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Two Generalized Zeta Functions

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Abstract

Two generalized zeta functions which can be associated with a numerical sequence $\{\lambda_k\}$ are investigated. These generalized zeta functions have the same functional equation as the usual Riemann zeta function. An elementary method for finding the Riemann zeta function zeros is devised (accurate to about one or two decimal points).
Keywords: Riemann zeta function

1. INTRODUCTION

Equation (3) in section 1.3 of Edward's [1] book is

$$\Pi(s) = \lim_{N\to\infty} \frac{1\cdot 2\cdots N}{(s+1)(s+2)\cdots(s+N)}(N+1)^s \tag{1}$$

This equation is valid for all s in the halfplane Re $s > -1$. (Edwards uses the notation $\Pi(s-1)$ instead of $\Gamma(s)$.)

2. THE CASE $\{\lambda_k = k^2\}$

The partition function is related to a Jacobi theta function. Equation 6.1 in Voros [2] article is

$$\Theta(t) = \sum_1^\infty e^{-tk^2} = (\Theta_3(0|\frac{it}{\pi}) - 1)/2 \tag{2}$$

Jacobi's identity is

$$\Theta_3(z|\tau) = (-i\tau)^{-1/2} e^{z^2}/i\pi\tau\Theta_3(\frac{z}{\tau}| - \frac{1}{\tau}) \tag{3}$$

Equation 6.5 (the reflection formula for $\Gamma(z)$) is

$$Z(s) = \sum_{k=1}^\infty k^{-2s} = \zeta(2s) \tag{4}$$

A function involving the reflection formula is

$$\zeta_1(s) = \frac{2\pi\zeta(s-1)}{\Pi(s-1)Z(s-1)} \tag{5}$$

A plot of this expression for the first non-trivial zeta function zero ($s = (0.5, 14.13472514173470)$) and $n \leq 100000$ is

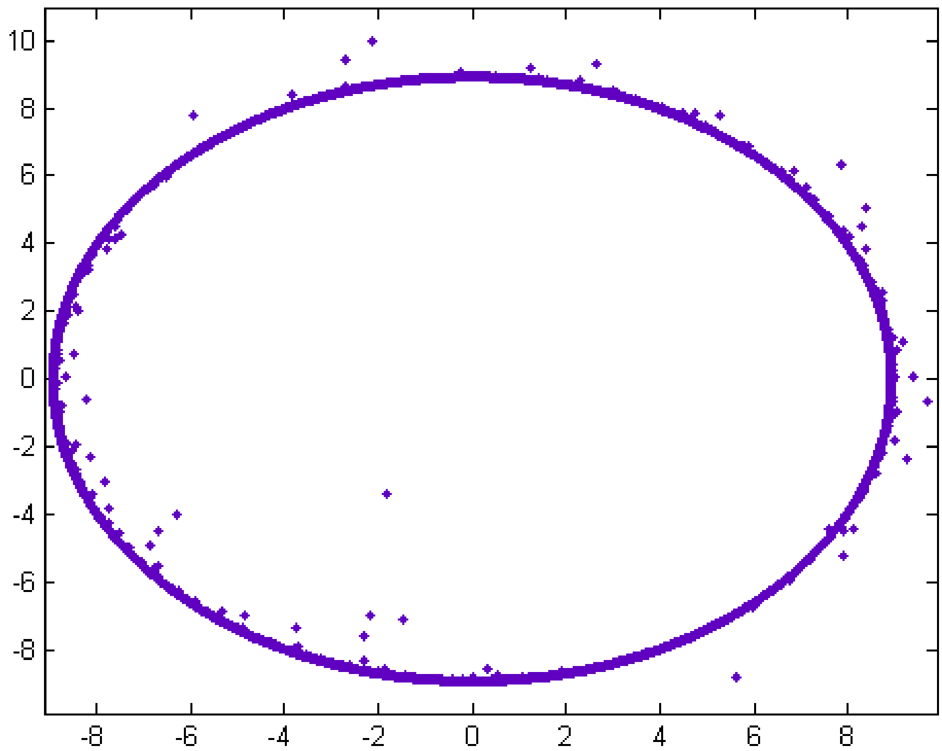

Figure 1:

The slopes and intercepts of the logarithms of the n values of the inflection points (where the curve decreases toward the x-axis and then increases) for the first ten zeta function zeros are (0.4458, 0.588), (0.2983, 1.297), (0.2512, 1.35), (0.2068, 1.719), (0.1908, 1.595), (0.1672, 1.902), (0.1537, 1.892), (0.145, 1.941), (0.1311, 1.87), and (0.1262, 1.943). A plot of the logarithms of the n values of the inflection points for the tenth zeta function zero ($s = (0.5, 49.77383247767230)$) and $n \leq 100000$ is

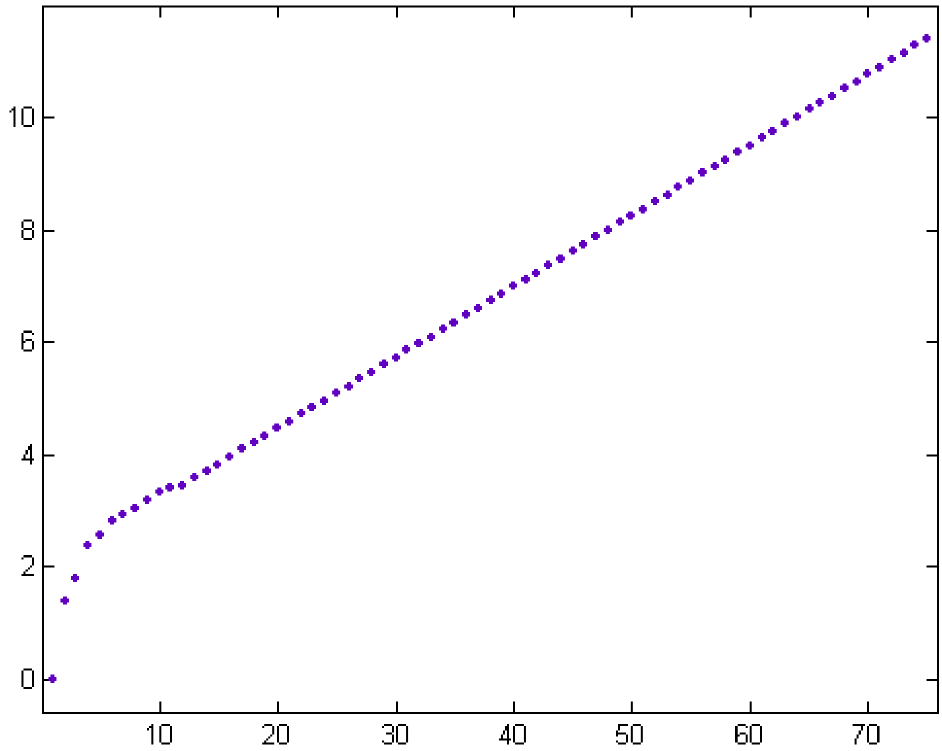

Figure 2:

The first sixteen n values will be disregarded in computing the slope. This is unusual for the usual zeta function zeros - at most one value has to be disregarded. In computing the above slopes 1, 2, 6, 6, 7, 9, 9, 9, 13, and 16 n values were discarded respectively. Other than this, the slopes are almost the same as for the usual zeta function zeros. The slopes for the usual zeta function zeros and $n \leq 1000000$ are 0.4444, 0.2988, 0.2512, 0.2064, 0.1909, 0.1673, 0.1535, 0.145, 0.1309, and 0.1263.

Riemann's functional equation (equation (5) in section 1.6 of Edwards' book) is

$$\Pi(\frac{s}{2} - 1)\pi^{-s/2}\zeta(s) = \Pi(\frac{1-s}{2} - 1)\pi^{-(1-s)/2}\zeta(1-s) \tag{6}$$

A plot of the left and right sides of the equation for the first function zero and $n \leq 100000$ is

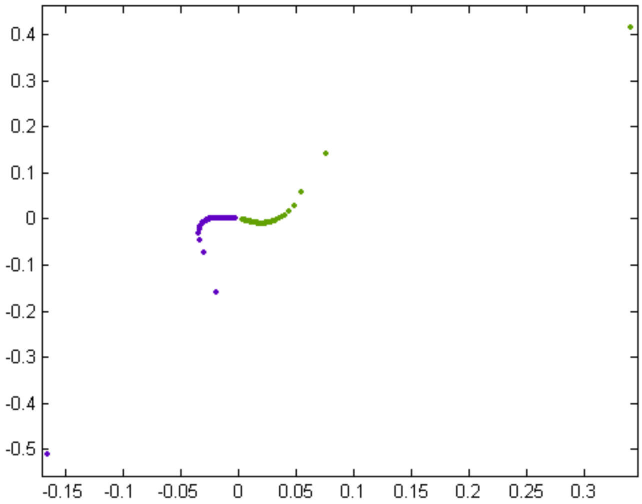

Figure 3:

Such curves are typical for zeta function zeros.

This variant zeta function can be substituted into Riemann's functional equation. A plot $\Pi(\frac{s}{2}-1)\pi^{-s/2}\zeta_1(s)$ for the first zeta function zero and $n \leq 100000$ is

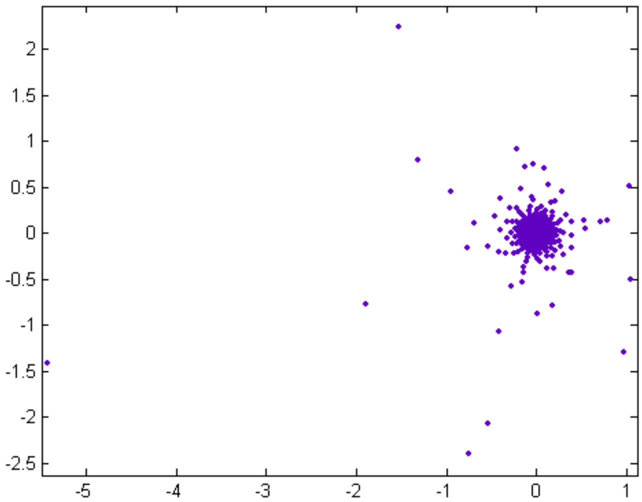

Figure 4:

A plot of $\Pi(\frac{1-s}{2}-1)\pi^{-(1-s)/2}\zeta_1(1-s)$ for the first zeta function zero and $n \leq$

100000 is

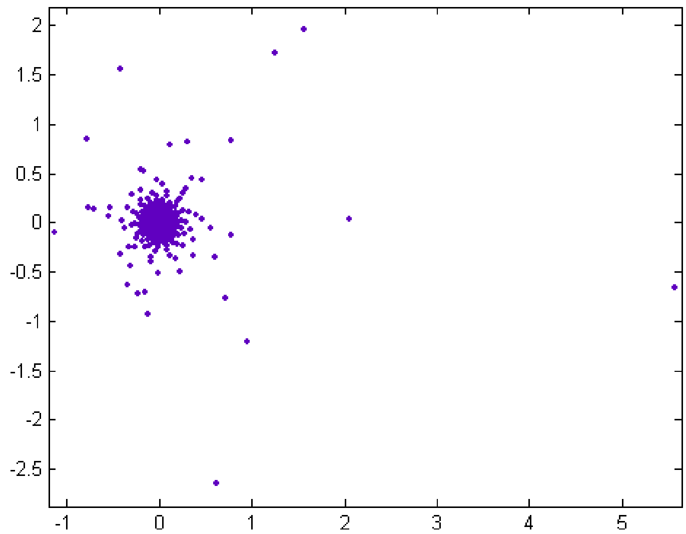

Figure 5:

A plot of the logarithms of the n values of the inflection points of $\Pi(\frac{s}{2}-1)\pi^{-s/2}\zeta_1(s)$ for the first zeta function zero and $n \leq 1000000$ is

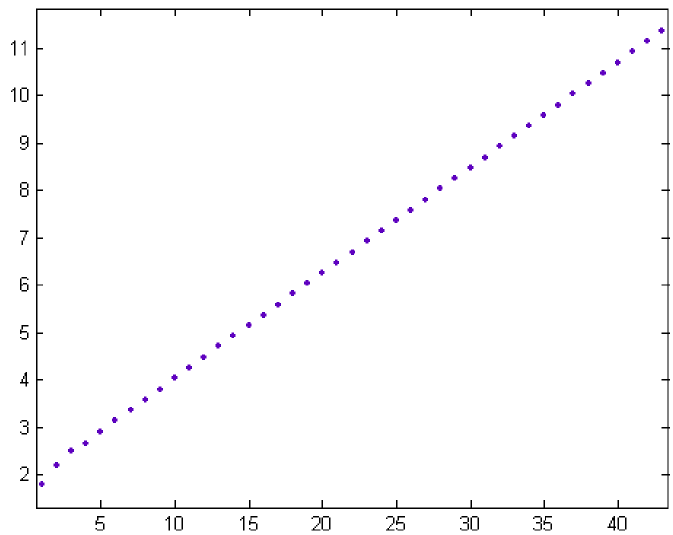

Figure 6:

For a linear least-squares fit of the curve (disregarding the first four n values), $p_1 = 0.2224$ with a 95% confidence interval of (0.2223, 0.2226), $p_2 = 1.804$ with a 95% confidence interval of (1.8, 1.808), SSE=0.0008746, R-squared=1.0, and RMSE=0.004862. The slope is about half of that for the usual zeta function zero.

A plot of the logarithms of the n values of the inflection points of $\Pi(\frac{1-s}{2} - 1)\pi^{-(1-s)/2}\zeta_1(1-s)$ for the first zeta function zero and $n \leq 1000000$ is

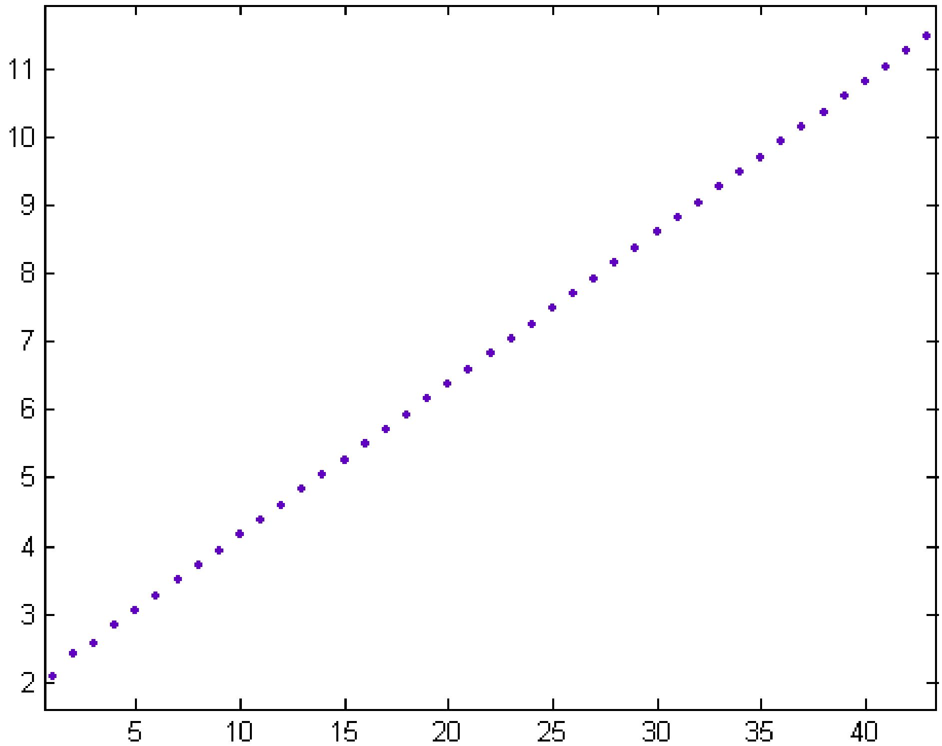

Figure 7:

For a linear least-squares fit of the curve (disregarding the first n value), $p_1 = 0.2222$ with a 95% confidence interval of (0.2221, 0.2224), $p_2 = 1.934$ with a 95% confidence interval of (1.929, 1.938), SSE=0.001959, R-squared=1, and RMSE=0.0069999. These parameters are almost the same as for the left-hand side of functional equation.

3. A FUNCTION DERIVED FROM $\zeta_1(s)$

Let $\alpha(s)$ denote $\zeta_1(s)\zeta(s)$. A plot of $\alpha(s)$ for the first zeta function zero and $n \leq 100000$ is

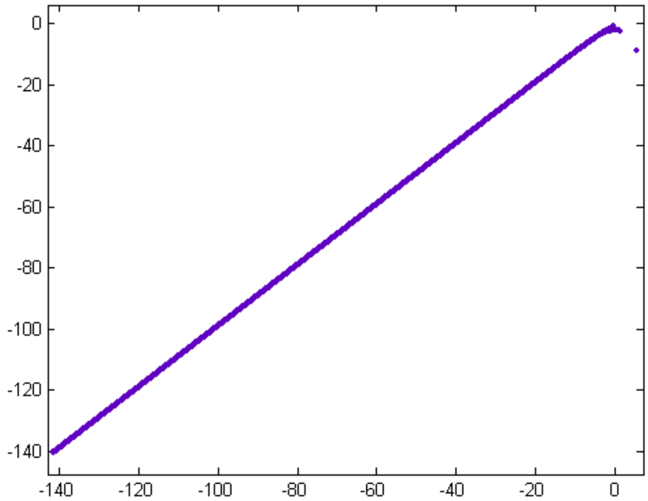

Figure 8:

A plot of $\alpha(s)$ for $s = (0.5, 14.0)$ and $n \leq 100000$ is

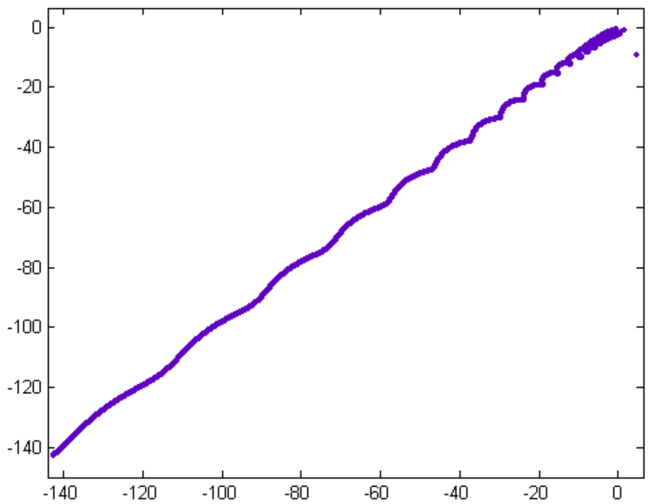

Figure 9:

There are three inflection points. A plot of the number of inflection points for imaginary components of t = 10.0, 10.05, 10.1, 10.15, 10.2,. . .,200.0 versus t is

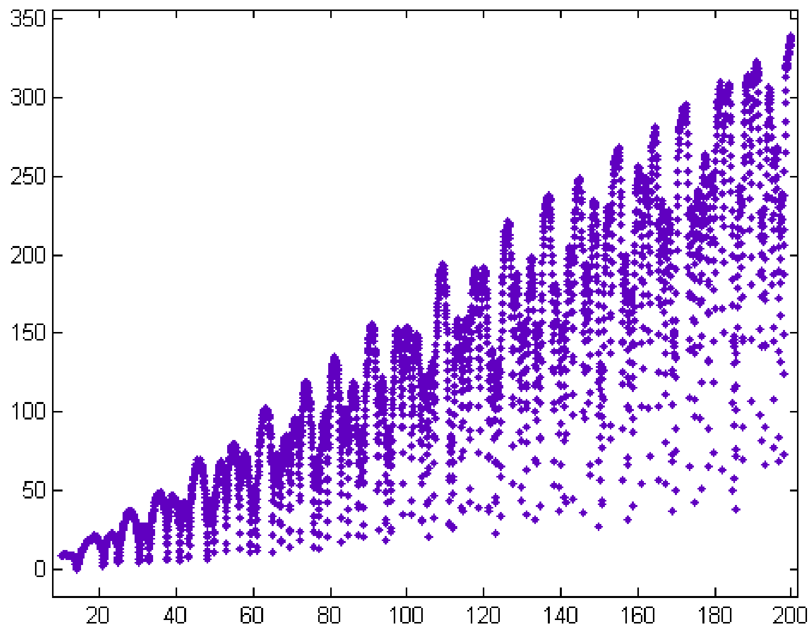

Figure 10:

Each peak corresponds to a zeta function zero. The t values of the lowest points between the peaks are 14.1 (or 14.15), 21.0, 25.0, 30.4, 32.9 (or 32.95), 37.55 (or 37.6), 40.9, 43.3, 48.0, 49.75 (or 49.8), 52.95, 56.45, 59.35, 60.85, 65.1, 67.1, 69.55, 72.05, 75.7, 77.15, 79.35, 82.9, 84.75, 87.4 (or 87.45), 88.8, 92.5, 94.65, 95.87, 98.85, 101.3, 103.7, 105.45, 107.15, 111.05, 111.85, 114.35, 116.25, 118.8, 121.35, 122.95, 124.25, 127.5, 129.6, 131.1, 133.5, 134.75, 138.1, 139.75, 141.15, 143.1, 146.0, 147.45, 150.05, 150.95, 153.05, 156.1, 157.6, 158.85, 161.2, 163.05, 165.55, 167.2, 169.1, 169.9, 173.4 174.75, 176.45, 18.4, 179.9, 182.2, 184.85, 185.6, 187.25, 189.4, 192.05, 193.1, 195.25, 196.9, and 198.0. These are the zeta function zeros accurate to one decimal place (the greatest absolute value of differences is 0.03).

The periodogram for a sequence $x_1, \ldots, x_n$ is given by the following formula;

$$S(e^{i\omega}) = \frac{1}{n} | \sum_{l=1}^{n} x_l e^{-i\omega l} |^2 \tag{7}$$

The periodogram uses a Fourier transform to compute the power spectral density as $S(e^{i\omega})/(2\pi)$. The range of the corresponding normalized frequencies is $[0, \pi]$ and the size of the Fourier transform is the smallest power of two greater than n. A periodogram of the above peaks is

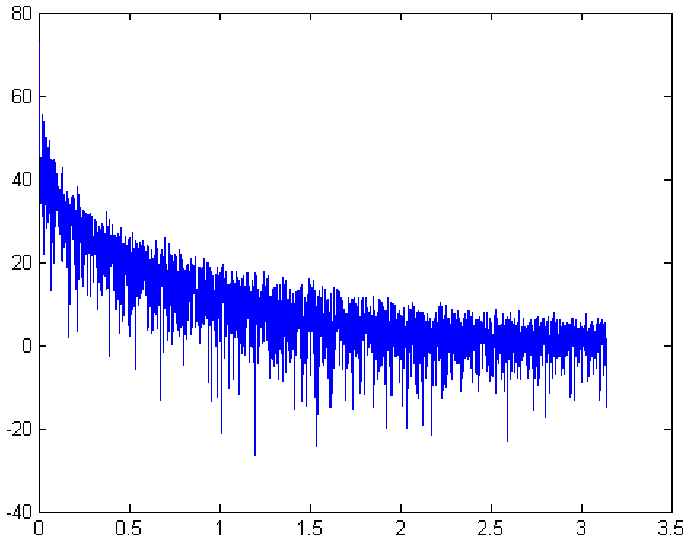

Figure 11:

The upper bound of the power spectral density decreases roughly logarithmically. A plot of the power spectral density versus the logarithm of the normalized frequencies is

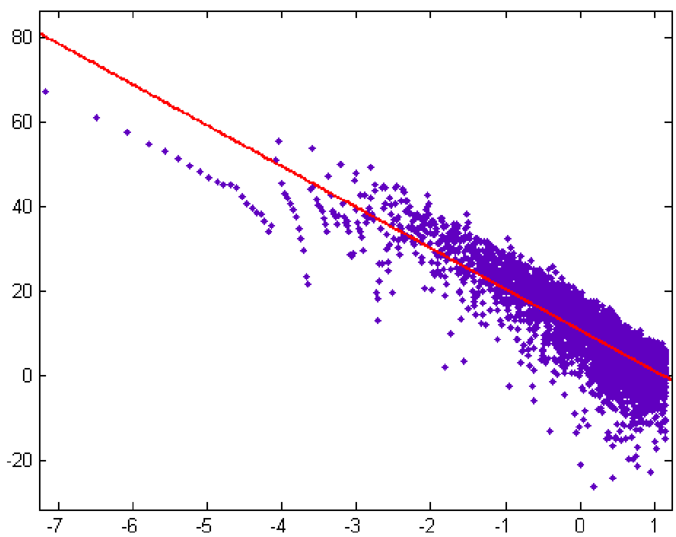

Figure 12:

A smaller t increment will have to be used for zeta function zeros that are closer together. A plot of the number of inflection points for imaginary components of $t = 16553.0, 16553.005, 16553.01, 16553.015, 16553.02, \ldots, 16560.0$ versus t is

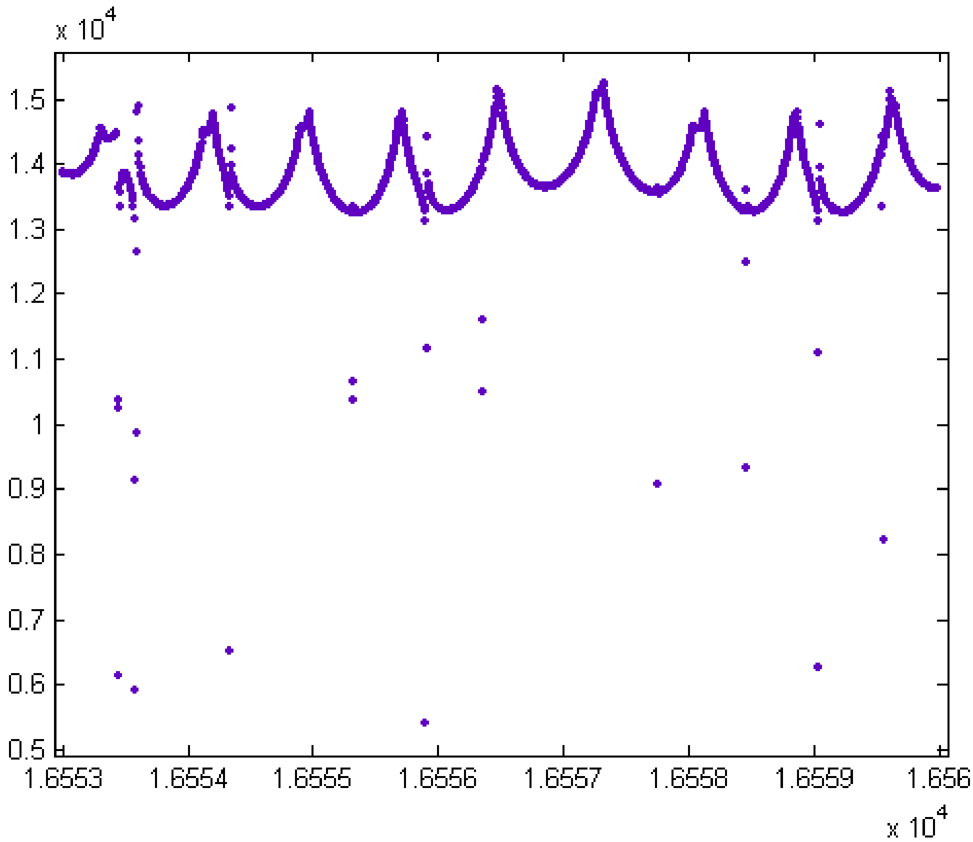

Figure 13:

The t values of the lowest points between the peaks are 16553.45, 16553.585, 16554.34, 16555.32, 16555.905, 16556.36, 16557.76, 16558.465, 16559.04, and 16559.56 (in two instances, the peaks occur on the left side of large peaks and are not discernible from the graph). The imaginary components of the zeta function zeros in this range are 16553.44967678656000, 16553.58447731021100, 16554.34049095428100, 16555.32211257643200, 16555.90568310858700, 16556.36199177242500, 16557.76084708020400, 16558.46353075773400, 16559.04099110432100, and 16559.55858182801600 (the 18,126th zero). The estimates are accurate to two decimal places.

A plot of the number of inflection points for imaginary components of $t = 74914.5, 74914.501, 74914.502, 74914.503, \ldots, 74921.0$ versus t is

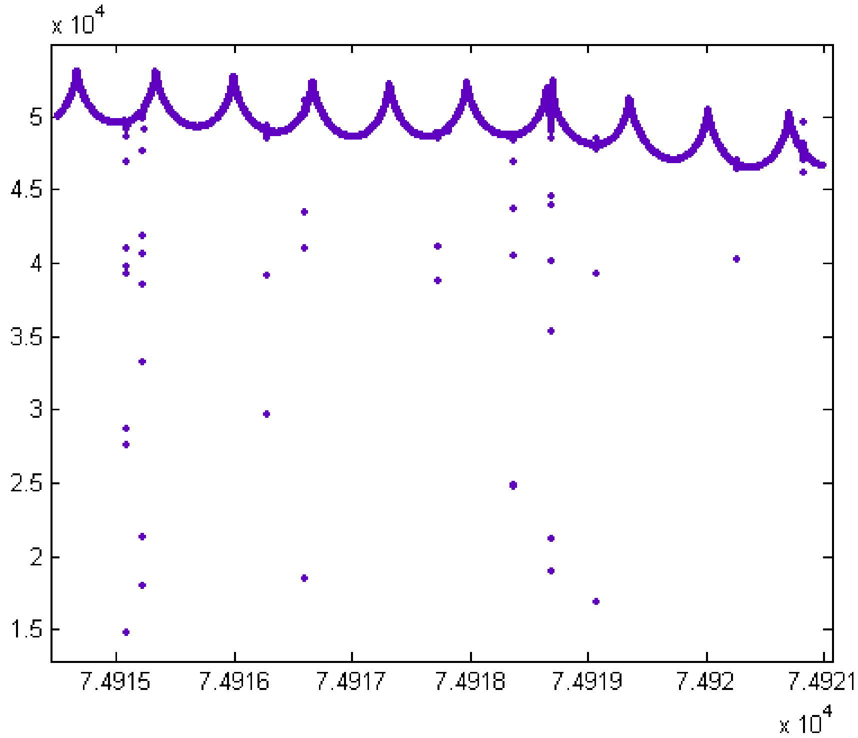

Figure 14:

The t values of the lowest points between the peaks are 74915.084, 74915.226, 74916.276, 74916.60, 74917.72, 74918.37, 74918.691, 74919.075, 74920.26, and 74920.827 (in one instance, the peak occurs on the left side of a large peak and is not discernible from the graph). The imaginary components of the zeta function zeros in this range are 74915.08423093339700, 74915.22633696218000, 74916.27645349281400, 74916.60013746743800, 74917.71941582848400, 74918.37058022667800, 74918.69143345370000, 74919.07516112076700, 74920.25979325889800, and 74920.82749899418600 (the hundred-thousandth zero). The estimates are accurate to three decimal places (taking rounding into account). It appears that larger zeta function zeros can be more accurately approximated by making the t increment sufficiently small. Attempting this with say the first zeta function zero just gives multiple "lowest points" (as mentioned above).

4. ANOTHER FUNCTION INVOLVING THE REFLECTION FORMULA

A similar function involving the reflection formula is

$$\zeta_2(s) = \frac{2\pi\zeta(s)}{\Pi(1-s)Z(1-s)} \tag{8}$$

A plot of this expression for the first zeta function zero and $n \leq 100000$ is

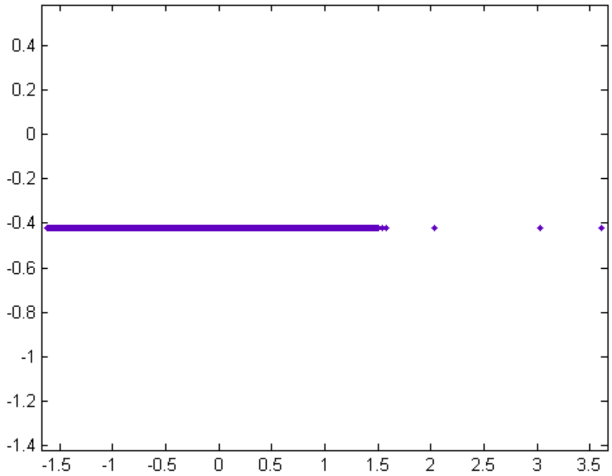

Figure 15:

This is an edge-on logarithmic spiral. A plot of the logarithms of the n values of the inflection points for $n \leq 1000000$ is

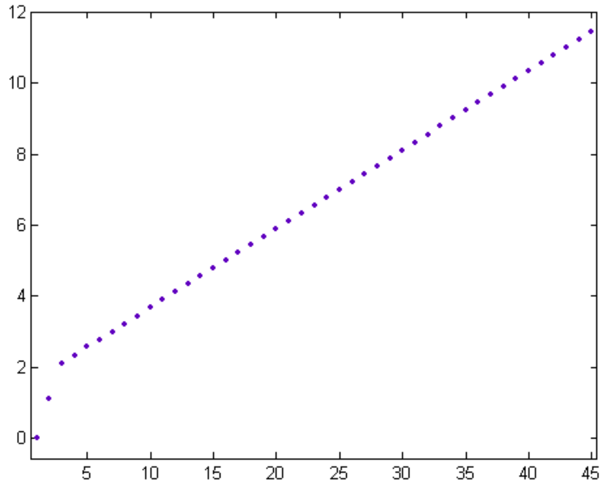

Figure 16:

For a linear least-squares fit of the curve (disregarding the first two n values), $p_1 = 0.2226$ with a 95% confidence interval of (0.2224, 0.2228), $p_2 = 1.438$ with a 95% confidence interval of (1.433, 1.433), SSE=0.0002274, R-squared=1, and RMSE=0.007447.

The slopes and intercepts for the first 10 zeta function zeros are (0.2224, 1.438), (0.1495, 1.951), (0.1256, 1.951), (0.1033, 2.314), (0.0954, 2.438), (0.08354, 2.629), (0.07682, 2.569, (0.07256, 2.648), (0.06545, 2.708), and (0.06315, 2.935). The slopes are about half of those for the usual zeta function. The number of n values disregarded in computing the slopes is 2, 5, 8, 9, 9, 10, 10, 8, 8, and 6 respectively.
This variant zeta function can also be substituted into Riemann's functional equation. A plot of $\Pi(\frac{s}{2}-1)\pi^{-s/2}\zeta_2(s)$ for the first zeta function zero and $40 \leq n \leq 100000$ is

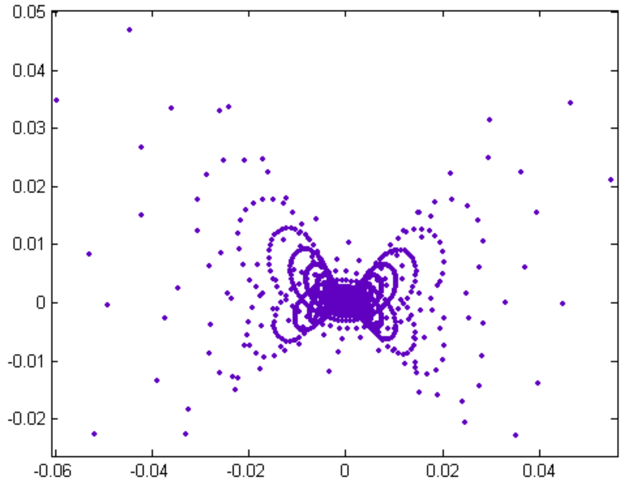

Figure 17:

A plot of the logarithms of the n values of the inflection points (for $n \leq 100000$) is

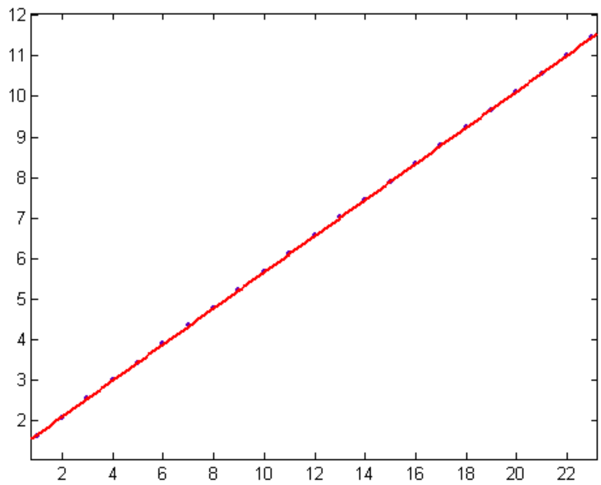

Figure 18:

For a linear least-squares fit of the curve, $p_1 = 0.4457$ with a 95% confidence interval of (0.4448, 0.4466), $p_2 = 1.209$ with a 95% confidence interval of (1.197, 1.221), SSE=0.003801, R-squared=1, and RMSE=0.01345.
A plot of $\Pi(\frac{1-s}{2}-1)\pi^{-(1-s)/2}\zeta_2(1-s)$ for the first zeta function zero and $40 \leq n \leq 100000$ is

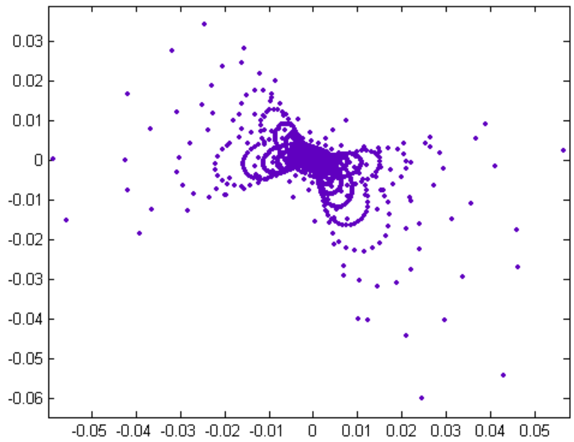

Figure 19:

A plot of the logarithms of the n values of the inflection points (for $n \leq 100000$) is

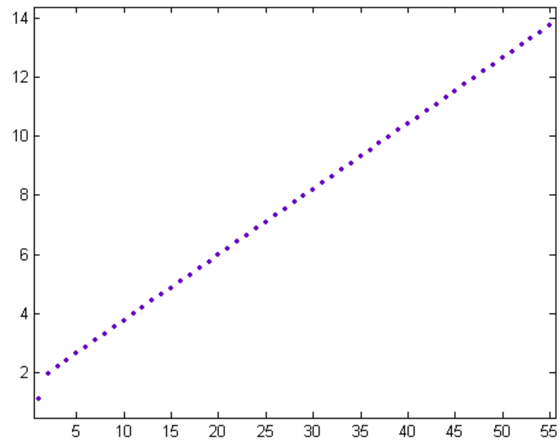

Figure 20:

For a linear least-squares fit of the curve (disregarding the first n value), $p_1 = 0.4436$ with a 95% confidence interval of (0.4423, 0.4449), $p_2 = 0.8399$ with a 95% confidence interval of (0.8213, 0.8586), SSE=0.008007, R-squared=1, and RMSE=0.01953. These parameters are about the same as those for the left-hand side of the functional equation.

5. A RELATIONSHIP BETWEEN THE TWO VARIANTS OF THE ZETA FUNCTION

A plot of $\alpha(s)$ and $\zeta_2(s)$ for the third zeta function zero ($s = (0.5, 25.01085758014569)$) and $n \leq 100000$ is

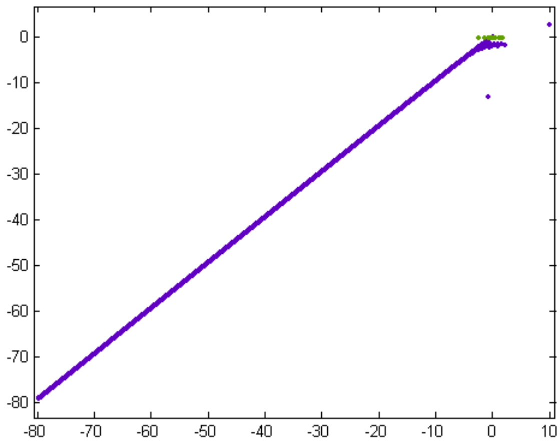

Figure 21:

For zeta function zeros, $\zeta_2(s)$ appears to be the derivative of $\alpha(s)$. A plot of $\alpha(s)$ and $\zeta_2(s)$ for $s = (0.5, 51.0)$ and $n \leq 100000$ is

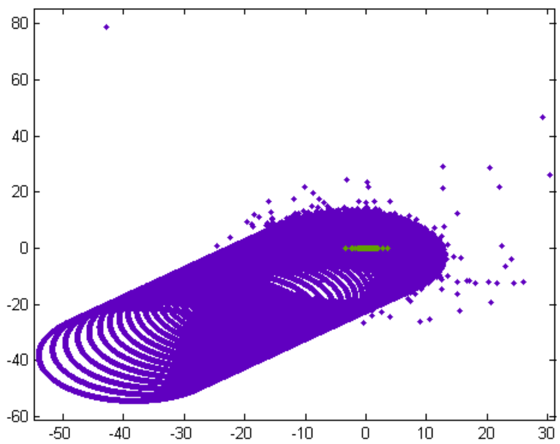

Figure 22:

A plot of $\alpha(s)$ and $\zeta_2(s)$ for $s = (0.5, 51.0)$ and $n \leq 500$ is

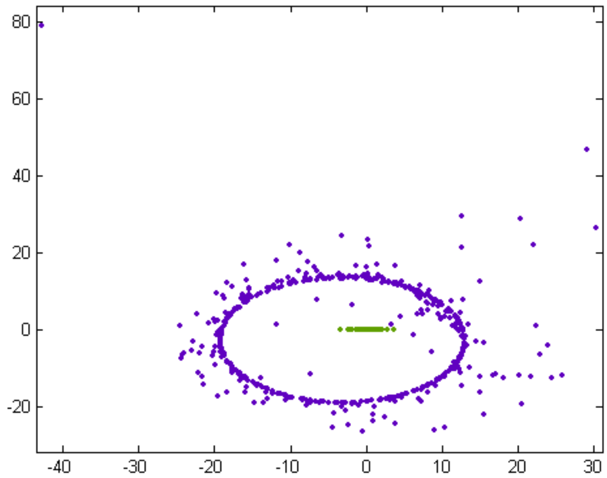

Figure 23:

A plot of $\alpha(s)$ and $\zeta_2(s)$ for the eleventh zeta function zero ($s = (0.5, 52.97032147771446)$) and $n \leq 100000$ is

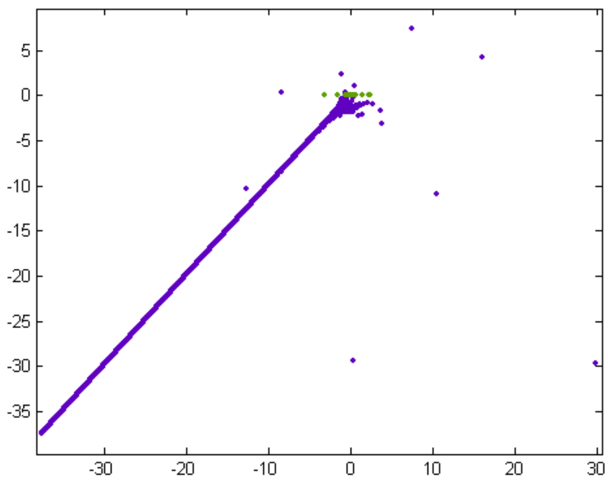

Figure 24:

6. METHODS

The following C code computes the two spectral functions. The prime look-up table contains the primes less than 1500000.

```
//
// compute Mobius function
//
#include <math.h>
#include <stdio.h>
#include ”table5.h” // prime look-up table
extern char *malloc();
int mobius(unsigned int a, unsigned int *table, unsigned int tsize) {
unsigned int i,count,p;
if (a==1)
    return(1);
count=0;
for (i=0; i<tsize; i++) {
    p=table[i];
    if (p>a)
        break;
    if (a==(a/p)*p) {
        a=a/p;
        if (a==(a/p)*p)
            return(0);
        count=count+1;
        if (a==1)
            break;
        }
    }
if ((count&1)==0)
    return(1);
else
    return(-1);
}
//
// compute Euler’s phi function
//
int mobius(unsigned int a, unsigned int *t, unsigned int tsize);
unsigned int nueuler(unsigned int n, unsigned int *table,
unsigned int tsize) {
unsigned int d;
int sum;
```

```
if (n==1)
    return(1);
sum=0;
for (d=1; d<=n; d++) {
    if (n==(n/d)*d)
        sum=sum+(n/d)*mobius(d, table, tsize);
    }
return((unsigned int)sum);
}
//
// 2πζ(s − 1)/(Z(s − 1)Π(s − 1))
//
unsigned int nueuler(unsigned int a, unsigned int *table, unsigned int tsize);
unsigned int max=100000;
double s=0.50; // usually set to 0.50
//double t=13.5;
double t=14.13472514173470;
//double t=21.02203963877156;
//double t=25.01085758014569;
//double t=30.42487612585951;
//double t=32.93506158773919;
//double t=37.58617815882568;
//double t=40.91871901214750;
//double t=43.32707328091500;
//double t=48.00515088116716;
//double t=49.77383247767230;
double pi=3.14159265359;
unsigned int n=1; // select n
unsigned int xmin=0;
unsigned int out=1; // 2 for top, 3 for left, 4 for bottom, 5 for right
// usually set to 4 when the spiral is not output
unsigned int skip=0; // if set, don't do final multiplication
unsigned int tsize=114155; // size of prime look-up table
void main() {
unsigned int temp,x;
double *rsave,*isave,suma,sumb,sumr,sumi;
double temp1,temps,tempt,prods,a,b,c,d,e,f,olds,oldt,sums,sumt;
double R,I;
FILE *Outfp;
```

```
Outfp = fopen("transs1.dat","w");
if (max>1500000) {
    printf("max too large \n");
    return;
    }
rsave=(double*) malloc(16000004);
if (rsave==NULL)
    return;
isave=(double*) malloc(16000004);
if (isave==NULL)
    return;
//
// compute Π(s − 1)
//
prods=1.0;
sums=0.0;
sumt=0.0;
for (x=1; x<=max; x++) {
    if (n==0)
        temp=nueuler(x,table,tsize);
    else
        temp=x;
    prods=prods*(double)temp/((double)temp+s);
    a=s-1.0;
    if (a>=0.0)
        temp1=pow((double)(x+1),a);
    else {
        temp1=pow((double)(x+1),-a);
        temp1=1.0/temp1;
        }
    temps=temp1*(cos(t*log(x+1)));
    tempt=temp1*(sin(t*log(x+1)));
    a=prods*temps-tempt;
    b=prods*tempt+temps;
    rsave[x-1]=a;
    isave[x-1]=b;
    }
//
// divide 2π/Z(s − 1) by Π(s − 1)
```

```
//
sumr=0.0;
sumi=0.0;
for (x=1; x<=max; x++) {
    if (n==0)
        temp=nueuler(x,table,tsize);
    else
        temp=x;
    a=2.0*(s-1);
    if (a>=0.0)
        temp1=pow((double)temp,a);
    else {
        temp1=pow((double)temp,-a);
        temp1=1.0/temp1;
        }
    R=temp1*cos(t*log((double)temp));
    I=temp1*sin(t*log((double)temp));
    temp1=R*R+I*I;
    sumr=sumr+R/temp1;
    sumi=sumi-I/temp1;
    a=sumr;
    b=sumi;
    c=2.0*pi;
    d=2.0*pi;
    temp1=a*a+b*b;
    e=(a*c+b*d)/temp1;
    f=-(a*d-b*c)/temp1;
    c=rsave[x-1];
    d=isave[x-1];
    temp1=c*c+d*d;
    temps=(c*e+d*f)/temp1;
    tempt=-(c*f-d*e)/temp1;
    rsave[x-1]=temps;
    isave[x-1]=tempt;
    }
//
// multiply by ζ(s − 1) (and ζ(s))
//
olds=0.0;
```

```
oldt=0.0;
sumr=0.0;
sumi=0.0;
suma=0.0;
sumb=0.0;
for (x=1; x<=max; x++) {
    if (n==0)
        temp=nueuler(x,table,tsize);
    else
        temp=x;
    a=s-1.0;
    if (a>0.0)
        temp1=pow((double)temp,a);
    else {
        temp1=pow((double)temp,-a);
        temp1=1.0/temp1;
        }
    R=temp1*cos(t*log((double)temp));
    I=temp1*sin(t*log((double)temp));
    temp1=R*R+I*I;
    sumr=sumr+R/temp1;
    sumi=sumi-I/temp1;
    if (s>0.0)
        temp1=pow((double)temp,s);
    else {
        temp1=pow((double)temp,-s);
        temp1=1.0/temp1;
        }
    R=temp1*cos(t*log((double)temp));
    I=temp1*sin(t*log((double)temp));
    temp1=R*R+I*I;
    suma=suma+R/temp1;
    sumb=sumb-I/temp1;
    c=rsave[x-1];
    d=isave[x-1];
    temps=c*sumr-d*sumi;
    tempt=c*sumi+d*sumr;
    if (skip==0) {
        a=temps;
```

```
        b=tempt;
        temps=a*suma-b*sumb;
        tempt=a*sumb+b*suma;
        }
    if (x>xmin) {
        if (out==1)
            fprintf(Outfp," %.10llf %.10llf \n",temps,tempt);
        if ((out==2)&&((olds>0.0)&&(temps<0.0)))
            fprintf(Outfp," %.10llf \n",log((double)x));
        if ((out==3)&&((oldt>0.0)&&(tempt<0.0)))
            fprintf(Outfp," %.10llf \n",log((double)x));
        if ((out==4)&&((olds<0.0)&&(temps>0.0)))
            fprintf(Outfp," %.10llf \n",log((double)x));
        if ((out==5)&&((oldt<0.0)&&(tempt>0.0)))
            fprintf(Outfp," %.10llf \n",log((double)x));
        olds=temps;
        oldt=tempt;
        }
    }
fclose(Outfp);
return;
}

//
// 2πζ(s)/(Π(s)Z(s))
//
unsigned int nueuler(unsigned int a, unsigned int *table, unsigned int tsize);
unsigned int max=100000;
double s=0.50; // usually set to 0.50
//double t=13.5;
double t=14.13472514173470;
//double t=21.02203963877156;
//double t=25.01085758014569;
//double t=30.42487612585951;
//double t=32.93506158773919;
//double t=37.58617815882568;
//double t=40.91871901214750;
//double t=43.32707328091500;
//double t=48.00515088116716;
```

```
//double t=49.77383247767230;
double pi=3.14159265359;
unsigned int n=1; // select n
unsigned int xmin=0;
unsigned int out=1; // 2 for top, 3 for left, 4 for bottom, 5 for right
// usually set to 4 when the spiral is not output
unsigned int skip=0; // if set to 1, output last part
// if set to 2, output first part
unsigned int tsize=114155; // size of prime look-up table
void main() {
unsigned int temp,x;
double temp1,tempr,temps,tempt,prods,a,b,c,olds,oldt,*rsave,*isave;
double sums,sumt,d,R,I,suma,sumb,sumr,sumi;
FILE *Outfp;
Outfp = fopen("transs2.dat","w");
if (max>1500000) {
    printf("max too large \n");
    return;
    }
rsave=(double*) malloc(16000004);
if (rsave==NULL)
    return;
isave=(double*) malloc(16000004);
if (isave==NULL)
    return;
//
// compute 2πζ(s)/Π(s)
//
prods=1.0;
sums=0.0;
sumt=0.0;
for (x=1; x<=max; x++) {
    if (n==0)
        temp=nueuler(x,table,tsize);
    else
        temp=x;
    prods=prods*(double)temp/((double)temp+s);
    if (s>=0.0)
        temp1=pow((double)(x+1),s);
```

```
    else {
        temp1=pow((double)(x+1),-s);
        temp1=1.0/temp1;
        }
    temps=temp1*(cos(t*log(x+1)));
    tempt=temp1*(sin(t*log(x+1)));
    a=prods*temps-tempt;
    b=prods*tempt+temps;
    if (s>=0.0)
        temp1=pow((double)x,s);
    else {
        temp1=pow((double)x,-s);
        temp1=1.0/temp1;
        }
    temps=temp1*(cos(t*log(x)));
    tempt=temp1*(sin(t*log(x)));
    temp1=temps*temps+tempt*tempt;
    c=temps/temp1;
    d=tempt/temp1;
    sums=sums+c;
    sumt=sumt-d;
    temp1=a*a+b*b;
    temps=(a*sums+b*sumt)/temp1;
    tempt=-(a*sumt-b*sums)/temp1;
    temps=temps*2.0*pi;
    tempt=tempt*2.0*pi;
    rsave[x-1]=temps;
    isave[x-1]=tempt;
    }
//
// compute 2πζ(s)/(Π(s) * Z(s))
//
olds=0.0;
oldt=0.0;
sumr=0.0;
sumi=0.0;
suma=0.0;
sumb=0.0;
for (x=1; x<=max; x++) {
```

```
if (n==0)
    temp=nueuler(x,table,tsize);
else
    temp=x;
a=2.0*s;
if (a>=0)
    temp1=pow((double)temp,a);
else {
    temp1=pow((double)temp,-a);
    temp1=1.0/temp1;
    }
R=temp1*cos(t*log((double)temp));
I=temp1*sin(t*log((double)temp));
temp1=R*R+I*I;
sumr=sumr+R/temp1;
sumi=sumi-I/temp1;
a=sumr;
b=sumi;
c=rsave[x-1];
d=isave[x-1];
temp1=a*a+b*b;
tempr=(a*c+b*d)/temp1;
temps=-(a*d-b*c)/temp1;
if (skip==1) {
    temps=a;
    tempt=b;
    }
if (skip==2) {
    temps=c;
    tempt=d;
    }
if (x>xmin) {
    if (out==1)
        fprintf(Outfp," %.10llf %.10llf \n",temps,tempt);
    if ((out==2)&&((olds>0.0)&&(temps<0.0)))
        fprintf(Outfp," %.10llf \n",log((double)x));
    if ((out==3)&&((oldt>0.0)&&(tempt<0.0)))
        fprintf(Outfp," %.10llf \n",log((double)x));
    if ((out==4)&&((olds<0.0)&&(temps>0.0)))
```

```
            fprintf(Outfp," %.10llf \n",log((double)x));
        if ((out==5)&&((oldt<0.0)&&(tempt>0.0)))
            fprintf(Outfp," %.10llf \n",log((double)x));
        olds=temps;
        oldt=tempt;
        }
    }
fclose(Outfp);
return;
}
```

REFERENCES

[1] H. M. Edwards, *Riemann's Zeta Function*, Dover, (1974)

[2] A. Voros, *Communications in Mathematical Physics*, 439-465(1987), Springer-Verlag