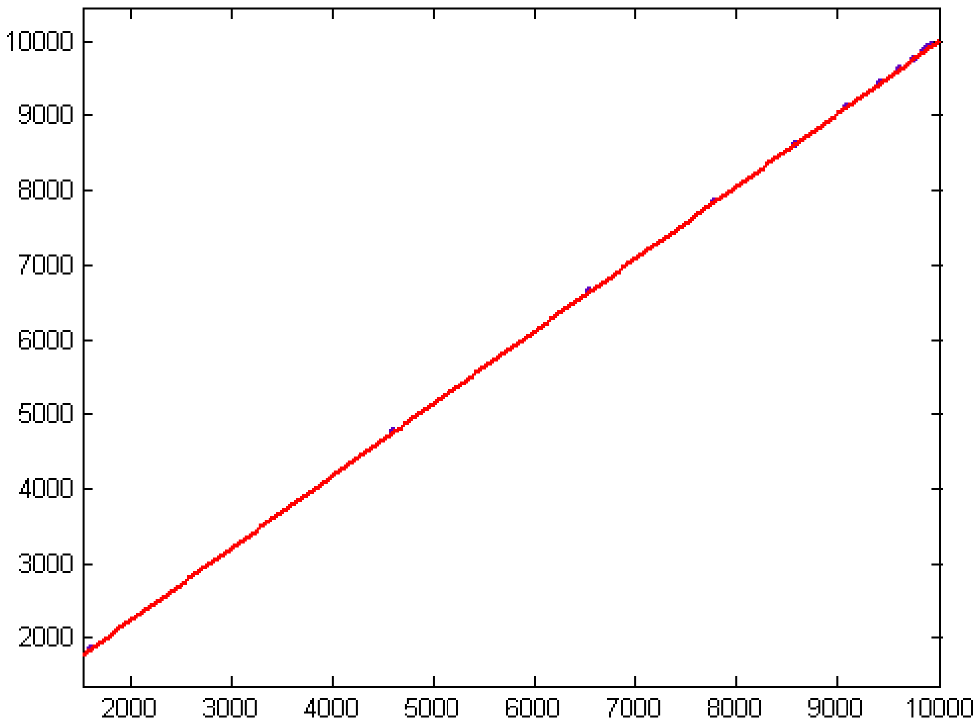

Figure 7

For a linear least-squares fit of the curve, $p_1 = 0.9703$ with a 95% confidence interval of (0.9699, 0.9704), $p_2 = 292.7$ with a 95% confidence interval of (289.1, 296.3), SSE=27.38, R-squared=1, and RMSE=1.655.

For the tenth zeta function zero and $n \leq 10000$, the n values of the inflection points of $C_n\{\frac{1}{(n-k+1)^z}\} - \zeta'(z,n)$ are 861, 1944, 2900, 3742, 4484, 5138, 5715 ,6223, 6671, 7066, 7414, 7721, 7991, 8229, 8439, 8625, 8788, 8932, 9058, 9170, 9269, 9355, 9432, 9499, 9559, 9611, 9657, 9698, 9734, 9766, 9794, 9818, 9840, 9859, 9876, 9891, 9904, 9915, 9925, 9934, 9942, 9949, 9955, 9961, 9966, 9970, 9973, 9977, 9980, 9982, 9984, 9986, 9988, 9991, and 9999. The n values of the inflection points of $X'(z,n)$ are 779, 1873, 2837, 3687, 4436, 5096, 5677, 6190, 6642, 7041, 7392, 7701, 7974, 8215, 8426, 8613, 8778, 8923, 9051, 9164, 9263, 9350, 9428, 9496, 9556, 9608, 9655, 9696, 9732, 9764, 9792, 9817, 9839, 9858, 9875, 9890, 9903, 9915, 9925, 9934, 9942, 9949, 9955, 9961, 9965, 9970, 9973, 9977, 9980, 9982, 9984, 9986, 9988, and 9991. A plot of the first fifty-four n values of the inflection points of $C_n\{\frac{1}{(n-k+1)^z}\} - \zeta'(z,n)$ (this excludes 9999) versus the n values of the inflection points of $X'(z,n)$ is

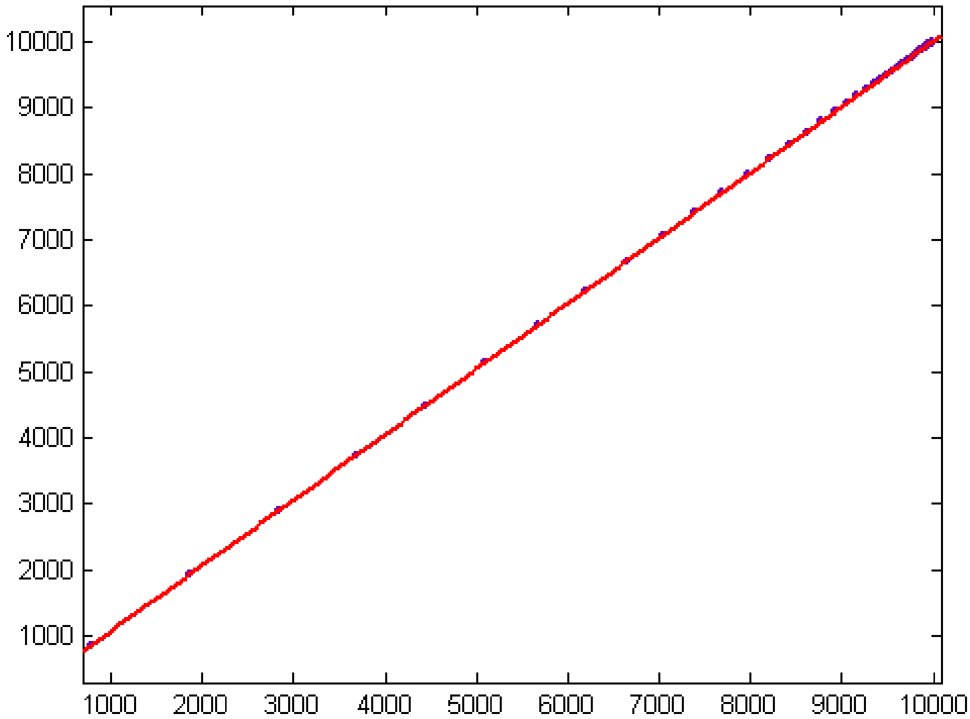

Figure 8

For a linear least-squares fit of the curve, $p_1 = 0.9912$ with a 95% confidence interval of (0.9911, 0.9913), $p_2 = 87.58$ with a 95% confidence interval of (87.05, 88.12), SSE=12.61, R-squared=1, and RMSE=0.4926. $X'(z,n)$ is equivalent to $C_n\{\frac{1}{(n-k+1)^z}\} - \zeta'(z,n)$ in that there is a linear relationship between the two.

Let $Y'(z,n)$ denote

$$(n-k+1)^{(1-\alpha)}\frac{1}{[(1-\alpha)^2+\beta^2]}[((1-\alpha)\cdot\cos(\beta\ln(n))+\beta\cdot\sin(\beta\ln)))+ \tag{12}$$

$$i(\beta\cdot\cos(\beta\ln(n))-(1-\alpha)\cdot\sin(\beta\ln(n)))]. \tag{13}$$

for $k = 1$ to n. A plot of $Y'(z, n)$ for the first zeta function zero and $n \leq 1000$ is

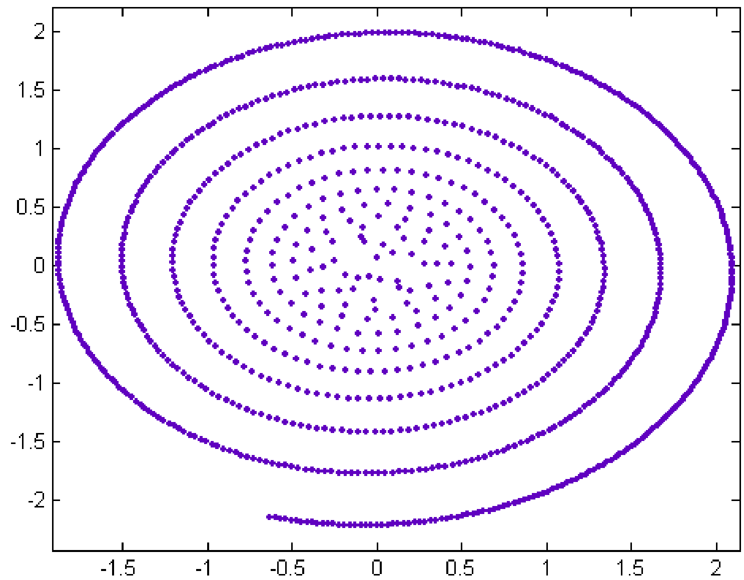

Figure 9

A plot of the n values of the inflection points is

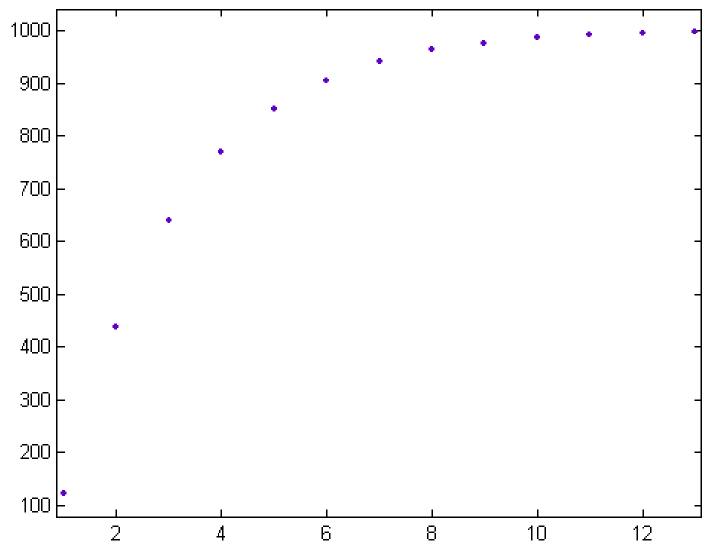

Figure 10

The values are 299, 551, 713, 816, 883, 925, 953, 970, 981, 989, 993, 996, and 998. The n values of the inflection points of $C_n\{\frac{1}{(n-k+1)^z}\} - \zeta'(z, n)$ are 122, 437, 639, 769, 852, 905, 940, 962, 976, 985, 991, 994, and 997. A plot of these values versus the first twelve n values of inflection points of the $Y'(z,n)$ values is

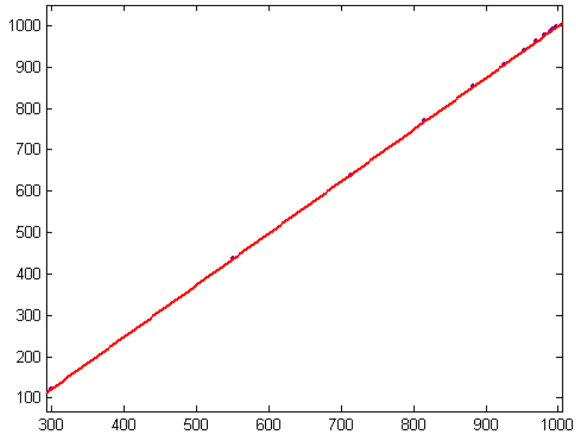

Figure 11

For a linear least-squares fit of the curve, $p_1 = 1.252$ with a 95% confidence interval of (1.25, 1.253), $p_2 = -252.8$ with a 95% confidence interval of $(-254.1, -251.4)$, SSE=2.816, R-squared=1, and RMSE=0.506. $Y'(z,n)$ is equivalent to $C_n\{\frac{1}{(n-k+1)^z}\} - \zeta'(z,n)$ in that there is a linear relationship between the two.

A plot of $X'(z,n) - Y'(z,n)$ for the fifth zeta function zero and $n \leq 10000$ is

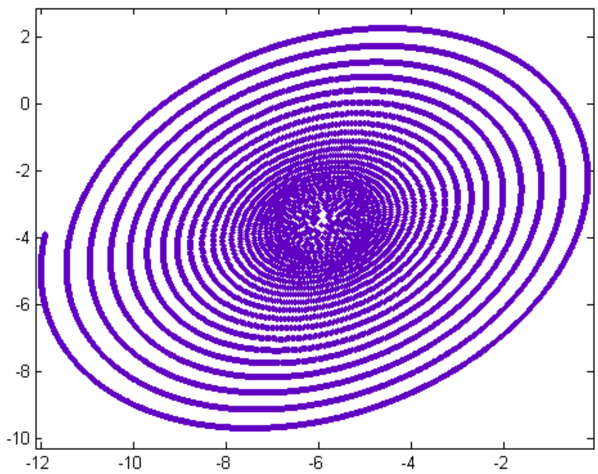

Figure 12

A plot of the n values of the inflection points is

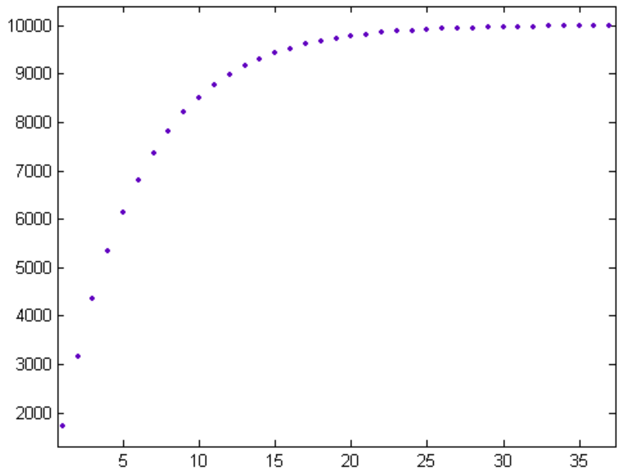

Figure 13

A plot of the real and imaginary components of $X'(z,n) - Y'(z,n)$ and the real and imaginary components of $C_n\{\frac{1}{(n-k+1)^z}\} - \zeta'(z,n)$ is

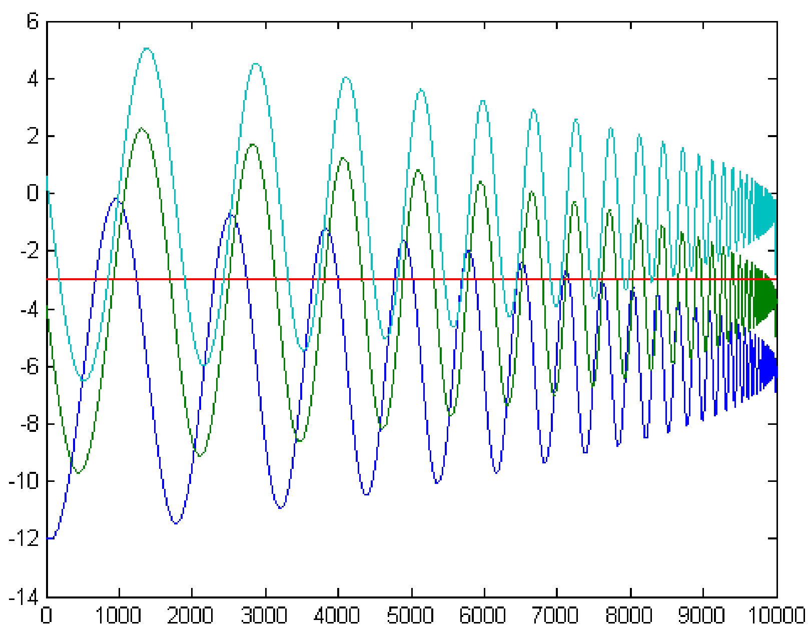

Figure 14

A plot of the imaginary components of $X'(z,n) - Y'(z,n)$ and the imaginary components of $C_n\{\frac{1}{(n-k+1)^z}\} - \zeta'(z,n)$ is

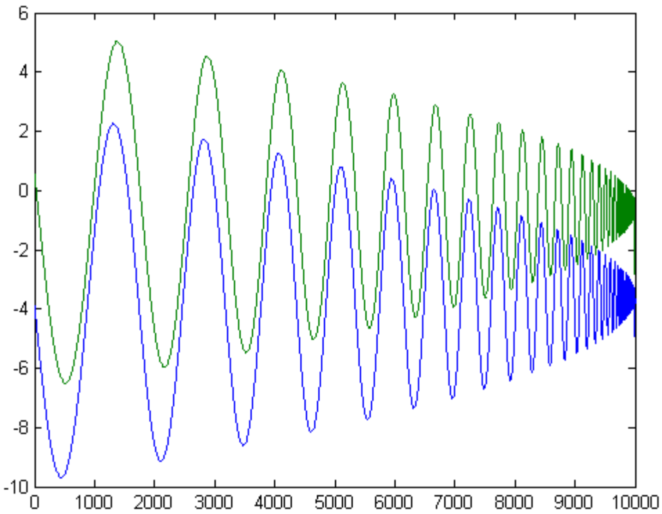

Figure 15

A plot of the first twenty-nine n values of the inflection points of the imaginary components of $C_n\{\frac{1}{(n-k+1)^z}\} - \zeta'(z,n)$ (there are thirty-eight in total) versus the n values of the inflection points of the imaginary components of $X'(z,n) - Y'(z,n)$ is

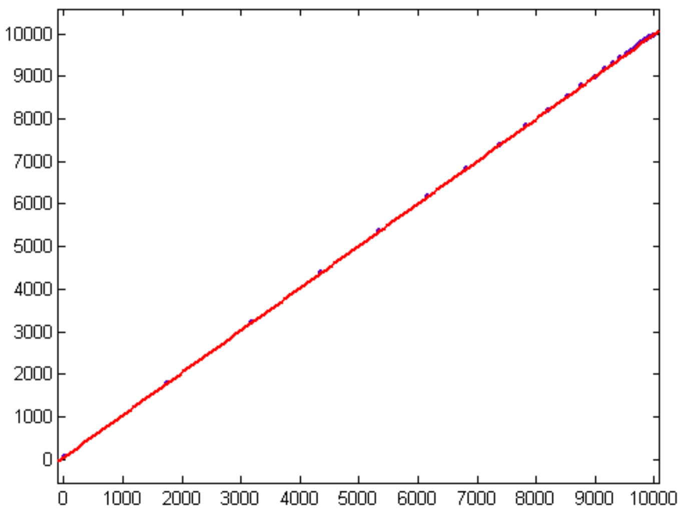

Figure 16

For a linear least-squares fit of the curve, $p_1 = 0.9947$ with a 95% confidence interval of (0.9936, 0.9957), $p_2 = 46.26$ with a 95% confidence interval of (37.31, 55.21), SSE=1451, R-squared=1, and RMSE=7.332. $X'(z,n) - Y'(z,n)$ is equivalent to $C_n\{\frac{1}{(n-k+1)^z}\} - \zeta'(z,n)$ in the sense that there is a linear relationship between them.

2. ABSOLUTE SQUARES

A "polynomial" form of $Y'(z,n)$ (similar to the one Caceres derived for $Y(z,n)$) is

$$|Y'(z,n)|^2 \geq (n-k+1)^{2(1-a)}/[(1-\alpha)^2 + \beta^2] \tag{14}$$

for $k = 1$ to n. Note that the left-hand side of the inequality is real-valued whereas the right-hand side is complex-valued. A plot $|Y'(z,n)|^2$ and the real and imaginary components of the right-hand side of the inequality for the first zeta function zero and $n \leq 1000$ is

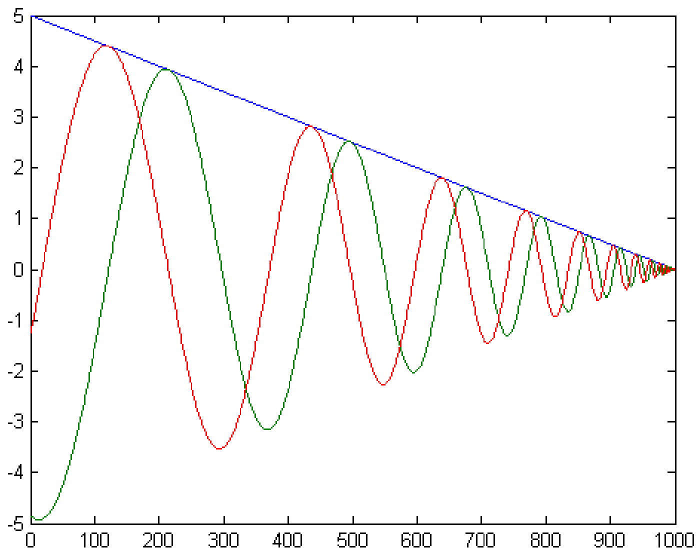

Figure 17

Caceres proved the following

Theorem 1. *When* $\alpha = 1/2$, $|Y(z,n)|^2 = \frac{n}{]\beta^2+\frac{1}{4}]}$

Let $s(z,n)$ denote the slope of $|Y'(z,n)|^2$. A plot of $\frac{1}{\sqrt{-s(z,n)}}$ for $z = (1/2, 1)$, (1/2, 2), (1/2, 3),. . ., (1/2, 10) and $n \leq 10000$ is

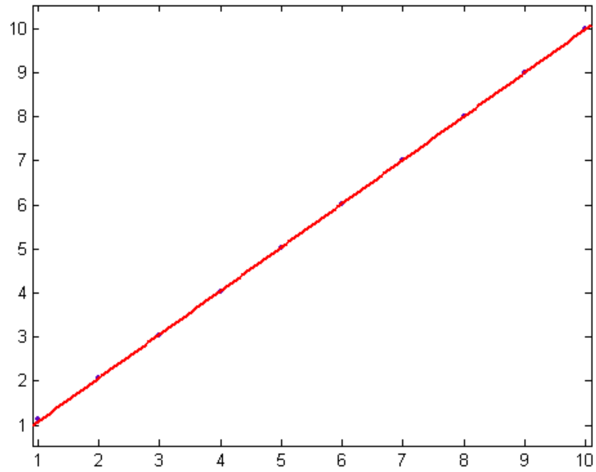

Figure 18

For a linear least-squares fit of the curve, $p_1 = 0.9911$ with a 95% confidence interval of (0.9861, 0.996), $p_2 = 0.08489$ with a 95% confidence interval of (0.054515, 0.1156), SSE=0.003046, R-squared=1, and RMSE=0.01951. The scaled slopes (multiplied by 10000) are -8000, -2353, -1081, -615.4, -396, -275.9, and -203. The corresponding intercepts (unscaled) are 8001, 2353, 1081, 615.4, 396.1, 275.9, and 203.1.

A plot of the slopes times 1000 (for $n \leq 1000$) versus the logarithms of β for the first ten zeta function zeros is

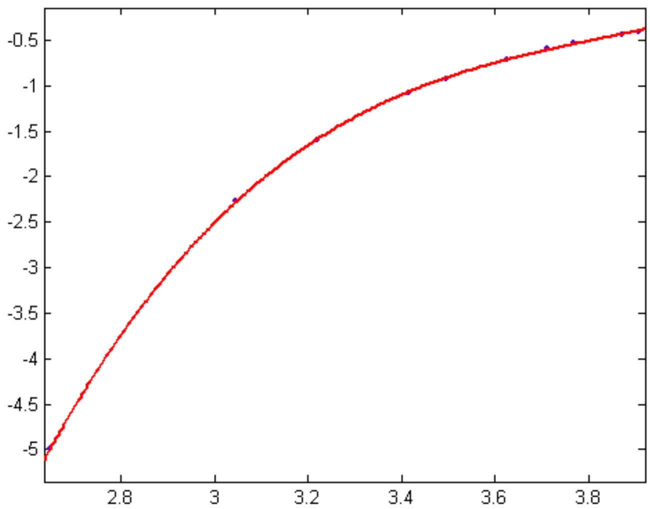

Figure 19

For a cubic least-squares fit of the curve, $p_1 = 1.94$ with a 95% confidence interval of (1.725, 2.154), $p_2 = -22.33$ with a 95% confidence interval of $(-24.45, -20.2)$, $p_3 = 86.73$ with a 95% confidence interval of (79.81, 93.65), $p_4 = -114.1$ with a 95% confidence interval of $(-121.6, -106.7)$, SSE=0.0009622, R-squared=0.9999 and RMSE=0.01266.

A plot of the intercepts versus the logarithms of β is

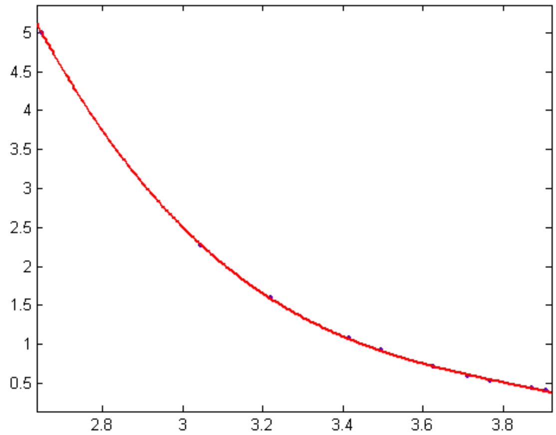

Figure 20

For a cubic least-squares fit of the curve, $p_1 = -1.942$ with a 95% confidence interval of $(-2.158, -1.726)$, $p_2 = 22.35$ with a 95% confidence interval of (20.21, 24.49), $p_3 = -86.82$ with a 95% confidence interval of $(-93.79, -79.85)$, $p_4 = 114.2$ with a 95% confidence interval of (106.7, 121.8), SSE=0.0009768, R-squared=0.9999, and RMSE=0.01276. The parameters of the intercepts are approximately equal to the parameters of the negatives of the slopes.

A plot of $n^{2(1-a)}/[(1-\alpha)^2 + \beta^2]$ for the tenth zeta function zero and $n \leq 1000$ is

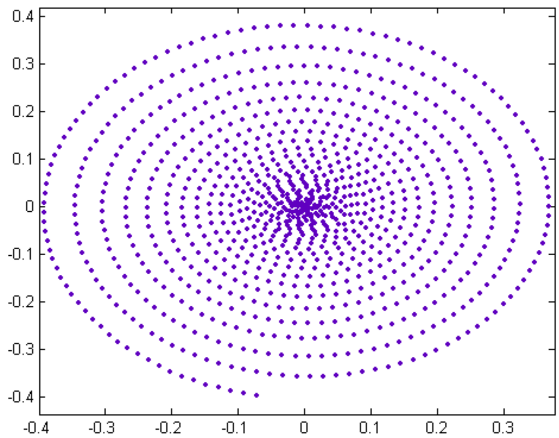

Figure 21

A plot of the n values of the inflection points is

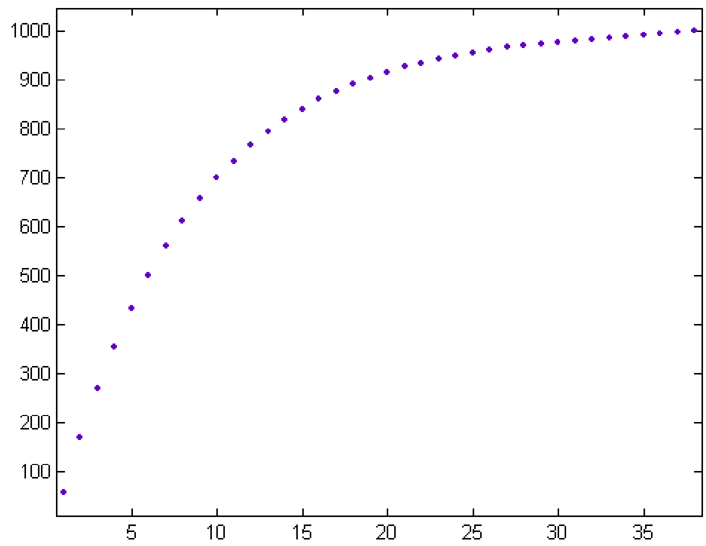

Figure 22

A plot of the n values of the inflection points for $C_n\{\frac{1}{(n-k+1)^z}\} - \zeta'(z,n)$ is

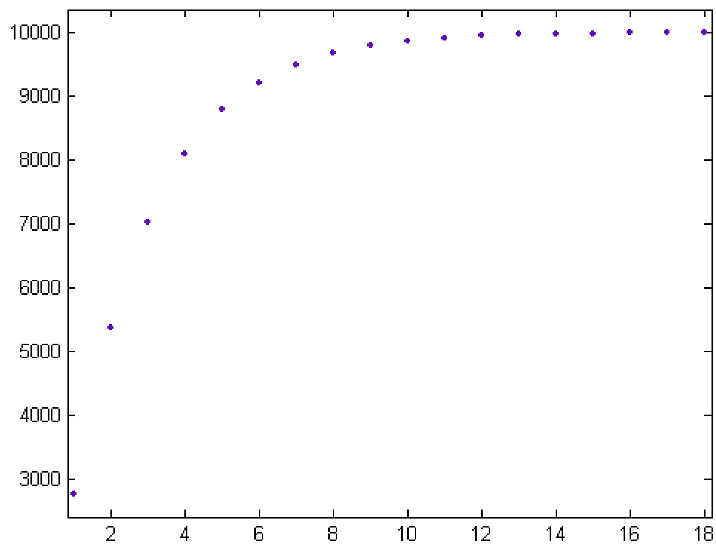

Figure 23

There is one more inflection point (say the first one) than for $n^{2(1-a)}/[(1-\alpha)^2+\beta^2]$.

A plot of the n values of the inflection points for $C_n\{\frac{1}{(n-k+1)^z}\} - \zeta'(z, n)$ versus the n values of the inflection points for $n^{2(1-a)}/[(1-\alpha)^2 + \beta^2]$ (disregarding the first one) is

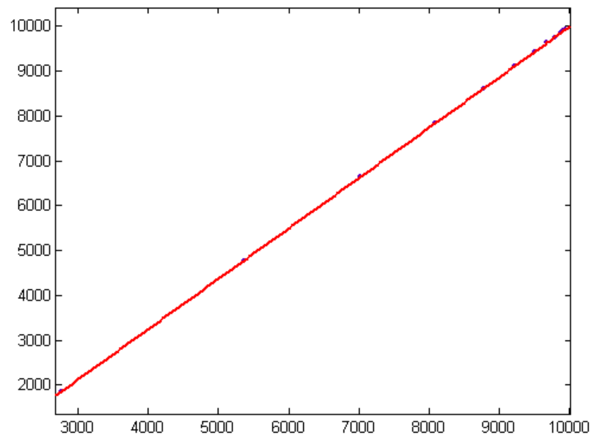

Figure 24

For a linear least-squares fit of the curve, $p_1 = 1.209$ with a 95% confidence interval of (1.208, 1.211), $p_2 = -210.4$ with a 95% confidence interval of $(-211.5, -209.2)$, SSE=23.73, R-squared=1, and RMSE=0.8355. $n^{2(1-a)}/[(1-\alpha)^2 + \beta^2]$ is equivalent to $C_n\{\frac{1}{(n-k+1)^z}\} - \zeta'(z, n)$ in that there is a linear relationship between the two.

A plot $|Y'(z, n)|^2$ and the real and imaginary components of the above polynomial form of $|Y'(z, n)|^2$ for $z = (0.5, 12.0)$ and $n \leq 10000$ is

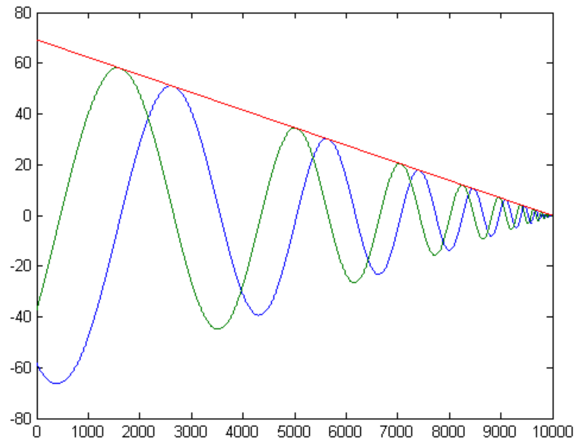

Figure 25

For real components of z other than 1/2, $|Y'(z,n)|^2$ is curved.

Caceres' equation [14] is

$$|X(z,n)|^2 = \frac{1}{4}n^{-2\alpha} + \sum_{k=1}^{n}\sum_{j=1}^{n} k^{-\alpha}\cdot j^{-\alpha}\cdot\cos(\beta\cdot\log(\frac{k}{j})) + 2n^{-\alpha}\cdot\sum_{k=1}^{n} k^{-\alpha}\cos(\beta\cdot\log(\frac{k}{n})) \quad (15)$$

A plot of $|X'(z,n)|^2$ (where $n-l+1$, $l=1$ to n, is substituted for n where a power of n is computed) for the second zeta function zero and $n \leq 1000$ is

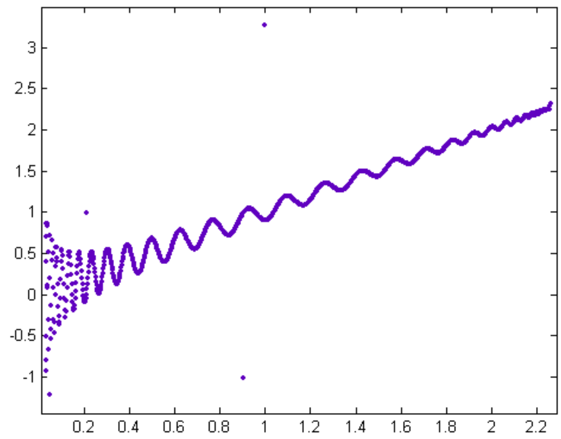

Figure 26

A plot of $|X'(z,n)|^2 \cdot X'(z,n)$ is

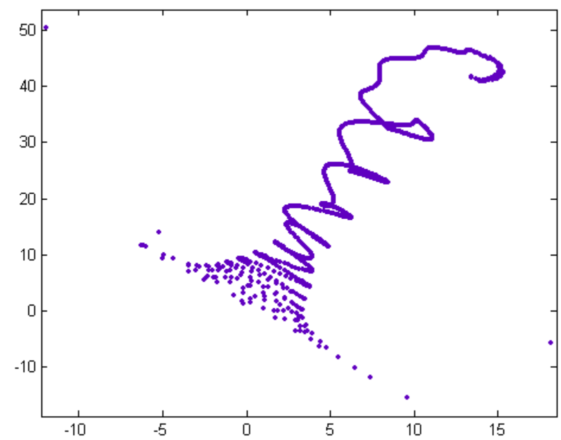

Figure 27

A plot of the n values of the inflection points is

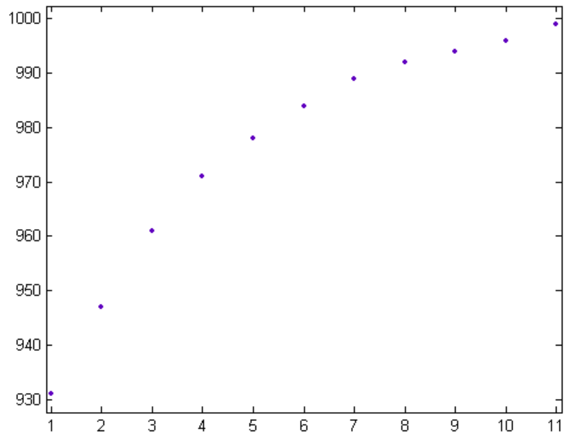

Figure 28

A plot of $|X'(z,n)|^2$ and $|X(z,n)|^2$ is

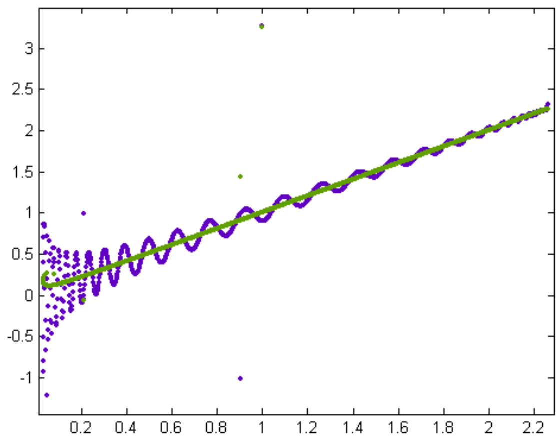

Figure 29

$|X'(z,n)|^2$ then has behavior similar to $|X(z,n)|^2$ for zeta function zeros.

3. PROPERTIES OF THE SQUARE OF ABSOLUTE VALUES OF $\zeta(z,n)$ AND $\zeta'(z,n)$

A plot of the slopes of $|\zeta(z,n)|^2$ times 1000 versus the logarithms of the zeta function zeros for the first ten zeta function zeros and $n \leq 1000$ is

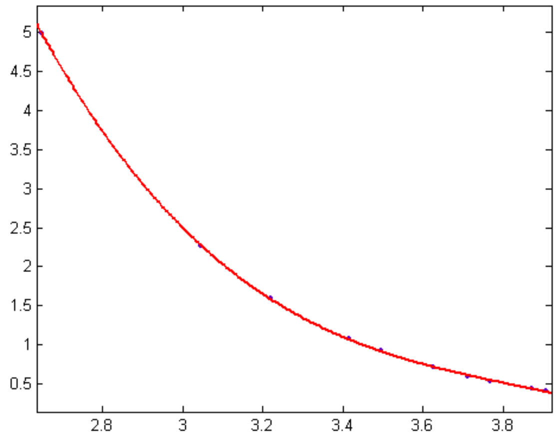

Figure 30

For a cubic least-squares fit of the curve, $p_1 = -1.944$ with a 95% confidence interval of $(-2.162, -1.725)$, $p_2 = 22.36$ with a 95% confidence interval of (20.2, 24.5), $p_3 = -86.85$ with a 95% confidence interval of $(-93.9, -79.81)$, $p_4 = 114.3$ with a 95% confidence interval of (106.7, 121.9), SSE=0.0009975 R-squared=0.9999, and RMSE=0.01289. These parameters are approximately equal to those of the above intercepts.

A plot of $|C_n\{\frac{1}{(n-k+1)^z}\} - \zeta'(z,n)|^2$ and $|\zeta(z,n)|^2$ for the tenth zeta function zero and $n \leq 10000$ is

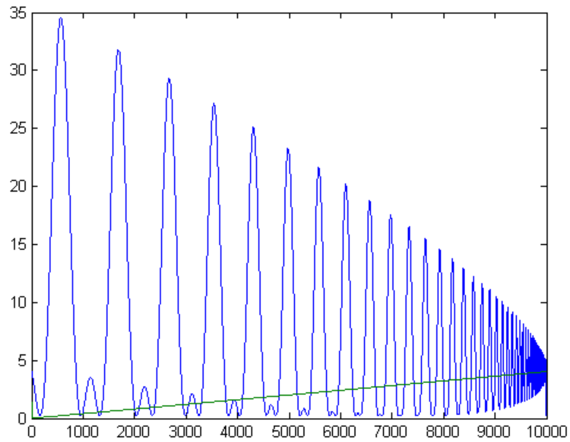

Figure 31

The curves appear to converge to the same value.

A plot of $|\zeta(z,n)|^2$ for $z = (0.5, 33.0)$ and $n \leq 10000$ is

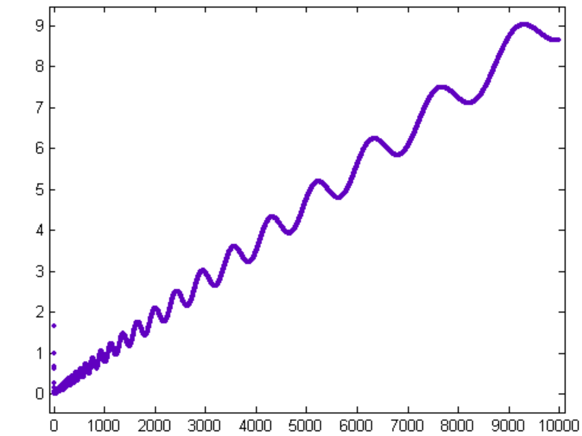

Figure 32

The curve does not converge to a straight line.

4. ANALOGUE OF CACERES' $\zeta(z) = X(z) - Y(z)$

A plot of $|\zeta'(z,n)|^2 - |C_n\{\frac{1}{(n-k+1)^z}\} - \zeta'(z,n)|^2$ and $n^{2(1-\alpha)}/[(1-\alpha)^2 + \beta^2]$ for $z = (0.5, 5.0)$ and $n \leq 10000$ is

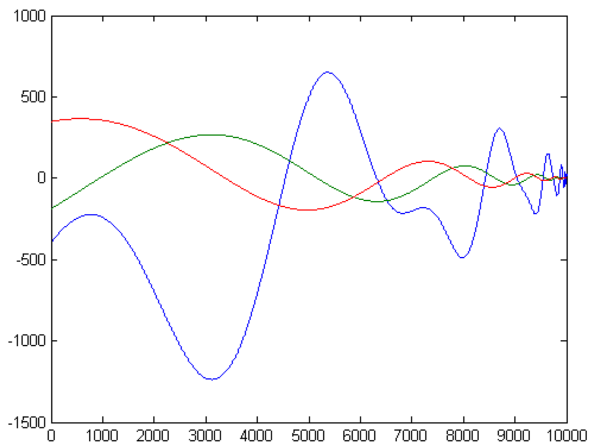

Figure 33

A plot of $|\zeta'(z,n)|^2 - |C_n\{\frac{1}{(n-k+1)^z}\} - \zeta'(z,n)|^2$ for $z = (0.5, 10.0)$ and $n \leq 10000$ is

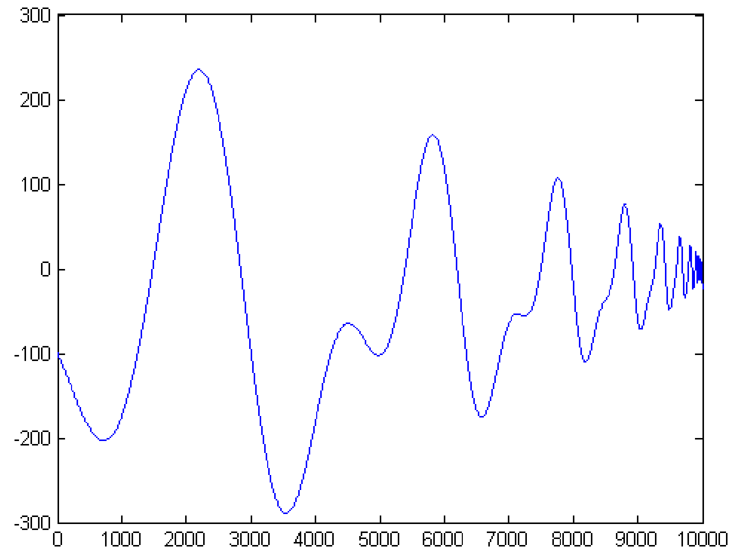

Figure 34

A plot of $|\zeta'(z,n)|^2 - |C_n\{\frac{1}{(n-k+1)^z}\} - \zeta'(z,n)|^2$ for $z = (0.4, 10.0)$ and $n \leq 10000$ is

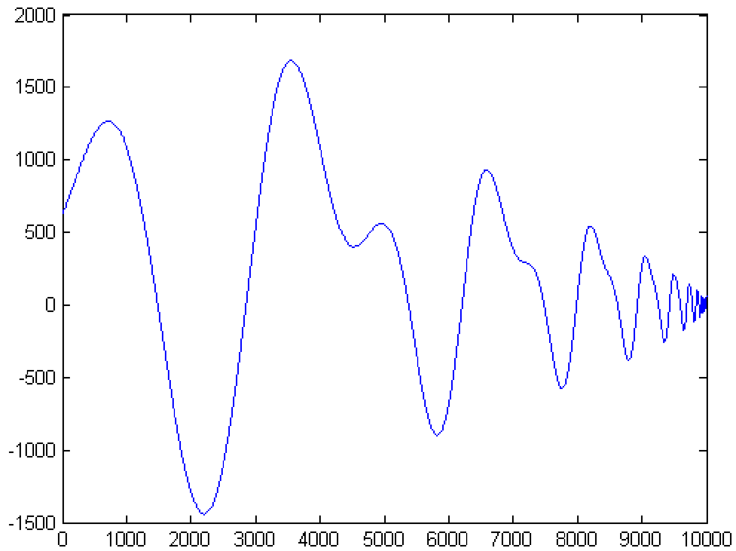

Figure 35

$|\zeta'(z,n)|^2 - |C_n\{\frac{1}{(n-k+1)^z}\} - \zeta'(z,n)|^2$ appears to converge to 0.

A plot of $X'(z, n) - Y'(z, n)$ for the fourth zeta function zero and $n \leq 1000$ is

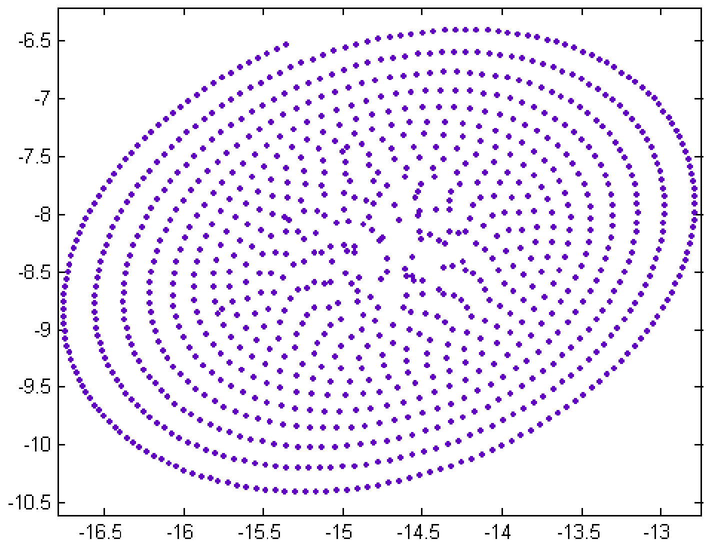

Figure 36

A plot of the average of the real and imaginary components of $X'(z, n) - Y'(z, n)$ and the average of the real and imaginary components of $\zeta'(z, n)$ is

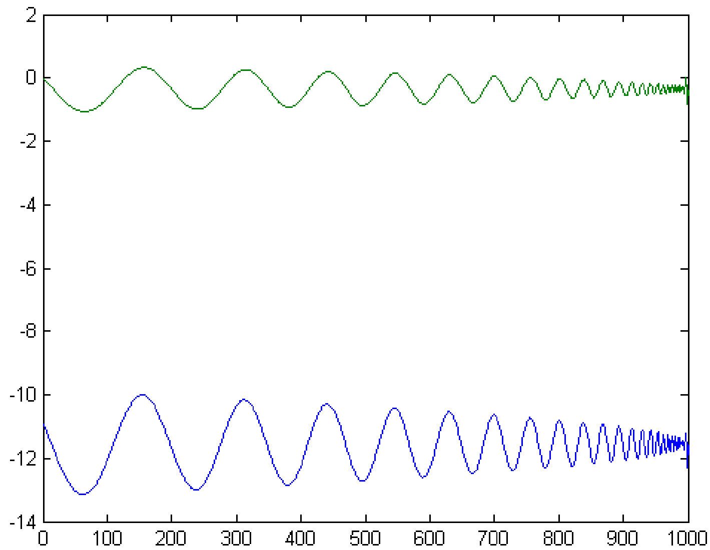

Figure 37

The average of the real and imaginary components of $\zeta'(z, n)$ is the upper curve.

A plot of the difference in the averages is

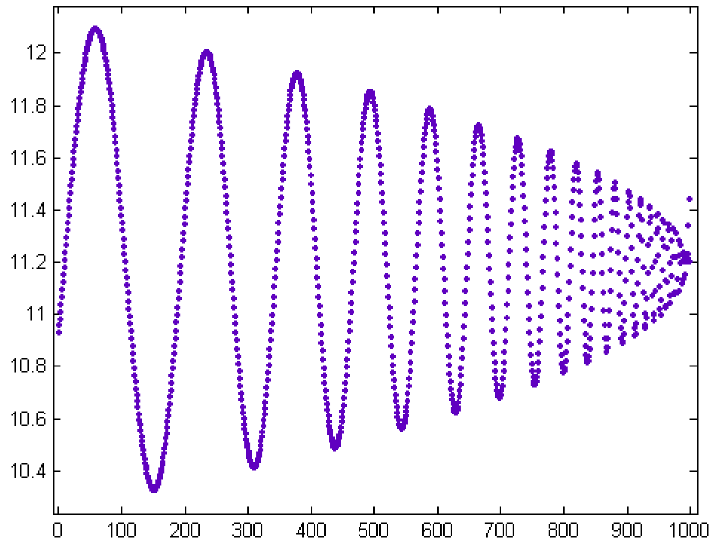

Figure 38

A plot of the n values of the inflection points is

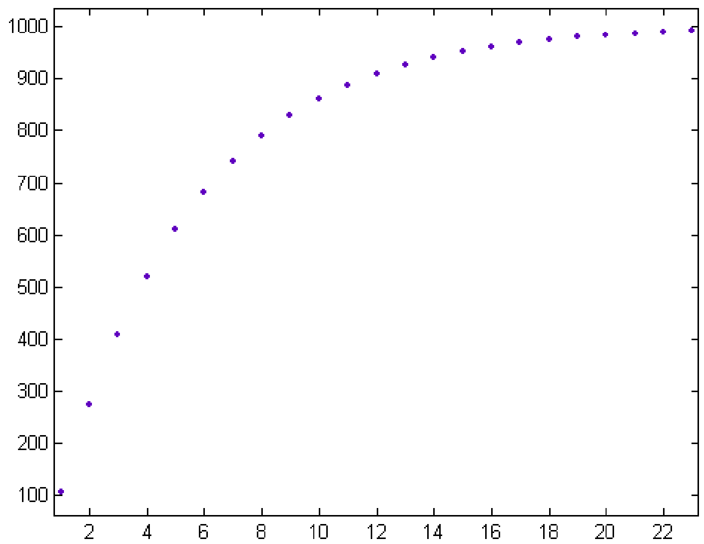

Figure 39

A plot of $X'(z,n) - Y'(z,n)$ and $\zeta'(z,n)$ for the first zeta function zero and $n \leq 10000$ is

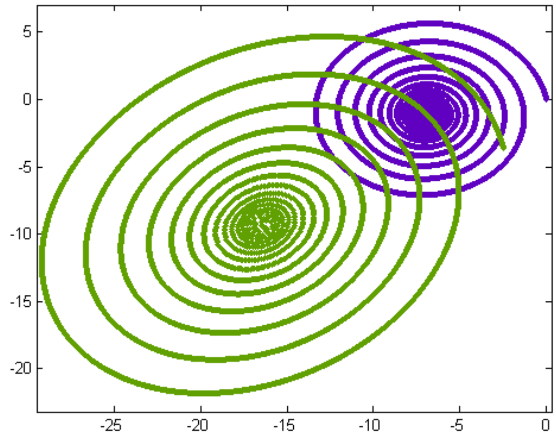

Figure 40

A plot of $\zeta'(z,n) - Y'(z,n)$ for the first zeta function zero and $n \leq 10000$ is

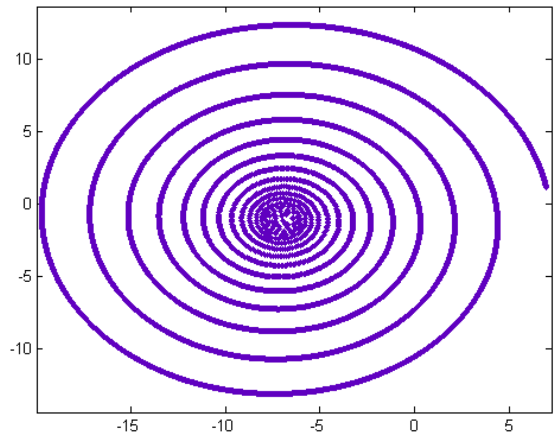

Figure 41

The n values of the inflection points of $\zeta'(z,n) - Y'(z,n) - (C_n\{\frac{1}{(n-k+1)^z}\} - \zeta'(z,n))$ are 1857, 4772, 6642, 7843, 8613, 9108, 9426, 9630, 9760, 9845, 9900, and 9935. The n values of the inflection points of $C_n\{\frac{1}{(n-k+1)^z}\} - \zeta'(z,n)$ are 1857, 4772, 6642, 7842, 8613, 9108, 9426, 9630, 9761, 9846, 9900, and 9935. The differences are at most 1.

A plot of the imaginary components of the curves is

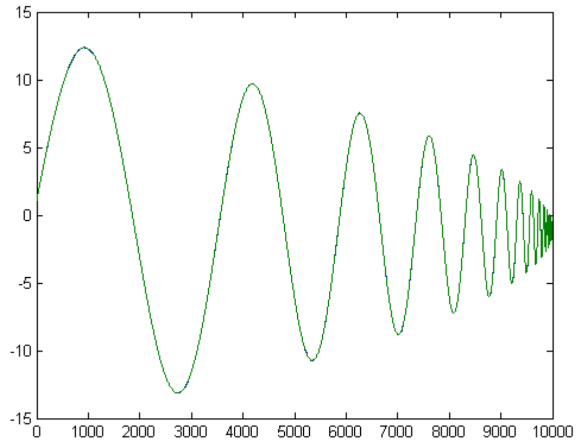

Figure 42

The two curves are almost equal.

5. A POLYNOMIAL FORM OF $|X(z,n)|^2$

The polynomial form of $|Y(z,n)|^2$ derived by Caceres is

$$|Y(z,n)|^2 \geq n^{2(1-a)}/[(1-\alpha)^2 + \beta^2] \tag{16}$$

Note that the right-hand side of the inequality has real and imaginary components. Let $X_p(z,n)$ denote the average of the real and imaginary components of $X(z,n)$. A plot of $X_p(z,n)$ and the real component of $n^{2(1-a)}/[(1-\alpha)^2+\beta^2]$ for the first zeta function zero and $n \leq 10000$ is

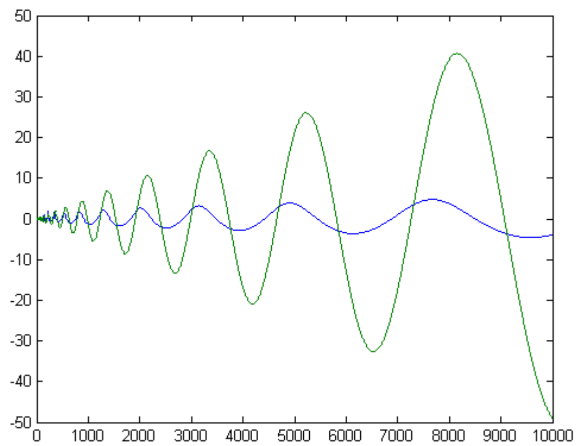

Figure 43

A plot of $X_p(z, n)$ minus the real component of $n^{2(1-a)}/[(1-\alpha)^2 + \beta^2]$ is

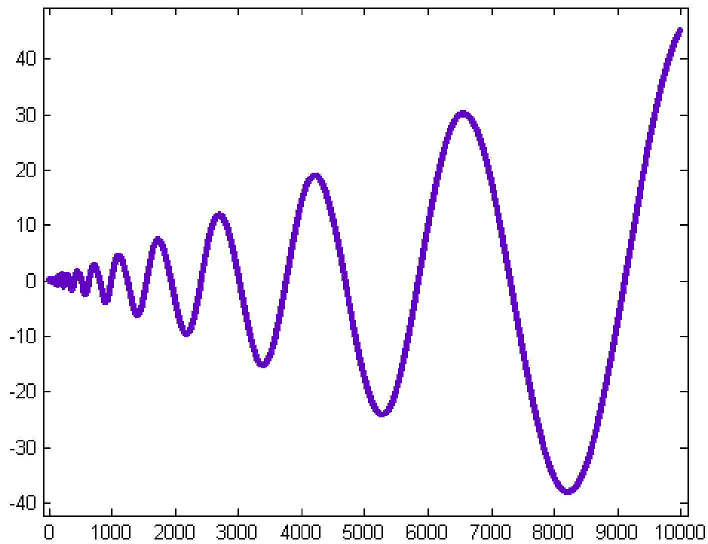

Figure 44

A plot of the logarithms of the n values of the inflection points is

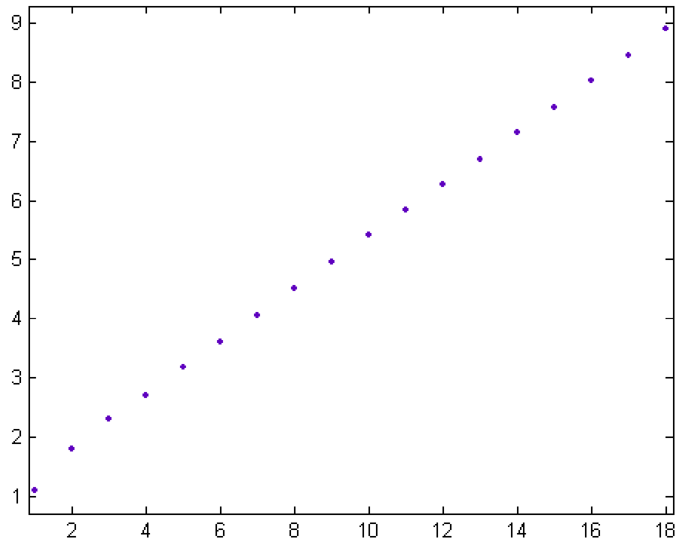

Figure 45

For a linear least-squares fit of the curve (disregarding the first point), $p_1 = 0.4415$ with a 95% confidence interval of (0.4391, 0.4439), $p_2 = 0.9634$ with a 95% confidence interval of (0.9367, 0.9901), SSE=0.007747, R-squared=0.9999, and RMSE=0.02273. These are almost the same parameters as given by $\zeta(z,n)$.

6. SUMMARY

$X'(z,n)$, $Y'(z,n)$, $X'(z,n) - Y'(z,n)$, and $n^{2(1-a)}/[(1-\alpha)^2+\beta^2]$ are equivalent to $C_n\{\frac{1}{(n-k+1)^z}\} - \zeta'(z,n)$ in the above sense. For z values with a real component of 1/2, $|Y'(z,n)|^2 \geq (n-k+1)^{2(1-a)}/[(1-\alpha)^2+\beta^2]$ and $|Y'(z,n)|^2$ is a straight line. For z values with a real component of 1/2, $\lim_{n\to\infty} |\zeta'(z,n)|^2 = \lim_{n\to\infty} |C_n\{\frac{1}{(n-k+1)^z}\} - \zeta'(z,n)|^2$. For zeta function zeros, $\lim_{n\to\infty} |\zeta(z,n)|^2 = \lim_{n\to\infty} |\zeta'(z,n)|^2$. For zeta function zeros, $|X'(z,n)|^2$ approaches a straight line as $n \to \infty$.

Let $X'(z)$ denote $\lim_{n\to\infty} X'(z,n)$ and $Y'(z)$ denote $\lim_{n\to\infty} Y'(z,n)$. Many of Caceres' criteria for a proof of the Riemann hypothesis appear to be met, but $\zeta'(z)$ is not equal to $X'(z) - Y'(z)$.

7. METHODS

The C code for computing X'(z,n), Y'(z,n), and $C_n\{\frac{1}{(n-k+1)^z}\}$ is as follows.

```
#include <math.h>
#include <stdio.h>
//
// zeta function variant (when noinc=1)
// X'(z,n) (when noinc=0)
//
unsigned int max=10000;
double s=0.50;
//double t=14.13472514173470;
//double t=21.02203963877156;
//double t=25.01085758014569;
//double t=30.42487612585951;
double t=32.93506158773919;
//double t=37.58617815882568;
//double t=40.91871901214750;
//double t=43.32707328091500;
//double t=48.00515088116716;
//double t=49.77383247767230;
unsigned int xmin=0; // usually set to 0
```

```
unsigned int noinc=0; // if set, don't add additional term
unsigned int out=1; // set to 1 when not finding inflection points
// set to 3 when noinc=0
// set to 3 or 4 when noinc=1
void main() {
unsigned int x,y;
double temp1,temps,tempt,sumr,sumi,a,b,olds,oldt;
FILE *Outfp;
Outfp = fopen("spirala.dat","w");
sumr=0.0;
sumi=0.0;
olds=0.0;
oldt=0.0;
for (x=1; x<=max; x++) {
    y=max-x+1;
    if (s>=0.0)
        temp1=pow((double)y,s);
    else {
        temp1=pow((double)y,-s);
        temp1=1.0/temp1;
        }
    a=temp1*(cos(t*log(y)));
    b=temp1*(sin(t*log(y)));
    temp1=a*a+b*b;
    sumr=sumr+a/temp1;
    sumi=sumi-b/temp1;
    if (s>=0.0)
        temp1=pow((double)max,s);
    else {
        temp1=pow((double)max,-s);
        temp1=1.0/temp1;
        }
    temps=temp1*(cos(t*log(max)));
    tempt=temp1*(sin(t*log(max)));
    if (noinc==0) {
        temps=sumr+temps/2.0;
        tempt=sumi+temps/2.0;
        }
    else {
```

```
        temps=sumr;
        tempt=sumi;
        }
    if (x>xmin) {
        if (out==1)
            fprintf(Outfp," %.10llf %.10llf \n",temps,tempt);
        if ((out==2)&&((olds>0.0)&&(temps<0.0)&&(x>1)))
            fprintf(Outfp," %d %.10llf %.10llf \n",x,temps,tempt);
        if ((out==3)&&((oldt>0.0)&&(tempt<0.0)&&(x>1)))
            fprintf(Outfp," %d %.10llf %.10llf \n",x,temps,tempt);
        if ((out==4)&&((olds<0.0)&&(temps>0.0)&&(x>1)))
            fprintf(Outfp," %d %.10llf %.10llf \n",x,temps,tempt);
        if ((out==5)&&((oldt<0.0)&&(tempt>0.0)&&(x>1)))
            fprintf(Outfp," %d %.10llf %.10llf \n",x,temps,tempt);
        olds=temps;
        oldt=tempt;
        }
    }
printf(" %.10llf %.10llf \n",temps,tempt);
fclose(Outfp);
return;
}
```

```
#include <math.h>
#include <stdio.h>
//
// Y'(z)
//
unsigned int maxn=1000;
unsigned int out=1;
double s=0.50;
double t=14.13472514173470;
//double t=21.02203963877156;
//double t=25.01085758014569;
//double t=30.42487612585951;
//double t=32.93506158773919;
//double t=37.58617815882568;
//double t=40.91871901214750;
//double t=43.32707328091500;
```

```
//double t=48.00515088116716;
//double t=49.77383247767230;
void main() {
double temp1,temps,tempt,x,olds,oldt;
unsigned int n,max;
FILE *Outfp;
Outfp = fopen("spiral2.dat","w");
olds=0.0;
oldt=0.0;
for (n=1; n<=maxn; n++) {
    max=maxn-n+1;
    x=1.0-s;
    if (x>=0.0)
        temp1=pow((double)max,x);
    else {
        temp1=pow((double)max,-x);
        temp1=1.0/temp1;
        }
    temps=temp1*(x*cos(t*log(max))+t*sin(t*log(max)));
    temps=temps/(x*x+t*t);
    tempt=temp1*(t*cos(t*log(max))-x*sin(t*log(max)));
    tempt=tempt/(x*x+t*t);
    if (out==1)
        fprintf(Outfp," %.10llf %.10llf \n",temps,tempt);
    if ((out==2)&&((olds>0.0)&&(temps<0.0)&&(n>1)))
        fprintf(Outfp," %d %.10llf %.10llf \n",n,temps,tempt);
    if ((out==3)&&((oldt>0.0)&&(tempt<0.0)&&(n>1)))
        fprintf(Outfp," %d %.10llf %.10llf \n",n,temps,tempt);
    if ((out==4)&&((olds<0.0)&&(temps>0.0)&&(n>1)))
        fprintf(Outfp," %d %.10llf %.10llf \n",n,temps,tempt);
    if ((out==5)&&((oldt<0.0)&&(tempt>0.0)&&(n>1)))
        fprintf(Outfp," %d %.10llf %.10llf \n",n,temps,tempt);
    olds=temps;
    oldt=tempt;
    }
printf(" %.10llf %.10llf \n",temps,tempt);
fclose(Outfp);
return;
}
```

```
#include <math.h>
#include <stdio.h>
//
// C-transformation
//
unsigned int max=1000;
double s=0.50;
double t=14.13472514173470;
//double t=21.02203963877156;
//double t=25.01085758014569;
//double t=30.42487612585951;
//double t=32.93506158773919;
//double t=37.58617815882568;
//double t=40.91871901214750;
//double t=43.32707328091500;
//double t=48.00515088116716;
//double t=49.77383247767230;
unsigned int out=1; // use out=3 otherwise
void main() {
unsigned int x,z;
double temp1,temps,tempt,sumr,sumi,a,b,olds,oldt,y;
FILE *Outfp;
Outfp = fopen(”ctrans.dat”,”w”);
sumr=0.0;
sumi=0.0;
olds=0.0;
oldt=0.0;
for (x=1; x<=max; x++) {
    z=max-x+1;
    if (s>=0.0)
        temp1=pow((double)z,s);
    else {
        temp1=pow((double)z,-s);
        temp1=1.0/temp1;
        }
    a=temp1*(cos(t*log(z)));
    b=temp1*(sin(t*log(z)));
    temp1=a*a+b*b;
```

```
    sumr=sumr+a/temp1;
    sumi=sumi-b/temp1;
    y=1-s;
    if (y>=0.0)
        temp1=pow((double)max,y);
    else {
        temp1=pow((double)max,-y);
        temp1=1.0/temp1;
        }
    temps=temp1*(y*cos(t*log(max))+t*sin(t*log(max)));
    tempt=temp1*(t*cos(t*log(max))-y*sin(t*log(max)));
    temps=temps/(y*y+t*t);
    tempt=tempt/(y*y+t*t);
    temps=sumr+temps;
    tempt=tempt-sumi;
    if (out==1)
        fprintf(Outfp," %.10llf %.10llf \n",temps,tempt);
    if ((out==2)&&((olds>0.0)&&(temps<0.0)&&(x>1)))
        fprintf(Outfp," %d %.10llf %.10llf \n",x,temps,tempt);
    if ((out==3)&&((oldt>0.0)&&(tempt<0.0)&&(x>1)))
        fprintf(Outfp," %d %.10llf %.10llf \n",x,temps,tempt);
    if ((out==4)&&((olds<0.0)&&(temps>0.0)&&(x>1)))
        fprintf(Outfp," %d %.10llf %.10llf \n",x,temps,tempt);
    if ((out==5)&&((oldt<0.0)&&(tempt>0.0)&&(x>1)))
        fprintf(Outfp," %d %.10llf %.10llf \n",x,temps,tempt);
    olds=temps;
    oldt=tempt;
    }
printf(" %.10llf %.10llf \n",temps,tempt);
fclose(Outfp);
return; }
```

REFERENCES

[1] Caceres, Pedro."$\zeta(z) = X(z) - Y(z)$ A decomposition of the Riemann Zeta Function for $Re(z) > 0$, $z \neq 1$", 2020. VIXRA:https://vixra.org/2003.0189, accessed March 2020.

[2] Caceres, Pedro, Proof of the Riemann Hypothesis using the decomposition Zeta(z)=X(z)-Y(z), RG/339841648, 2020