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Some Multiplicative Indices of Unitary Cayley Graphs

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Abstract

The unitary Cayley graph X_n has vertex set $Z_n = \{0, 1, 2, .. n-1\}$.Vertices a, b are adjacent, if $gcd\ (a-b, n) = 1$. The status $\sigma(u)$ of a vertex u in a graph G is the sum of distances of all other vertices from u in G. In this paper, we obtain the values of the multiplicative first and second status indices, multiplicative first and second hyper status indices, multiplicative sum connectivity status index, multiplicative product connectivity status index, general multiplicative first and second status indices of unitary Cayley graph X_n.

Keywords: Unitary cayley graphs, Multiplicative status indices, multiplicative first and second hyper status indices, multiplicative sum connectivity status index, multiplicative Product connectivity status index.

Mathematics Subject Classification: 05C09**,** 05C25, 05C50

Preliminaries:

A graph $G = (V, E)$ in which $V\ (G)$ is the set of vertices of, and $E\ (G)$ is the set of edges. An edge $\{u, v\}$ is an unordered pair of distinct vertices; the vertices u and v are the endpoints of the edge. The vertices of a graph can be pictured as dots, and the edges as line segments whose endpoints are vertices. An edge e is said to be incident at a vertex v if v is an endpoint of e. Two vertices are said to be adjacent if there is an edge between them. Adjacent vertices are sometimes referred to as neighbours. The degree deg (u) of a vertex u in a graph G is the number of vertices adjacent to u. A graph is simple if there is at most one edge between any two vertices, and the endpoints of each edge are distinct.

Researchers have introduced many regular graphs using Cayley graphs with more number of vertices for a given diameter and for a given number of edges per vertex than were studied previously. This lead to the construction of larger networks, while meeting design criteria of a fixed number of nearest neighbours and a fixed maximum communication time between arbitrary vertices. An immediate example of Cayley graph is a unitary cayley graph. More information about Cayley graphs can be found in the books on algebraic graph theory by Biggs [1] and by Godsil and Royle [2]. Unitary Cayley graphs are highly symmetric. They have some remarkable properties connecting graph theory and number theory.

All graphs discussed in this paper are simple graphs. The unitary Cayley graph X_n has vertex set $Z_n = \{0, 1, 2, .. n-1\}$ where the vertices a, b are adjacent, if $gcd\,(a-b, n) = 1$.

If we represent the elements of Z_n by the integers $0, 1, \ldots, n - 1$, then it is well known that X_n has vertex set $V(X_n) = Z_n = \{0, 1, \ldots .. n-1\}$ and edge set

$$E(X_n) = \{\{a, b\}: a, b \in Z_n, \gcd(a-b, n) = 1\}.$$

In section 2 we study some new multiplicative indices of unitary cayley graphs. The multiplicative indices are based on the distances of vertices in a graph.A numerical parameter mathematically derived from the graph structure is called as a topological index or graph index. Topological indices uniquely characterize molecular topology and is extensively applied in chemical graph theory. Application of ideas from one scientific field to another one often gives a new view on the problems.

Mostly we can find the applications of graph indices [3] towards various fields of Science and Technology in [4, 5]. One can find that graph indices are extensively studied in [6, 7, 8]. The status, denoted by$\sigma(u)$, of a vertex u in G is the sum of distances of all other vertices from u in G.

The paper proceeds further on the successful consideration of multiplicative Zagreb indices [9-10]. Kulli.V.R. introduced the first and second multiplicative hyper Zagreb indices [11] as

$$HII_1\,(G) = \prod_{uv \in E\,(G)} [d_G(u) + d_G(v)]^2$$

$$HII_2\,(G) = \prod_{uv \in E\,(G)} [d_G(u) d_G(v)]^2$$

The multiplicative status indices given in [16] are as follows:

The multiplicative first and the second status indices of a graph *G* are defined as

$$SII_1\,(G) = \prod_{uv \in E\,(G)} [\sigma(u) + \sigma(v)]$$

$$SII_2\,(G) = \prod_{uv \in E\,(G)} [\sigma(u)\sigma(v)]$$

The multiplicative first and second hyper status indices of a graph *G* are defined as

$$HSII_1\,(G) = \prod_{uv \in E\,(G)} [\sigma(u) + \sigma(v)]^2$$

$$HSII_2\,(G) = \prod_{uv \in E\,(G)} [\sigma(u)\sigma(v)]^2$$

The multiplicative sum connectivity status index of a graph *G* is defined as

$$SSII(G) = \prod_{uv \in E\,(G)} \frac{1}{\sqrt{[\sigma(u) + \sigma(v)]}}$$

The multiplicative product connectivity status index of a graph *G* is defined as

$$PSII(G) = \prod_{uv \in E\,(G)} \frac{1}{\sqrt{[\sigma(u)\sigma(v)]}}$$

The reciprocal multiplicative product connectivity status index of a graph *G* is defined as

$$RPSII(G) = \prod_{uv \in E\,(G)} \sqrt{[\sigma(u)\sigma(v)]}$$

The general multiplicative first and second status indices of a graph *G*, are defined as

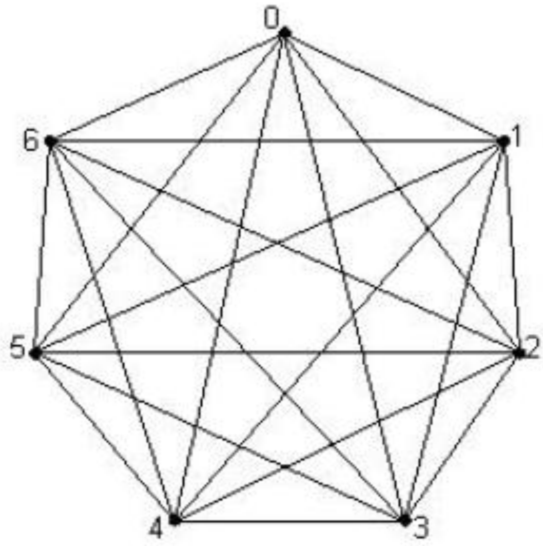
$$S_1^a II(G) = \prod_{uv \in E\,(G)} [\sigma(u) + \sigma(v)]^a$$

$$S_2^a II(G) = \prod_{uv \in E\,(G)} [\sigma(u)\sigma(v)]^a$$

Where a is a real number.
Some of the research works on status indices can be found in [12, 13, 14, 15].In this paper, the values of these newly proposed indices for unitary cayley graphs are determined.

Results for Unitary cayley graphs:
Let X_n be a unitary cayley graph where $n = p$ is a prime number, then $X_n = K_p$ is a

complete graph on p vertices.



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Theorem 2.1: The general multiplicative first status index of a unitary cayley graph where $n = p$ is a prime number is

$$S_1^a II(X_n) = 2\ (n-1)^{\frac{an\ (n-1)}{2}}.$$

Proof: Given X_n is a unitary cayley graph where $n = p$ is a prime number, then $X_n = K_p$ is a complete graph on p vertices. In a complete graph we have $\frac{n\ (n-1)}{2}$ edges and the status of every vertex $\sigma(u) = n - 1$.Thus from [16] we get

$$S_1^a II(X_n) = 2\ (n-1)^{\frac{an\ (n-1)}{2}}.$$

Observations 2.2 [16]:
2.2.1 The multiplicative first index is

$$S_1 II(X_n) = 2\ (n-1)^{\frac{n\ (n-1)}{2}}, \text{ where a=1.}$$

2.2.2 The multiplicative first hyper status index is

$$HS_1 II(X_n) = 2\ (n-1)^{n\ (n-1)}, \text{ where } a = 2.$$

2.2.3 The multiplicative sum connectivity status index is

$$SSII(X_n) = \left[\frac{1}{\sqrt{2\,(n-1)}}\right]^{\frac{n\,(n-1)}{2}}, \text{ where } a = -1/2.$$

Theorem 2.3: The general multiplicative second status index of a unitary cayley graph where $n = p$ is a prime number is

$$S_2^a II(X_n) = (n-1)^{an\,(n-1)}.$$

Proof: Given X_n is a unitary cayley graph where $n = p$ is a prime number, then $X_n = K_p$ is a complete graph on p vertices. In a complete graph we have $\frac{n\,(n-1)}{2}$ edges and the status of every vertex $\sigma(u) = n - 1$.Thus from [16] we get

$$S_2^a II(X_n) = (n-1)^{an\,(n-1)}.$$

Observations 2.4 [16]:
2.4.1 The multiplicative second index is

$$S_2 II(X_n) = (n-1)^{n\,(n-1)} \text{ where } a = 1.$$

2.4.2 The multiplicative second hyper status index is

$$HS_2 II(X_n) = (n-1)^{2n\,(n-1)}, \text{ where } a = 2.$$

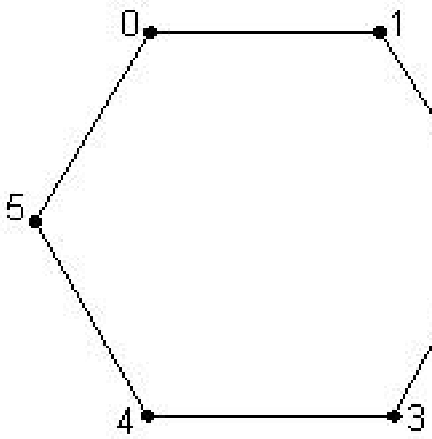
2.4.3 The multiplicative product connectivity status index is

$$PSII(X_n) = \left[\frac{1}{\sqrt{(n-1)}}\right]^{n\,(n-1)}, \text{ where } a = -1/2.$$

2.4.4 The reciprocal multiplicative product connectivity status index is

$$RPSII(X_n) = (n-1)^{\frac{1}{2}n\,(n-1)}, \text{ where } a = 1/2.$$

We proceed further the study of indices of unitary cayley graphs for n as any even number. Specifically when $n = 6$, X_n becomes a cycle as shown below:



Z_6

Theorem 2.5: The general multiplicative first status index of a unitary cayley graph where

$$n = 6 \text{ is } S_1^a II(X_n) = (\frac{n^2}{2})^{an}.$$

Proof: When $n = 6$, X_n is a cycle, then it has n edges and the status of every vertex is $\sigma(u) = \frac{n^2}{4}$.Thus from [16], $S_1^a II(X_n) = (\frac{n^2}{2})^{an}$.

Observations 2.6[16**]:**

2.6.1 The multiplicative first index when $n = 6$ is $S_1 II(X_n) = (\frac{n^2}{2})^n$, where $a = 1$.

2.6.2 The multiplicative first hyper status index when $n = 6$ is $HS_1 II(X_n) = (\frac{n^2}{2})^{2n}$, where $a = 2$.

2.6.3 The multiplicative sum connectivity status index when $n = 6$ is $SSII(X_n) = \left[\frac{\sqrt{2}}{n}\right]^n$, where $a = -1/2$.

Theorem 2.7:The general multiplicative second status index of a unitary cayley graph where $n = 6$ is $S_2^a II(X_n) = (\frac{n}{2})^{4an}$.

Proof: When $n = 6$, X_n is a cycle, then it has n edges and the status of every vertex is $\sigma(u) = \frac{n^2}{4}$.Thus from [16], $S_2^a II(X_n) = (\frac{n}{2})^{4an}$.

Observations 2.8 [16]:

2.8.1 The multiplicative second index when $n = 6$ is $S_2 II(X_n) = (\frac{n}{2})^{4n}$ where $a = 1$.

2.8.2 The multiplicative second hyper status index when $n = 6$ is $HS_2II(X_n) = (\frac{n}{2})^{8n}$, where $a = 2$.
2.8.3 The multiplicative product connectivity status index when $n = 6$ is $PSII(X_n) = \left[\frac{2}{n}\right]^{2n}$, where $a = -1/2$.
2.8.4 The reciprocal multiplicative product connectivity status index when $n = 6$ is

$RPSII(X_n) = \left[\frac{n}{2}\right]^{2n}$, where $a = 1/2$.

Moving ahead with the study of indices of unitary cayley graphs for n as any even number such that $n = 2p$ where p is a prime and $p \neq 2, 3$.

Theorem 2.9:The general multiplicative first status index of a unitary cayley graph where $n = 2p$ where p is a prime and $p \neq 2, 3$ is $S_1^aII(X_n) = 3n^{2an}$.

Proof: These type of graphs has $2n$ edges, every vertex has $\deg(u) = p - 1$ and the status of every vertex u in X_n is $\sigma(u) = \frac{3}{2}n$. Thus

$$S_1^aII(X_n) = \prod_{uv\in E(X_n)}[\sigma(u) + \sigma(v)]^a = \left[\frac{3}{2}n + \frac{3}{2}n\right]^{a2n} = 3n^{2an}.$$

Corollary 2.10:If X_n is a unitary cayley graph where $n = 2p$ where p is a prime and $p \neq 2, 3$ then
2.10.1 The multiplicative first index is $S_1II(X_n) = (3n)^{2n}$, where a=1.
2.10.2 The multiplicative first hyper status index is $HS_1II(X_n) = (3n)^{4n}$, where a=2.
2.10.3 The multiplicative sum connectivity status index is $SSII(X_n) = \frac{1}{(3n)^n}$, where a= -1/2.

Theorem 2.11:The general multiplicative second status index of a unitary cayley graph where $n = 2p$ where p is a prime and $p \neq 2, 3$ is

$$S_2^aII(X_n) = (\frac{3}{2}n)^{2a}.$$

Proof: These type of graphs has *2n* edges, every vertex has $\deg(u) = p - 1$ and the status of every vertex u in X_n is $\sigma(u) = \frac{3}{2}n$.Thus

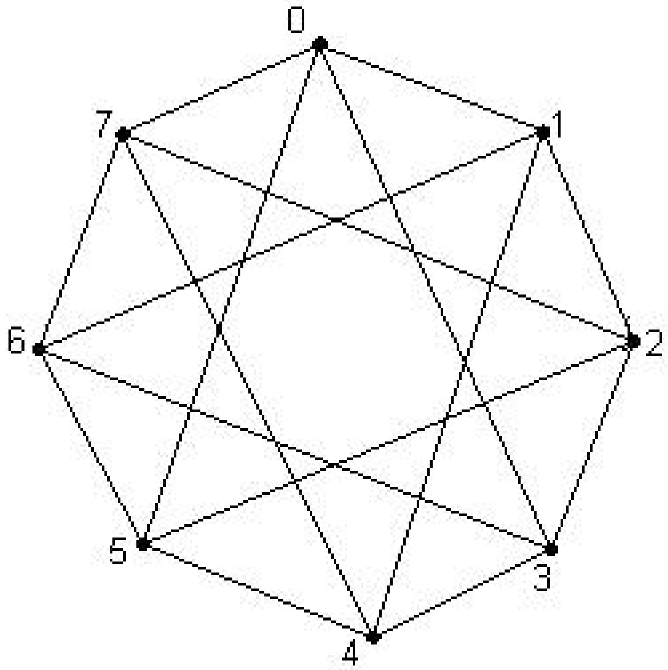
$$S_2^aII(X_n) = \prod_{uv\in E\ (X_n)}[\sigma(u)\sigma(v)]^a = \left(\frac{3}{2}n \times \frac{3}{2}n\right)^a = \left(\frac{3}{2}n\right)^{2a}.$$

Corollary 2.12: If X_n is a unitary cayley graph where $n = 2p$ where p is a prime and $p \neq 2, 3$ then
2.12.1 The multiplicative second index is $S_2II(X_n) = \left(\frac{3}{2}n\right)^2$ where *a=1.*

2.12.2 The multiplicative second hyper status index is $HS_2II(X_n) = \left(\frac{3}{2}n\right)^4$, where $a = 2$.
2.12.3 The multiplicative product connectivity status index is $PSII(X_n) = \frac{2}{3n}$, where $a = -1/2$.
2.12.4 The reciprocal multiplicative product connectivity status index is

$$RPSII(X_n) = \frac{3}{2}n, \text{ where } a = 1/2.$$

Lastly we study the indices of unitary cayley graphs for n as any even number such that $n = p^\alpha$ where p is a prime, $p = 2$ *and* $\alpha = 3, 4$ ....



Z_8

Theorem 2.13: The general multiplicative first status index of a unitary cayley graph where $n = p^\alpha$ where p is a prime, $p = 2$ *and* $\alpha = 3, 4$ .... is

$$S_1^a II(X_n) = (3n - 4)^{2an}.$$

Proof: These type of graphs has *2n* edges, every vertex has $\deg(u) = \frac{n}{2}$ and the status of every vertex u in X_n is $\sigma(u) = \frac{3}{2}n - 2$.Thus

$$S_1^a II(X_n) = \prod_{uv \in E(X_n)} [\sigma(u) + \sigma(v)]^a = \left[\frac{3}{2}n - 2 + \frac{3}{2}n - 2\right]^{a2n} = (3n - 4)^{2an}.$$

Corollary 2.14:If X_n is a unitary cayley graph where $n = p^\alpha$ where p is a prime, $p = 2$ and $\alpha = 3, 4$ .... then
2.14.1 The multiplicative first index is $S_1II(X_n) = (3n-4)^{2n}$, where $a = 1$.
2.14.2 The multiplicative first hyper status index is $HS_1II(X_n) = (3n-4)^{4n}$, where $a = 2$.
2.14.3 The multiplicative sum connectivity status index is $SSII(X_n) = \frac{1}{(3n-4)^n}$, where $a = -1/2$.

Theorem 2.15: The general multiplicative second status index of a unitary cayley graph where $n = p^\alpha$ where p is a prime, $p = 2$ and $\alpha = 3, 4$ .... is

$$S_2^a II(X_n) = \left(\frac{3}{2}n - 2\right)^{2a}.$$

Proof: These type of graphs has *2n* edges, every vertex has $\deg(u) = \frac{n}{2}$ and the status of every vertex *u* in X_n is $\sigma(u) = \frac{3}{2}n - 2$.Thus

$$S_2^a II(X_n) = \prod_{uv\in E\ (X_n)} [\sigma(u)\sigma(v)]^a = \left(\frac{3}{2}n - 2 \times \frac{3}{2}n - 2\right)^a = \left(\frac{3}{2}n - 2\right)^{2a}$$

Corollary 2.16: If X_n is a unitary cayley graph where $n = p^\alpha$ where p is a prime, $p = 2$ and $\alpha = 3, 4$ .... then
2.16.1 The multiplicative second index is $S_2II(X_n) = \left(\frac{3}{2}n - 2\right)^2$ where $a = 1$.
2.16.2 The multiplicative second hyper status index is $HS_2II(X_n) = \left(\frac{3}{2}n - 2\right)^4$, where $a = 2$.
2.16.3 The multiplicative product connectivity status index is $PSII(X_n) = \frac{2}{3n-4}$, where $a = -1/2$.
2.16.4 The reciprocal multiplicative product connectivity status index is

$$RPSII(X_n) = \frac{3n-4}{2}, \text{ where } a = 1/2.$$

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