Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 19, Number 2 (2023), pp. 743-751

http://www.ripublication.com

Common Fixed Point Theorems in *S*-Metric Spaces Using the Property E.A. with an Application

[1]Surabhi Bhatt, [2]Rashmi Tiwari, [1]Kamal Wadhwa,
[1]*Department of Mathematics, Govt. Narmada College, Narmadapuram*
[2]*Department of Mathematics, Govt. M.G.M. P.G. College, Itarsi*

ABSTRACT

The aim of this paper is to prove some common fixed point theorems for two pairs of weakly compatible mappings using the property E.A. in S-metric spaces. Our results generalize and improve the results of Manro et al [8] and Gugnani et al [7] in *S*-metric spaces. Also we give an application of our results in proving existence of unique solution to a class of integral equations.

Keywords: *S*-metric space, weakly compatible mappings, property E.A.

Mathematics Subject Classification: Primary 47H10, Secondary 54H25.

INTRODUCTION:

The study of common fixed points of mappings satisfying certain contractive conditions has been an area of continuous and intense research. In 1988, Jungck and Rhoades [2] introduced the concept of weakly compatible mappings and weakened the concept of compatibility without using continuity of the mappings involved in the metric spaces. In 2002, Aamri and Moutawakil[3] introduced a new notion of the property E.A. in order to generalize the concept of non compatible mappings in metric spaces and proved common fixed point theorems. Since then many mathematicians proved several common fixed point theorems for contraction mappings in metric spaces by using different notions such as compatible mappings, weakly compatible mappings, property E.A., and by now there exists an extensive literature. On the other hand, a number of generalizations of metric spaces have been done, and one such generalization is an *S*-metric space.In 2012, Sedghi et al [6], introduced the concept of *S*-metric space. After that various authors proved fixed point theorems in these spaces. Some of these works have been noted in [9-12]. Several other studies relevant to metric spaces are being extended to *S*-metric spaces. In the present paper, we prove some common fixed point theorems for four weakly compatible mappings in *S*-metric spaces using the property E.A. As an application of our main result we prove a theorem for existence of solution of a non linear integral equation in the context of S-metric spaces. Now we give some preliminaries and basic definitions which will be used in this paper.

Definition1.1: [6] Let X be a nonempty set. A function $S: X^3 \to [0,\infty)$ is said to be an S-metric on X, if for each $x, y, z, a \in X$,

1. $S(x,y,z) \geq 0$;
2. $S(x,y,z) = 0$ if and only if $x = y = z$;
3. $S(x,y,z) \leq S(x,x,a) + S(y,y,a) + S(z,z,a)$.

The pair (X,S) is called an S-metric space.Some examples of S-metric spaces are:

Example1.2:[6]Let $X = R^n$and $\|.\|$ a norm on , then $S(x,y,z)$=$\|y+z-2x\| + \|y - z\|$ is an S-metric on X.

Example 1.3:[6] Let $X = R^n$and $\|.\|$ a norm on , then $S(x,y,z) = \|x-z\| + \|y-z\|$ is an S-metric on X.

Definition 1.4:[6] Let(X,S) be an S-metric space and $A \subset X$.

1. A sequence $\{x_n\}$ in X converges to x if $S(x_n,x_n,x) \to 0$ as $n \to \infty$, that is for every $\varepsilon > 0$ there exists $n_0 \in N$ such that for $n \geq n_0$, $S(x_n,x_n,x) < \varepsilon$. We denote this by $\lim_{n\to\infty} x_n = x$ and we say that x is the limit of $\{x_n\}$ in X.

2. A sequence $\{x_n\}$ in X is said to be cauchy sequence if for each $\varepsilon > 0$, there exists $n_0 \in N$ such that $S(x_n,x_n,x_m) < \varepsilon$ for each $n,m \geq n_0$.

3. The S-metric space (X,S) is said to be complete if every Cauchy sequence is convergent.

Lemma1.5:[6] Let(X,S) be an S-metric space. Then, we have

$S(x,x,y) = S(y,y,x),\ x,y \in X.$

Lemma 1.6:[6] Let(X,S) be an S-metric space. If there exist sequences $\{x_n\}$ and $\{y_n\}$ such that $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} y_n = y$, then $\lim_{n\to\infty} S(x_n,x_n,y_n) = S(x,x,y)$.

Definition1.7:[2] Let (X,S) be an S-metric space and *f* and g be two self mappings on X. Then *f* and g are said that to be weakly compatible if they commute at coincidence points.

Definition1.8:[3] Let(X,S) be an S-metric space and *f* and g be two self mappings on X. Then *f* and g are said to satisfy the property E.A. if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$ for some $t \in X$.

Definition1.9: Let Φ denote the set of all non decreasing continuous functions $\varphi: R^+ \to R^+$ satisfying,

1. $\varphi(0) = 0$;
2. $0 < \varphi(t) < t$ for all $t > 0$;
3. The series $\sum_{n\geq 1} \varphi^n(t)$converges for all $t > 0$.

MAIN RESULT:

Theorem 2.1: Let(X, S) be an S-metric space and $A, B, f, g : X \to X$ be four self mappings such that

1.$A(X) \subseteq g(X)$ and $B(X) \subseteq f(X)$;

2.one of the pairs (A, f) and (B, g) satisfies the property E.A.;

3.$S(Ax, Ax, By) \leq \varphi(\max\{ S(fx, fx, gy), S(fx, fx, By), S(gy, gy, By)\})$ for all

$x, y \in X$, where $\varphi \in \Phi$;

4.one of $A(X), B(X), f(X)$ and $g(X)$ is a complete subset of *X*.

Then the pairs (A, f) and (B, g) have a coincidence point each. Furthermore if pairs (A, f) and (B, g) are weakly compatible, then A, B, f and g have a unique common fixed point in *X*.

Proof: Suppose the pair (B, g) satisfies the property E.A. Then there exists a sequence $\{x_n\}$ in X such that

$\lim_{n\to\infty} Bx_n = \lim_{n\to\infty} gx_n = t$ for some $t \in X$.

Since $B(X) \subseteq f(X)$, there exists a sequence $\{y_n\}$ in X such that $Bx_n = fy_n$.

Hence, $\lim_{n\to\infty} fy_n = t$. Now, we will show that $\lim_{n\to\infty} Ay_n = t$. From (3) we have,

$S(Ay_n, Ay_n, Bx_n) \leq$
$\varphi(\max\{ S(fy_n, fy_n, gx_n), S(fy_n, fy_n, Bx_n), S(gx_n, gx_n, Bx_n)\})$.

Taking the limit as $n \to \infty$ and using the fact that $\varphi(r)$ is continuous at r =0, we get

$\lim_{n\to\infty} S(Ay_n, Ay_n, t) \leq \varphi(\max\{S(t,t,t), S(t,t,t), S(t,t,t)\}) = \varphi(0) = 0.$

Hence, $\lim_{n\to\infty} Ay_n = t.$ Thus we have,

$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} fy_n = \lim_{n\to\infty} gx_n = t.$

Suppose that $f(X)$ is a complete subspace of *X*. Then $t=fu$ for some $u \in X$.

Now we will show that $Au = fu = t$. From (3) we have,

$S(Au, Au, Bx_n) \leq \varphi(\max\{S(fu, fu, gx_n), S(fu, fu, Bx_n), S(gx_n, gx_n, Bx_n)\})$.

Taking the limit as $n \to +\infty$ by the property of φ,we get

$S(Au, Au, fu) \leq \varphi(\max\{S(t,t,t), S(t,t,t), S(t,t,t)\}) = \varphi(0) = 0$,

which implies, $S(Au, Au, fu) = 0$. Hence, $Au = fu$. Thus, u is a coincidence point of pair (A, f). The weak compatibility of A and f implies that $AAu = fAu$ and hence, $AAu = Afu = fAu = ffu$. Since $A(X) \subseteq g(X)$, there exists $v \in X$ such that $Au = gv$. Now we will prove that $gv = Bv$. Suppose not, then from (3) and using the fact that $\varphi(r) < r$, we have

$$S(Au,Au,Bv) \leq \varphi(\max\{S(fu,fu,gv),S(fu,fu,Bv),S(gv,gv,Bv)\})$$
$$= \varphi(\max\{S(0,S(Au,Au,Bv),S(Au,Au,Bv)\}$$
$$= \varphi(S(Au,Au,Bv))$$
$$< S(Au,Au,Bv).$$

This implies that $Au = Bv$ and hence $gv = Bv$. Thus, v is a coincidence point of the pair (B,g).Thus, $Au = fu = gv = Bv$. Now, if B and g are weakly compatible, then we obtain, $Bgv = gBv = ggv = BBv$. And we show that Au is a common fixed point of A,B,f and g. For this firstly we show that $fAu = AAu = Au$. Suppose on the contrary, $AAu \neq Au$. Then from (3) and the property of φ, we get,

$$S(AAu,AAu,Au) \leq S(AAu,AAu,Bv)$$
$$\leq \varphi(\max\{S(fAu,fAu,gv),S(fAu,fAu,Bv),S(gv,gv,Bv)\})$$
$$= \varphi(\max\{S(fAu,fAu,Bv),S(AAu,AAu,Bv),S(Bv,Bv,Bv)\})$$
$$= \varphi(\max\{S(AAu,AAu,Bv),0\})$$
$$= \varphi(S(AAu,AAu,Bv)$$
$$< S(AAu,AAu,Bv)$$
$$= S(AAu,AAu,Au).$$

Which is a contradiction. Thus, $AAu = Au$. Hence, $Au = AAu = fAu$ is a common fixed point of A and f. In the similar manner we can prove that Bv is a common fixed point of B and g. Since $Au = Bv$, we conclude that A,B,f and g have a common fixed point Au. Finally we show that the common fixed point is unique. Suppose to the contrary, A,B,f and g have two common fixed points w and z, with $w \neq z$. Then, from (3) and the property of φ, we have

$$S(w,w,z) = S(Aw,Aw,Bz)$$
$$\leq \varphi(\max\{S(fw,fw,gz),S(fw,fw,Bz),S(gz,gz,Bz)\})$$
$$= \varphi(\max\{S(w,w,z),S(w,w,z),S(z,z,z)\}$$
$$= \varphi\big(S(w,w,z)\big)$$
$$< S(w,w,z),$$

which is a contradiction. Thus, $w = z$. Hence, A,B,f and g have a common fixed point.

Theorem 2.2: Let(X,S) be an S-metric space and $A,B,f,g : X \to X$ be four self mappings such that

1.$\overline{A(X)} \subseteq g(X)$ and $\overline{B(X)} \subseteq f(X)$;

2. One of the pairs (A,f) and (B,g) satisfies the property E.A.;

3.$S(Ax,Ax,By) \leq \varphi(\max\{ S(fx,fx,gy),S(fx,fx,By),S(gy,gy,By)\})$;

for all $x,y \in X$, where $\varphi \in \Phi$.

Then the pairs (A,f) and (B,g) have a coincidence point each. Furthermore if pairs

(A,f) and (B,g) are weakly compatible, then A,B,f and g have a unique common fixed point in X.

Proof: Suppose the pair (B,g) satisfies the property E.A. Then there exists a sequence $\{x_n\}$ in X such that

$\lim_{n\to\infty} Bx_n = \lim_{n\to\infty} gx_n = t$ for some $t \in X$.

Since $\overline{B(X)} \subseteq f(X)$, there exists a sequence $\{y_n\}$ in X such that $Bx_n = fy_n$.

Hence, $\lim_{n\to\infty} fy_n = t$.

Now, we will show that $\lim_{n\to\infty} Ay_n = t$. From (3), we have

$S(Ay_n, Ay_n, Bx_n) \le \varphi(\max\{S(fy_n, fy_n, gx_n), S(fy_n, fy_n, Bx_n), S(gx_n, gx_n, Bx_n)\})$

Taking the limit as $n \to \infty$ and using the fact that $\varphi(r)$ is continuous at r =0, we get

$\lim_{n\to\infty} S(Ay_n, Ay_n, t) \le \varphi(\max\{S(t,t,t), S(t,t,t)S(t,t,t)\}) = \varphi(0) = 0.$

Hence, $\lim_{n\to\infty} Ay_n = t$. Thus, we have

$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} fy_n = \lim_{n\to\infty} gx_n = t,$ for some $t \in X$

Since, $t = \lim_{n\to\infty} Bx_n \in \overline{B(X)}$ and $\overline{B(X)} \subseteq f(X)$, there exists a point $u \in X$ such that $t = fu$.As proved in the previous theorem, we can show that u is a coincidence point of pair (A,f). Since $t = \lim_{n\to\infty} Ay_n \in \overline{A(X)}$ and $\overline{A(X)} \subseteq g(X)$, there exists a point $v \in X$ such that $t = gv$. As proved in the previous theorem, we can show that v is a coincidence point of the pair (B,g). The rest of the proof is similar to similar to the proof of the previous theorem, hence it is omitted.

Remarks 2.3:

1. The results of theorem 2.1 are true if we replace condition (1) of theorem 2.1 with the condition (1) of theorem 2.2 and omit the condition (4) of theorem 2.1 which requires completeness of one of the subspaces. Hence theorem 2.2 is an improved version of theorem 2.1..

2. Theorem 2.2 generalizes theorem 2.1 in Gugnani et al[7].

3. Theorem 2.2 is an improved version of the result of Manro et al [Theorem 2.1,8] for two pairs of weakly compatible pairs in the S-metric space setting which does not require completeness of the subspaces.

Theorem 2.4: Let(X,S) be an S-metric space and $A,B,f,g : X \to X$ be four self mappings such that

1.$\overline{A(X)} \subseteq g(X)$ and $\overline{B(X)} \subseteq f(X)$;

2. One of the pairs (A,f) and (B,g) satisfy the property E.A.;

3.$S(Ax, Ax, By) \le \varphi(S(fx, fx, gy))$, for all $x, y \in X$, where $\varphi \in \Phi$.

Then the pairs (A,f) and (B,g) have a coincidence point each. Furthermore if pairs (A,f) and (B,g) are weakly compatible, then A,B,f and g have a unique common fixed point in *X*.

Proof: Suppose the pair (B, g) satisfies the property E.A. Then there exists a sequence $\{x_n\}$ in X such that

$\lim_{n\to\infty} Bx_n = \lim_{n\to\infty} gx_n = t$ for some $t \in X$.

Since $\overline{B(X)} \subseteq f(X)$, there exists a sequence $\{y_n\}$ in X such that $Bx_n = fy_n$.

Hence, $\lim_{n\to\infty} fy_n = t$.

Now, we will show that $\lim_{n\to\infty} Ay_n = t$. From (3) we have,

$S(Ay_n, Ay_n, Bx_n) \leq \varphi\big(S(fy_n, fy_n, gx_n)\big)$.

Taking the limit as $n \to \infty$ and using the fact that $\varphi(r)$ is continuous at *r* =0, we get

$\lim_{n\to\infty} S(Ay_n, Ay_n, t) \leq \varphi(S(t, t, t)) = \varphi(0) = 0.$ Hence, $\lim_{n\to\infty} Ay_n = t$. Thus, we have $\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} fy_n = \lim_{n\to\infty} gx_n = t$, for some $t \in X$. Since, $t = \lim_{n\to\infty} Bx_n \in \overline{B(X)}$ and $\overline{B(X)} \subseteq f(X)$, there exists a point $u \in X$ such that $t = fu$. Again from (3), we have

$S(Au, Au, Bx_n) \leq \varphi(S(fu, fu, gx_n))$.

Taking $n \to \infty$ yields, $S(Au, Au, fu) \leq \varphi\big(S(fu, fu, fu)\big) = \varphi(0) = 0$.

Thus, $Au = fu$ and u is a coincidence point of the pair (A, f). Since the pair (A, f) is weakly compatible. Thus, $Afu = fAu$ and $AAu = Afu = fAu = ffu$. In the similar manner we can prove that v is a coincidence point of the pair (B, g). Since the pair (B, g) is weakly compatible. Also, weak compatibility of B and g implies that $Bgv = gBv$ and hence $BBv = ggv = Bgv = gBv$. And now we show that Au is a common fixed point of A, B, f and g. For this firstly we show that $fAu = AAu = Au$. Suppose on the contrary, $AAu \neq Au$. Then from (3) and the property of φ, we get,

$$\begin{aligned}
S(AAu, AAu, Au) &= S(AAu, AAu, Bv) \\
&\leq \varphi(S(fAu, fAu, gv)) \\
&= \varphi(S(AAu, AAu, Bv)) \\
&= \varphi(S(AAu, AAu, Au)) \\
&< S(AAu, AAu, Au),
\end{aligned}$$

Which is a contradiction.Thus, $AAu = Au$. Hence, $Au = AAu = fAu$ is a common fixed point of A and f. In the similar manner we can prove that Bv is a common fixed point of B and g. Since $Au = Bv$, we conclude that A, B, f and g have a common fixed point. Finally we can show that the common fixed point is unique as shown in the previous theorem.

Corollary 2.5: Let(X, S) be an S-metric space and $A, B, f, g : X \to X$ be four self mappings such that

1. $\overline{A(X)} \subseteq g(X)$ and $\overline{B(X)} \subseteq f(X)$; 2.

One of the pairs (A, f) and (B, g) satisfies the property E.A.;
3. $S(Ax, Ax, By) \leq q(S(fx, fx, gy))$, for all $x, y \in X$ where $q \in [0,1)$.

Then the pairs (A, f) and (B, g) have a coincidence point each. Furthermore if pairs (A, f) and (B, g) are weakly compatible, then A, B, f and g have a unique common fixed point in X.

Proof: Define $\varphi: [0, \infty) \to [0, \infty)$ by $\varphi(t) = qt$ for $q \in [0,1)$. Clearly $\varphi \in \Phi$. Hence the result easily follows from Theorem 2.4.

Corollary 2.6: Let (X, S) be an S-metric space and $A, f: X \to X$ be two self mappings satisfying
1. $\overline{A(X)} \subseteq f(X)$;

2. The pair (A, f) satisfies the property E.A.;

3. $S(Ax, Ax, Ay) \leq \varphi(S(fx, fx, fy))$, for all $x, y \in X$, where $\varphi \in \Phi$.

If pair (A, f) is weakly compatible, then A and f have a unique common fixed point in X.

Proof: By taking $f = g$ and $A = B$. The proof readily follows from theorem 2.4.

Corollary 2.7: Let(X, S) be an S-metric space and $A : X \to X$ be a self mapping satisfying$S(Ax, Ax, Ay) \leq \varphi(S(x, x, y))$, for all $x, y \in X$ .and where $\varphi \in \Phi$ then A has a unique fixed point in X.

Proof: By taking $A = B$ and $f = g = I$. The proof readily follows from theorem 2.4.

Corollary 2.8: Let (X, S) be an S-metric space and $A : X \to X$ be a self mapping satisfying $S(Ax, Ax, Ay) \leq qS(x, x, y)$, for all $x, y \in X$ .and where $q \in [0,1)$ then A has a unique fixed point in X.

Proof: Define $\varphi: [0, \infty) \to [0, \infty)$ by $\varphi(t) = qt$ for $q \in [0,1)$. Clearly, $\varphi \in \Phi$. Hence the result easily follows from Theorem 2.4.

Applications to Integral Equations

The existence and uniqueness of solution of integral equations has been studied by several authors (see [1, 4, 5]). The aim of this section is to prove an existence theorem for solution of a nonlinear integral equation in S-metric space using corollary 2.8. Consider the integral equation.

$u(t) = \int_0^T K(t, s, u(s))ds + h(t),\ t \in [0, T]$, Where $T > 0$ ……(1)

Let $X = C([0, T])$ be the set of all continuous functions defined on $[0, T]$. Define $S: X \times X \times X \to R^+$ by $S(x, y, z) = sup_{t\in[0,T]}|y(t) + z(t) - 2x(t)|$+$sup_{t\in[0,T]}\ |y(t) - z(t)|$. Clearly (X, S) is a complete S-metric space. Now, we shall prove theorem for existence of the solution the solution of the integral equation.

Theorem 3.1: Suppose the following assumptions hold a) $K: [0, T] \times [0, T] \to R$and

$h: R \to R$ are continuous;b)There exists a continuous function $G: [0, T] \times [0, T] \to R^+$ such that $|K(t, s, u) - K(t, s, v)| \leq G(t, s)|u - v|$ for each $u, v \in R$ and $s, t \in [0, T]$;

c) $sup_{s,t\in[0,T]} \int_0^T G(t, s)ds \leq q$for some $q \in [0,1)$.

Then the integral equation (1) has a solution $u \in X$.

Proof: Define $F: X \to X$ by

$$F\big(x(t)\big) = \int_0^T K\big(t, s, x(s)\big)ds + h(t), \quad t \in [0, T]$$

$$\begin{aligned}
S(Fx, Fx, Fy) &= sup_{t\in[0,T]}|Fx(t) + Fy(t) - 2Fx(t)| + sup_{t\in[0,T]}|Fx(t) - Fy(t)| \\
&= sup_{t\in[0,T]}|Fy(t) - Fx(t)| + sup_{t\in[0,T]}|Fy(t) - Fx(t)| \\
&= 2sup_{t\in[0,T]}|Fy(t) - Fx(t)| \\
&= 2sup_{t\in[0,T]}\left|\int_0^T (K\big(t, s, x(s)\big) - K\big(t, s, y(s)\big))ds\right| \\
&\leq 2sup_{t\in[0,T]} \int_0^T \left|K\big(t, s, x(s)\big) - K\big(t, s, y(s)\big)\right| ds \\
&\leq 2sup_{t\in[0,T]}|x(t) - y(t)|sup_{t\in[0,T]} \int_0^T G(t, s)ds \quad \text{(using condition (b))} \\
&= S(x, x, y)sup_{t\in[0,T]} \int_0^T G(t, s)ds \quad \text{(by (2))} \\
&\leq qS(x, x, y).
\end{aligned}$$

By using condition (c) there exists a $q \in [0,1)$ such that $sup_{t\in[0,T]} \int_0^T G(t, s)ds \leq q$.

Thus, $S(Fx, Fx, Fy) \leq qS(x, x, y)$ where $q \in [0,1)$.

Thus, all the required conditions of corollary 1.5 are satisfied. Hence F has a unique fixed point $u \in X$. And hence there exists a continuous solution $u \in X$ of the integral equation.

CONCLUSION:

In this paper, we proved some common fixed point theorems in S-metric spaces for four weakly compatible mappings using the weakly compatible mappings using the property E.A., without using the condition of continuity of mappings involved. Our results essentially generalize several results existing in the literature. Theorem 1.2 is an improved version of the result of Manro et al [Theorem 2.1,8] for two pairs of weakly compatible pairs without condition of completeness of subspaces in the context of S-metric spaces. Our results relaxed the continuity requirement of mappings completely and weakened the completeness requirement of the space. In the end, as an application of our result, we present a theorem for existence of unique solution of a non linear integral equation in S-metric space.

REFERENCES:

[1] Nieto,J.J.,1997,"An abstract monotone iterative technique", Non linear analysis: Theory Methods and Applications,28,1923-1933.

[2] Jungck,G.,Rhoades,B.E.,1998, "Fixed point for set valued functions without continuity",Indian J. Pure Appl. Math. 29(3), 227-238 MR1617919.

[3] Aamri,M. and Moutawakil,D. EI.,2002, "Some New Common Fixed Point Theorems under strict Contractive conditions", J. Math. Anal. Appl., 270,181188.

[4] Altun,I. and Simsek,H., 2010, "Some fixed point theorems on ordered metric spaces and application,Fixed point theory and applications, Article ID 6214469,17 pages.

[5] Shatanawi,W.,2011, "Some fixed point theorems in ordered G-metric spaces and Applications", Abst. Appl. Anal., Article ID 126205.

[6] Sedghi,S., Shobe,N., Aliouche,A.,2012, "A generalization of fixed point theorems in S-metric spaces", Mat. Vesn. 64, 258-266.

[7] Gugnani,M., Aggarwal,M., Chugh,R.,2012, "Common fixed point results in G-metric spaces and Applications" International Journal of Comp. Appl."43(12).

[8] Manro,S., Bhatia,S., Kumar,S., Vetro,C.,2013, "A Common fixed point theorem for two weakly compatible pairs in G-metric spaces using the property E.A." Fixed Point theory and Appl., Article ID:2013:41, 9 pages.

[9] Sedghi,S., Dung,N.V.,2014, "Fixed point theorems on S-metric spaces", Mat. Vesn. 66, 113-124.

[10] Sedghi,S., Shobe,N., Dosenovic,T.,2015, "Fixed point results in S-metric spaces". Non linearFunct. Anal. Appl. 20(1), 55-67.

[11] Sedghi,S., Dosenovic,T., Mahdi Rezaee,M.,2016,"Radenovic,S.,"Common fixed point theorems for contractive mappings satisfying φ-maps in S-metric spaces", Acta Univ. Sapientiae Math. 8(2), 298311.

[12] Sedghi,S., Shobe, N., Shahraki,M., Dosenovic,2018,"Common fixed point of four maps in S-metric spaces", Math. Sci., 12, 137-143.