Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 19, Number 2 (2023), pp. 727-743
© Research India Publications
http://www.ripublication.com

A Note on the Loop Product Difference Technique for Optimizing the IBFS of a Linear Transportation Problem

Dr. R. Murugesan

Associate Professor, Department of Mathematics, St. John's College, Palayamkottai-627002
(*Affiliated to Manonmaniam Sundaranar University Tirunelveli-627012, Tamil Nadu, India*)

Abstract

Opara Jude et al. (2019) developed and published a new and efficient approach (called Loop Product Difference Technique) for optimizing the Initial Basic Feasible Solution (IBFS) of a linear Transportation Problem (TP). Actually, it is a new and easy technique for optimizing the IBFS of a TP. The authors have given seven numerical examples of balanced category to illustrate the proposed technique. However, while optimizing the IBFSs of certain TPs, we have experienced that the proposed technique do not present optimal solution at all times. This statement is established by few numerical examples.

Keywords: Transportation problem, IBFS, Optimal Solution, I-SOFT Method, MODA Method**.**

Mathematics Subject Classification: 90C08, 90C90

INTRODUCTION

The readers of this article know the basics of Transportation problems (TPs) and the key methods such as NWCM, LCM, VAM, MODI and Stepping Stone [2, 8, 9] available to solve them. Now, we shall observe the very recent developments and publications in the area of TPs.

In 2021, Esakkiammal T. and Murugesan R. [1] proposed an innovative zero allocations approach named SOFTMIN which produces optimal solutions to a good number of the TPs. In 2022, Murugesan R. [6] established that the SOFTMIN method performs much better than the IASM method, but not a direct method to produce optimal solution to any given TP.

We further analyzed the process of allocation due to the SOFTMIN method on the near optimal solutions obtained for some ‘More Challenging’ TPs, and identified that very few changes made in the allocation process have improved the solution. This resulted in the ‘Improved SOFTMIN” (or briefly I-SOFT) method [3]. As for as our knowledge and search is concerned so far, no competing methods for generating best initial basic feasible solution (IBFS) on the identified some ‘More Challenging’ TPs are not available in the literature and thereby, the I-SOFT method may be the best one to produce the best IBFS to a given TP.

In 2022, Murugesan R. [4] proposed an innovative method named MODA (Modified Allocation) which tests the optimality of a solution and also optimizes the solution, if it is not optimal. By our further research we have identified the extra efforts made in the MODA method to trace and consider all possible loops starting and ending at an identified basic cell and passing through a non-basic cell. Consequently, we have simplified this difficulty by introducing the new idea of *Solution Improvement Loops* only to consider. This resulted in a revised version of the existing MODA method [5]. Here after, we call the revised version of the MODA method simply as MODA method.

In 2019, Opara Jude and et al. [7] proposed a new approach called Loop Product Difference for optimizing the IBFS of balanced TPs. The authors have tested and proved the efficiency of the technique by solving several numbers of costs minimizing balanced category of TPs and discovered that their technique gives the same result as that of the optimal solution obtained via MODI/Stepping Stone methods.

In this paper, we have tried to reveal that the proposed Loop Product Difference technique for optimizing the IBFS of TPs due to Opara Jude and et al. [7] do not present optimal solution at all times. We give examples of the TPs where the Loop Product Difference technique fails to find the optimal solution.

The paper is structured as follows: Section 1 briefs the introduction. Section 2 presents the algorithms of the existing I-SOFT method, MODA method and the Loop Product Difference Technique. Section 3 illustrates with three numerical examples for which the MODA method has produced optimal solution from the IBFS, whereas the Loop Product Difference technique has not produced optimal solution from the IBFS. Section 4 draws the conclusion.

ALGORITHMS OF THE EXISTING FOCUSED METHODS

In this section, the algorithms of the existing I-SOFT method, MODA method and the Loop Product Difference technique have been presented.

As the I-SOFT method generates the solution to a TP based on the *reduced cost matrix,* first we explain its derivation from the given cost matrix.

Row Minimum Subtraction (RMS) operation: Select the minimum element from each row and subtract it from each element in the corresponding row so that each row will contain at least one 0-entry.

Column Minimum Subtraction (CMS) operation: Select the minimum element from each column and subtract it from each element in the corresponding column so that each column will contain at least one 0-entry.

Reduced Cost Matrix (RCM): The matrix derived by applying the RMS/CMS or CMS/RMS operations on the cost matrix of the given TP is called the reduced cost matrix (RCM). It is obvious that there will be at least one 0-entry in each row and in each column of an RCM. In an RCM, the cells with only 0-entries are called*0-entry cells*.

ALGORITHM FOR THE EXISTING I-SOFT METHOD

The existing method is named as 'I-SOFT' (abbreviated from Improved SOFTMIN)because the individual allocations by this method is placed at a 0-entry cell on the basis of the ***S****um* ***O****f* ***F****irst* ***T****hree* ***min****imum* (SOFTMIN) elements computed for every row and every column of the RCM derived from the given TP. The following are the sequence of eight simple steps involved in it:

Step 1: Compute the Total supply as well as the Total demand for the given TP.

Step 2: Construct the Reduced Cost Matrix (RCM).

a) If (Total supply = Total demand), then Do the RMS operation first followed by the CMS operation on the resultant matrix. This will generate an RCM and go to Step-3 for making individual allocations.

b) If (Total demand < Total supply) then, introduce an additional dummy column to the transportation table to absorb the excess supply. The unit transportation cost for each of the cells in this dummy column is set to zero. Do the CMS operation only on the obtained balanced TP. This will generate an RCM and go to Step-3 for making individual allocations.

c) If (Total Supply < Total demand) then, introduce an additional dummy row to the transportation table to absorb the excess demand. The unit transportation cost for each of the cells in this dummy row is set to zero. Do the RMS operation only on the obtained balanced TP. This will generate an RCM and go to Step-3 for making individual allocations.

Step 3: Build the Allocations one by one in the RCM by means of the Tie Breaking Techniques.

(i) For each row, find the ***s****um* ***o****f* ***f****irst* ***t****hree* (soft) ***min****imum* (min) elements. Write the resulting sum under the Soft Min column by enclosing it in parentheses against the respective row. Similarly, do the same computation for each column.

(ii) Mark by *, the maximum among the Soft Min elements computed for rows and columns, along the corresponding row(s) and/ column(s).

(iii) Select the row or column which is marked by * and allocate the maximum possible quantity to that cell having 0-entry in that row or column. If tie occurs among certain 0-enty cells in that selected row or column, select the 0-entry cell which has the smallest original unit cost figure for allocation. If tie occurs among the smallest original unit cost figure, break the tie by using the technique given in (v).

(iv) Again, if tie occurs in case of (ii) amongst *, then select all these row and column (marked by *). For each such row and column list all the 0-entry cells. For each such 0-entry cell write its original unit cost figure. Select the 0-entry cell for allocation for which the original unit cost figure is the *minimum*.

Priority rule: While selecting the 0-entry cell corresponding to the original minimum unit cost, give priority to the cell which is not dummy in nature.

(v) Yet again, if tie occurs among certain 0-entry cells with the same minimum unit cost figure in (iv), then select the 0-entry cell for which the "*minimum*" possible quantity of allocation can be made.

(vi) Over again, if tie occurs among certain 0-entry cells with the same "*minimum*" allocation quantity in (v), then consider each such 0-entry cell for allocation as a separate case and finally choose the best solution among them. Such a situation may produce an alternative solution to the given TP.

Step 4: Reduce the RCM further. After performing Step 3, delete the row or column for further calculation where the supply from a given source is exhausted or the demand for a given destination is satisfied.

Step 5: Check the Reducibility of the Resultant Matrix. Check whether the resultant matrix obtained in Step 4 possesses at least one 0-entry in each row and in each column. If yes, go to Step3 for making the next allocation; otherwise, go to Step 2(a) for constructing a further reduced RCM.

Step 6: Repeat Steps 3 to 5 until and unless all the supplies are exhausted and all the demands are satisfied.

Step 7: Write the solution. Write the allocated quantities along with its cell and unit transportation cost one by one row-wise or by its order of allocation.

Step 8: Compute the Overall Transportation Cost. Finally, calculate the overall transportation cost, which is the sum of the product of unit transportation cost (from the original TP) and the corresponding quantity of allocation.

Important Note:

During the allocation process if the size of a particular RCM is of size 2 × n or m × 2, then only the individual allocations by this method is placed at a 0-entry cell on the basis of the ***S**um **Of** **F**irst "**Two**" **min**imum* (Soft Min) elements computed for every row and every column of that RCM. The allocations are very obvious, when the size of an RCM is with 1 × n or m × 1.

ALGORITHM FOR THE EXISTING 'MODA METHOD'

The term MODA has been coined from the first three letters of the word 'Modified' and the first one letter of the word 'Allocation'. MODA is an iterative method which can be used for testing the optimality of an IBFS and also optimize the IBFS, if it is not optimal, for transportation problems. The innovative way of improving a non-optimal solution to an optimal solution by the MODA method is based on redistributing the allocation available at a currently allocated cell (basic cell) with largest 'unit transportation cost' (UTC) to another un-allocated cell (non-basic cell) and its subsequent induced reallocations. The algorithm of the MODA method consists of two stages. In Stage #1, an IBFS is obtained to the given TP. In Stage #2, optimality testing of the obtained IBFS and also optimizing it, if it is not optimal, is carried out.

We use the following notations and abbreviations in the development of the algorithm of the MODA method:

$m \times n$	Size of the unit cost matrix of the given TP
TT	Transportation table
BTP	Balanced transportation problem
UTP	Unbalanced transportation problem
UTC	Unit transportation cost
C_{ij}	UTC available at the cell (i, j)
Optimal Solution	Initial basic feasible solution
$X = [x_{ij}]$	A solution
X^*	An optimal solution
TTC	Total transportation cost
Z(X)	TTC corresponding to the solution X.
$Z(X^*)$	Minimum TTC
NBC	Non-Basic Cell
IBC	Identified Basic Cell
NCC	Net Cost Change
SI	Solution Improvement

STAGE#1: OBTAIN AN IBFS TO THE GIVEN TP

For the given TP, first obtain an IBFS say $X^{(0)}$ with its associated total transportation cost $Z(X^{(0)})$ using any available method in TPs. We use the I-SOFT method [3] to obtain an IBFS because at present day it has been identified and established as the best method to find the best IBFS to TPs.

STAGE #2: TEST THE OPTIMALITY OF THE OBTAINED IBFS

Step 1: Construct the current solution table

Consider the transportation table (TT) en-squared with the obtained allocations (IBFS) $X^{(0)}= [x_{ij}]$as the current solution table. Also, compute the corresponding TTC $Z(X^{(0)})$.

Step 2: Ensure the Non-degeneracy condition

Ensure the numbers of basic cells in the TT exactly equal to (m+n-1).

Step 3: Perform the Optimality Test on the IBFS $X^{(0)}$

(a) Determine $C(X^{(0)}) = \text{Max}\{c_{ij} : x_{ij} > 0\}$ and the corresponding basic cell as the identified basic cell (IBC). Let it be (h, k).

- (i) If the IBC is unique, then go to Step (b) directly.
- (ii) If there is two or more basic cells having the same largest UTC $C(X^{(0)})$, then select the basic cell having the maximum quantity of allocation as the IBC. Let it be (h, k) and go to Step (b).
- (iii) If there is two or more basic cells having the same largest UTC $C(X^{(0)})$ and with the same maximum allocation quantity, then select any one such basic cell asthe IBC. Let it be (h, k) and go to Step (b).

(b)Trace a Solution Improvement (SI) loop starting and ending at the IBC (h, k) and passing through a non-basic cell. As it is a SI loop, it will have the Net Cost Change (NCC) value as non-positive (≤ 0). If there is a tie between two or more than two SI loops with the same NCC value, then select any one loop. Such a situation may generate alternative solutions to the given TP. If the NCC value of the SI loop is zero, then this will also indicate the existence of an alternative solution to the given TP.

(c) Implement this loop and obtain the better BFS, say $X^{(1)}$ with its associated TTC $Z(X^{(1)})$.

(d) If it is not possible to trace a SI loop starting and ending at the current IBC, then consider the next basic cell having UTC next to $C(X^{(0)})$ as the new IBC (h, k) and go to Step (a(i)).

Step 4: Repeat Steps 3(a) to (d) until no SI loop can be traced starting and ending at the new IBC with the current largest UTC. At this level, the solution under optimality test is the optimal one. Write the optimal solution X^* with its minimum TTC as $Z(X^*)$.

ALTERNATIVE OPTIMAL SOLUTION

At the 'optimal level', if the NCC value of the SI loop is zero, then this indicates that the given TP has an alternative optimal solution. By implementing this loop we can get the alternative optimal solution to the given TP.

IMPORTANT NOTE

1. One cannot restrict a SI loop with corner cells having UTCs less than or equal to the UTC of the identified basic cell.
2. One cannot restrict the place (even position or odd position) of the non-basic cell in a SI loop.

ALGORITHM FOR THE EXISTING TEST FOR OPTIMALITY USING THE LOOP PRODUCT DIFFERENCETECHNIQUE (OPARA JUDE ET AL. 2019)

The new algorithm for obtaining the optimal solution of the linear transportation problem indirectly is discussed below.

Step 1: Form a closed path for the entire non basic cell. Having formed the closed path, mark the identified empty cell as positive and each occupied cell at the corners of the path alternately –ve, +ve, –ve, +ve and so on.

Step 2: For each non basic cell, determine P_{ts}=C_{ts} C_{+mc}–C_{-mc} C_{-nmc}= C_{tq} and if $P_{ts} \geq 0$, stop. Otherwise obtain P_{ts}=min[C_{ts} C_{+mc}–C_{-mc} C_{-nmc}]= C_{tq} and go to step 3, where C_{ts} is the cost of the entering variable in the closed path with a positive sign, C_{+mc} is the minimum cost in the closed path with a positive sign, C_{-mc} is the minimum cost in the closed path with a negative sign, and C_{-nmc} is the next minimum cost in the closed path with a negative sign.

Step 3: The non basic variable say C_{tq} enters the basis since $C_{tq} < 0$. Allocate $x_{tq} = \theta$, (where θ is found as in the linear transportation case) in the concerned closed loop, which when modified by the $x_{tq} = \theta$ value will keep a_i and b_j values unchanged. Determine the leaving variable say x_{Btk}, where x_{Btk} is the basic variable which turns to zero while making the modification, and $x_{tq} = \theta$ becomes the new basic variable, and go to Step 1.

NUMERICAL ILLUSTRATIONS

Suitable illustrative solution makes the readers to understand the unworthiness of the Loop Product Difference technique which has failed to generate the optimal solution from an IBFS of a given TP. Bearing in mind three TPs from the literature have been illustrated.

Example-1: Consider the following cost minimization type BTP with three sources and five destinations, as given in Table 1.

Table 1: The given BTP

Sources	Destinations					Supply
	D1	**D2**	**D3**	**D4**	**D5**	
S1	1	9	13	36	51	**50**
S2	24	12	16	20	1	**100**
S3	14	33	1	23	26	**150**
Demand	**100**	**70**	**50**	**40**	**40**	**300**

SOLUTION BY THE 'MODA METHOD'

Stage #1: Obtain an IBFS

In Stage #1, we solve the given BTP by using the I-SOFT method and obtain the IBFS ($X^{(0)}$) table. This is shown in Table 2.

Stage #2: Optimizing the obtained solution by the MODA method

Construct the current solution table

Consider the transportation table (TT) en-squared with the obtained allocations (solution) as the current solution table. This is the IBFS and is shown in Table 2.

Table 2: The IBFS table obtained by the I-SOFT method

Sources	Destinations					Supply
	D1	D2	D3	D4	D5	
S1	**50** 1	9	13	36	51	**50**
S2	24	**60** 12	16	20	**40** 1	**100**
S3	**50** 14	**10** 33	**50** 1	**40** 23	26	**150**
Demand	**100**	**70**	**50**	**40**	**40**	**300**

Writing the IBFS $X^{(0)}$

As of Table 2, the IBFS is $X^{(0)}$ = {x_{11} = 50, x_{22} = 60, x_{25} = 40, x_{31} = 50, x_{32} = 10, x_{33} = 50, x_{34} = 40}and the associated TTC is Z($X^{(0)}$) = $2810.

Optimality Testing for the IBFS $X^{(0)}$by the MODA method [First Iteration]

(a) Determine C($X^{(0)}$**)** **=** Max{c_{ij} ; $x_{ij} > 0$} = Max {1, **12**, 1, 14, 33, 1, 23} = 33 at the unique cell (h, k) = (S3, D2). Therefore, the IBC is (S3, D2).

(b) Trace a SI loop starting and ending at the IBC (S3, D2) and passing through one opt NBC. There is only one SI loop.

Loop = {(S3, D2), (S1, D2), (S1, D1), (S3, D1)} passing through the NBC (S1, D2) with c_{12} = 9 and NCC value as -11 (= 9-1+14-33).

(c) By implementing this loop we obtain the following better BFS shown in Table 3.

Table 3: A better BFS (Optimal Solution) table

Sources	Destinations					Supply
	D1	D2	D3	D4	D5	
S1	**40** 1	**10** 9	13	36	51	**50**
S2	24	**60** 12	16	20	**40** 1	**100**
S3	**60** 14	33	**50** 1	**40** 23	26	**150**
Demand	**100**	**70**	**50**	**40**	**40**	**300**

Writing the better BFS $X^{(1)}$

As of Table 3, the new BFS is $X^{(1)}$ = {x_{11} = 40, x_{12} = 10, x_{22} = 60, x_{25} = 40, x_{31} = 60, x_{33} = 50, x_{34} = 40}and the associated TTC is Z($X^{(0)}$) = $2700. Note that $X^{(1)}$ is a better BFS than $X^{(0)}$ as Z($X^{(1)}$) < Z($X^{(0)}$).

Optimality Testing for the BBFS $X^{(1)}$ by the MODA method [Second Iteration]

As it is not possible to improve the BBFS $X^{(1)}$, the current solution is the optimal solution one to the given TP.

Writing the Optimal Solution

The optimal solution (X^*) to the given TP is $X^* = \{x_{11} = 40, x_{12} = 10, x_{22} = 60, x_{25} = 40, x_{31} = 60, x_{33} = 50, x_{34} = 40\}$ with the minimum TTC of $Z(X^*)$ = \$2700.

OPTIMALITY TEST ON THE IBFS $X^{(0)}$BY THE LOOP PRODUCT DIFFERENCE TECHNIQUE DUE TO OPARA JUDE ET AL. (2019)

Consider the IBFS shown in Table 2. For this IBFS, we are going to apply the Loop Product Difference technique for optimizing the IBFS. The computation of Loop Product Difference of each of the loops starting and ending at every NBC is shown in Table 4.

Table 4: Optimality test on the IBFS $X^{(0)}$ by the Loop Product Difference technique

Non Basic Cell(NBC)	Loop starts and ends at the NBC	Loop Product Difference
(S1, D2)	{(S1, D2), (S3, D2), (S3, D1), (S1, D1)}	9(14) –1(33) = 093
(S1, D3)	{(S1, D3), (S3, D3), (S3, D1), (S1, D1)}	13(14) – 1(1) = 182
(S1, D4)	{(S1, D4), (S3, D4), (S3, D1), (S1, D1)}	36(14) – 1(23) = 481
(S1, D5)	{(S1, D5), (S2, D5), (S2, D2), (S3, D2), (S3, D1), (S1, D1)}	51(12) – 1(1) = 611
(S2, D1)	{(S2, D1), (S2, D2), (S3, D2), (S3, D1)}	24(33) – 14(12) = 624
(S2, D3)	{(S2, D3), (S3, D3), (S3, D2), (S2, D2)}	16(33) – 1(12) = 516
(S2, D4)	{(S2, D4), (S3, D4), (S3, D2), (S2, D2)}	20(33) – 23(12) = 384
(S3, D5)	{(S2, D3), (S3, D2), (S2, D2), (S2, D5)}	26(12) – 1(33) = 279

Note that, the Loop Product Difference for each of the NBCs is non-negative. Therefore, the current solution is the optimal one.

Writing the Optimal Solution (X^*)

As of Table 2, the optimal solution is $X^* = \{x_{11} = 50, x_{22} = 60, x_{25} = 40, x_{31} = 50, x_{32} = 10, x_{33} = 50, x_{34} = 40\}$, and the associated TTC is $Z(X^*)$ = \$2810.

Comment

By the MODA method we have found that the optimal solution to the given problem is $X^* = \{x_{11} = 40, x_{12} = 10, x_{22} = 60, x_{25} = 40, x_{31} = 60, x_{33} = 50, x_{34} = 40\}$ with the minimum TTC of $Z(X^*)$ = \$2700. This shows that the Loop Product Difference technique fails to produce the optimal solution to the given TP.

Example-2: Consider the following cost minimization type BTP with four sources and six destinations, as given in Table 5.

Table 5: The given BTP

Sources	Destinations						Supply
	D1	**D2**	**D3**	**D4**	**D5**	**D6**	
S1	1	2	1	4	5	2	**30**
S2	3	3	2	1	4	3	**50**
S3	4	2	5	9	6	2	**75**
S4	3	1	7	3	4	6	**20**
Demand	**20**	**40**	**30**	**10**	**50**	**25**	**175**

SOLUTION BY THE 'MODA METHOD'

Stage #1: Obtain an IBFS

In Stage #1, we solve the given BTP by using the I-SOFT method and obtain the IBFS ($X^{(0)}$) table. This is shown in Table 6.

Stage #2: Optimizing the obtained solution by the MODA method

Construct the current solution table

Consider the transportation table (TT) en-squared with the obtained allocations (solution) as the current solution table. This is the IBFS and is shown in Table 6.

Table 6: The IBFS table obtained by the I-SOFT method

Sources	Destinations						Supply
	D1	D2	D3	D4	D5	D6	
S1	[20] 1	2	[10] 1	4	5	2	**30**
S2	3	3	[20] 2	[10] 1	[20] 4	3	**50**
S3	4	[20] 2	5	9	[30] 6	[25] 2	**75**
S4	3	[20] 1	7	3	4	6	20
Demand	**20**	**40**	**30**	**10**	**50**	**25**	**175**

Writing the IBFS $X^{(0)}$

As of Table 6, the IBFS is $X^{(0)}$ = {x_{11} = 20, x_{13} = 10, x_{23} = 20, x_{24} = 10, x_{25} = 20, x_{32} = 20, x_{35} = 30, x_{36} = 25, x_{41} = 20}and the associated TTC is Z($X^{(0)}$) = $440.

Optimality Testing for the IBFS $X^{(0)}$ by the MODA method [First Iteration]

(a) Determine C($X^{(0)}$**)** **=** Max{c_{ij} ; $x_{ij} > 0$} = Max {1, 1, 2, 1, 4, 2, **6**, 2, 1} = 6 at the unique cell (h, k) = (S3, D5). Therefore, the IBC is (S3, D5).

(b) Trace a SI loop starting and ending at the IBC (S3, D5) and passing through one opt NBC. There is only one SI loop.

Loop = {(S3, D5), (S4, D5), (S4, D2), (S3, D2)} passing through the NBC (S4, D5) with c_{45} = 4 and NCC value as -1 (= 4-1+2-6).

(c) By implementing this loop we obtain the following better BFS shown in Table 7.

Table 7: The better BFS (Optimal Solution) table

Sources	Destinations						Supply
	D1	D2	D3	D4	D5	D6	
S1	**20** 1	2	**10** 1	4	5	2	**30**
S2	3	3	**20** 2	**10** 1	**20** 4	3	**50**
S3	4	**40** 2	5	9	**10** 6	**25** 2	**75**
S4	3	1	7	3	**20** 4	6	20
Demand	**20**	**40**	**30**	**10**	**50**	**25**	**175**

Writing the new BFS $X^{(1)}$

As of Table 7, the new BFS is $X^{(1)}$ = {x_{11} = 20, x_{13} = 10, x_{23} = 20, x_{24} = 10, x_{25} = 20, x_{32} = 40, x_{35} = 10, x_{36} = 25, x_{45} = 20}, and the associated TTC is Z($X^{(0)}$) = $430.

Optimality Testing for the IBFS $X^{(1)}$ by the MODA method [Second Iteration]

(a) Determine C($X^{(0)}$**)** **=** Max{c_{ij} ; $x_{ij} > 0$} = Max {1, 1, 2, 1, 4, 2, **6**, 2, 4} = 6 at the unique cell (h, k) = (S3, D5). Therefore, the IBC is (S3, D5).

(b) Trace a SI loop starting and ending at the IBC (S3, D5) and passing through one opt NBC. There is only one SI loop.

Loop = {(S3, D5), (S3, D1), (S1, D1), (S1, D3), (S2, D3), (S2, D5)} passing through the NBC (S3, D1) with c_{31} = 4 and NCC value as 0 (= 4-1+1-2+4-6).

This shows that there is no improvement in the current solution. Further, if we consider basic cells (S2, D5), (S4, D5) and so on as the next IBCs then also no further improvement in the current solution. Hence the current solution is the optimal solution.

Writing the Optimal Solution (X^*)

As of Table 7, the optimal solution is $X^* = \{x_{11} = 20, x_{13} = 10, x_{23} = 20, x_{24} = 10, x_{25} = 20, x_{32} = 40, x_{35} = 10, x_{36} = 25, x_{45} = 20\}$, and the associated minimum TTC is $Z(X^{(*}) = \$430$.

Alternative Optimal Solution (X^{**})

At the optimal level, as the SI loop starting and ending at the IBC (S3, D5) and passing through the NBC (S3, D1), that is,

Loop = {(S3, D5), (S3, D1), (S1, D1), (S1, D3), (S2, D3), (S2, D5)} has NCC value as 0, the given TP has an alternative optimal solution. By implementing this loop we obtain the alternative optimal solution, which is shown in Table 8.

Table 8: The Alternative Optimal Solution table

Sources	Destinations						Supply
	D1	D2	D3	D4	D5	D6	
S1	**10** 1	2	**20** 1	4	5	2	**30**
S2	3	3	**10** 2	**10** 1	**30** 4	3	**50**
S3	**10** 4	**40** 2	5	9	6	**25** 2	**75**
S4	3	1	7	3	**20** 4	6	20
Demand	**20**	**40**	**30**	**10**	**50**	**25**	**175**

Writing the Alternative Optimal Solution (X^{**})

As of Table 8, the alternative optimal solution is $X^{**} = \{x_{11} = 10, x_{13} = 20, x_{23} = 10, x_{24} = 10, x_{25} = 30, x_{31} = 10, x_{32} = 40, x_{36} = 25, x_{45} = 20\}$, and the associated minimum TTC is $Z(X^{**}) = \$430$.

OPTIMALITY TEST ON THE IBFS $X^{(0)}$BY THE LOOP PRODUCT DIFFERENCE TECHNIQUE DUE TO OPARA JUDE ET AL. (2019)

Consider the IBFS shown in Table 6. For this IBFS, we are going to apply the Loop Product Difference Technique for optimizing the IBFS. The working out of Loop Product Difference of each of the loops starting and ending at every NBC is shown in Table 9.

Table 9: Optimality test on the IBFS $X^{(0)}$by the Loop Product Difference technique

NBC	Loop starts and ends at the NBC	Loop Product Difference
(S1, D2)	{(S1, D2), (S1, D3), (S2, D3), (S2, D5), (S3, D5), (S3, D2)}	2(2) – 1(2) = 2
(S1, D4)	{(S1, D4), (S3, D4), (S2, D3), (S1, D3)}	4(2) – 1(1) = 7
(S1, D5)	{(S1, D5), (S2, D5), (S2, D3), (S1, D3)}	5(2) – 1(4) = 6
(S1, D6)	{(S1, D6), (S3, D6), (S3, D5), (S2, D5), (S2, D3), (S1, D3)}	2(2) – 1(2) = 2
(S2, D1)	{(S2, D1), (S2, D3), (S1, D3), (S1, D1)}	3(1) – 1(2) = 1
(S2, D2)	{(S2, D2), (S2, D5), (S3, D5), (S3, D2)}	3(6) – 4(2) = 10
(S2, D6)	{(S2, D6), (S3, D6), (S3, D5), (S2, D5)}	3(6) – 2(4) = 10
(S3, D1)	{(S3, D1), (S3, D5), (S2, D5), (S2, D3), (S1, D3), (S1, D1)}	4(1) – 1(2) = 2
(S3, D3)	{(S3, D3), (S3, D5), (S2, D5), (S2, D3)}	5(4) – 2(6) = 8
(S3, D4)	{(S3, D4), (S3, D5), (S2, D5), (S2, D4)}	9(4) – 1(6) = 30
(S4, D1)	{(S4, D1), (S4, D2), (S3, D2), (S3, D5) , (S2, D5), (S2, D3), (S1, D3), (S1, D1)}	3(1) – 1(1) = 2
(S4, D3)	{(S4, D3), (S2, D3), (S2, D5), (S3, D5) , (S3, D2), (S4, D2)}	7(2) – 1(2) = 12
(S4, D4)	{(S4, D4), (S2, D4), (S2, D5), (S3, D5) , (S3, D2), (S4, D2)}	3(2) – 1(1) = 5
(S4, D5)	{(S4, D5), (S3, D5), (S3, D2), (S4, D2)}	4(2) – 1(6) = 2
(S4, D6)	{(S4, D6), (S3, D6), (S3, D2), (S4, D2)}	6(2) – 1(2) = 10

Note that, the Loop Product Difference for each of the NBCs is non-negative. Therefore, the current solution is the optimal one to the given TP.

Writing the Optimal Solution (X^*)

As of Table 6, the optimal solution is $X^* = \{x_{11} = 20, x_{13} = 10, x_{23} = 20, x_{24} = 10, x_{25} = 20, x_{32} = 20, x_{35} = 30, x_{36} = 25, x_{41} = 20\}$, and the associated TTC is $Z(X^*) =$ \$440.

Comment

By the MODA method we have found that there are two optimal solutions to the given problem:

X^* = {x_{11} = 20, x_{13} = 10, x_{23} = 20, x_{24} = 10, x_{25} = 20, x_{32} = 40, x_{35} = 10, x_{36} = 25, x_{45} = 20}, and the associated minimum TTC is $Z(X^{(*})$ = \$430 and

X^{**} = {x_{11} = 10, x_{13} = 20, x_{23} = 10, x_{24} = 10, x_{25} = 30, x_{31} = 10, x_{32} = 40, x_{36} = 25, x_{45} = 20}, and the associated minimum TTC is $Z(X^{**})$ = \$430.

This shows that the Loop Product Difference technique fails to produce the optimal solution to the given TP.

Example-3: Consider the following cost minimization type Unbalanced TP with three sources and three destinations, as given in Table 10.

Table 10: The given Unbalanced TP

Sources	Destinations			Supply
	D3	**D4**	**D5**	
S1	6	10	14	**50**
S2	12	19	21	**50**
S3	15	14	17	**50**
Demand	**30**	**40**	**55**	**125/150**

For the Unbalanced TP shown in Table 10, the MODA method has produced an optimal solution X^* = {x_{11} = 5, x_{12} = 40, x_{13} = 5, x_{21} = 25, x_{24} = 25, x_{33} = 50} with the minimum TTC of \$1650 from the IBFS $X^{(0)}$ = {x_{11} = 30, x_{12} = 20, x_{23} = 25, x_{24} = 25, x_{32} = 20, x_{33} = 30}obtained using the I-SOFT method with the TTC of \$1695 where as the Loop Product Difference technique has produced an optimal solution X^* = {x_{11} = 30, x_{12} = 20, x_{23} = 25, x_{24} = 25, x_{32} = 20, x_{33} = 30}with the minimum TTC of \$1695. This shows that the Loop Product Difference technique fails to produce the optimal solution to the given Unbalanced TP.

CONCLUSION

In 2019, Opara Judeet al. proposed a new approach named Loop Product Difference Technique for optimizing the IBFS of linear TPs. In fact, it is a new and easy technique for optimizing the IBFS of a TP. To illustrate the new technique, the authors have given seven numerical examples from the balanced category. But, at the same time as optimizing the IBFSs of certain TPs, we have experienced that the proposed technique do not generate optimal solution at all. We have given the IBFS of TPs where the Loop Product Difference technique does not generate optimal solutions at all times. Also, it does not produce the alternative optimal solution of a

TP, if it exists to the TP. Another drawback of this technique is that it is not extended to optimize the IBFS of unbalanced TPs. Hence it is not intelligent to depend on the solutions found by proposed technique.

REFERENCES

[1] Esakkiammal T., Murugesan R., *SOFTMIN – An Innovative and Efficient Zeros Allocation Method to Solve Transportation Problems*,1st International Conference on Mathematical Modeling and its Soft Computing Applications (ICMMSCA-2021, November 29-30, 2021, Puttaparthi, Andhra Pradesh, India, Paper ID C-11/ISBN 978-81-952716-2-7 (2021).

[2] Kanti Swarup, Gupta P.K. and Man Mohan, *Operations Research*, 19th Edition, Sultan Chand & Sons, Educational Publishers, New Delhi, (2017).

[3] Murugesan R., *I-SOFT – The Best Method for Finding the Best IBFS to Transportation Problems*, Global Journal of Pure and Applied Mathematics, Volume 18, Number 01, pp. 377-391, (2022).

[4] Murugesan R., *MODA – An Innovative Method for Optimality Testing and Optimizing a Solution in Transportation Problems*, Global Journal of Pure and Applied Mathematics, Volume 18, Number 02, pp. 479-498, (2022).

[5] Murugesan R., *MODA Method – A Revised Version: For Optimality Testing and Optimizing a Solution in Transportation Problems*, Global Journal of Pure and Applied Mathematics, Volume 19, Number 02, pp. 387-404, (2022).

[6] Murugesan R., *SOFTMIN Method versus IASM Method on Some Challenging Transportation Problems*, Global Journal of Pure and Applied Mathematics, Volume 18, Number 1, pp. 341-351, (2022).

[7] Opara Jude, Oruh Ben, Ihekuna Stephen, Okenwe Idochi, *A New and Efficient Proposed Approach for Optimizing the Initial Basic Feasible Solution of a Linear Transportation Problem*, International Journal of Mathematics Trends and Technology (IJMTT), Volume 65, Issue 6, pp. 104-118, (2019).

[8] Sharma J.K., *Operations Research-Theory and applications*, Trinity Press, New Delhi, (2017).

[9] Srinivasan G., Operations Research - Principles and Applications, PHI Learning Private Limited, New Delhi, (2012).