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Pulsatile Effect on Couple Stress Fluid Model for Blood Flow through Stenosed Artery

Gade Madhava Rao[1*], Thota Nagaraju[2] and Prathigadapa Anuradha[3]

[1]Professor, Department of Mathematics, Mallareddy University,
Hyderabad, Telangana, India, 50010.
Corresponding author E-mail:: rao.gmr.madhav@gmail.com
[2]Assistant Professor of Mathematics, Anurag Engineering College and
Research Scholar of Department of Mathematics,
Mallareddy University, Hyderabad, Telangana, India, 500100.
E-mail: nagarajuthota678@gmail.com
[3]Assistant Professor of Mathematics,
SR & BGNR Govt Arts and Science (Autonomous) College and
Research Scholar of Department of Mathematics,
Mallareddy University, Hyderabad, Telangana, India, 500100.
E-mail: anuprathigadapa@gmail.com

Abstract

This article deals about flow of blood through artery with mild constriction in it. The artery under consideration is presumed to be a straight cylinder of restricted length and is symmetric about its axis. Blood is model as non-newtonian couple stress fluid under the influence of applied magnetic field. Governing equations of couple stress are simplified with mild stenosis case. Geometry of considered artery is swapped to a rectangular with particular coordinate transformation. The important physical quantities like shear stress, flow rate and impedance at the peak of the stenosis are numerically computed with respect to pertinent parameters and analysed through graphs.

Keywords: Artery; Stenosis, Couple stress fluid, Womersley number.

1. Introduction

In many developed and developing nations, one of the major health peril is atherosclerosis or lumen blockage. Research on blood flow through stenosed artery is importance in many cardiovascular bugs, particularly atherosclerosis. Stenosis is a blockage of the blood vessels due to the acquisition of cholesterol, fat and unnatural growth of tissue on the walls of artery. Characteristics of blood flow through arteries are strongly depends on nature of blood, size and shape of the artery. A reasonable number of experimental and theoretical investigations have been performed on blood flow through the complex arterial system in last few decades. Development of stenosis with time for a Newtonian blood fluid model through an axially symmetric tube has been initially investigated by Young [1]. Mukesh Roy *et.al.* [2] explored that the stenosis is responsible for growing maximum wall shear stress on the wall. Srinivasacharya and Madhava Rao [3] noticed that due to back flow at flow divider, impedance and shear stress have been drastically changed. Shape of stenosis and its influence on blood flow through an artery has been reviewed by Asha1 and Srivastava [4]. Sharma *et.al.*[5] mentioned that the permeability and curvature of the arterial wall increase, while heat source on the blood flow decrease the risk of atherosclerosis formation.

An antisymmetric stress has been produced by spin of freely suspended particles (red blood cells or erythrocytes) because of rotational field which is termed as a couple-stress. This flow is leads to existence of couple stress fluid theory. The number of models has been introduced to explain the properties of non-Newtonian fluids. Out of these, Stokes [6] has proposed the micro-continuum theory of couple stress fluid which reveals the presence of couple stresses and body couples along with non-symmetric stress tensor. The size dependent effect is the major feature of couple stresses. In the classical continuum mechanics, size effect of material particles is neglect within the continua. Couple stress fluid model plays a significant role to understand the important properties of blood. Sinha and Singh [7] revealed that impedance and shear stress increases for larger values of couple stress parameter. Srinivasacharya and Madhava Rao [8] examined pulsatile nature of couple stress fluid model for blood through a bifurcated artery with mild stenosis in parent lumen. The tiny size gold particles drift is added to couple stress fluid and results are analyzed by Rahmat et.al. [9].

Magnetohydrodynamics (MHD) is branch of science which deals with dynamics of electrically conducting fluids in the presence of applied magnetic field without considering polarization effects of the fluid. Xenos [10] studied MHD Effects on Blood Flow in a Stenosis mentioned that there is a opportunity for new avenues of flow control at stenotic regions without going for the vivo testing of patient. The axial velocity of blood flow in an artery is reducing for more intensity of Magnetic field has been proposed by Devendra Kumar *et.al.* [11]. Veera Krishna and Chamkha [12] noticed that magnetic parameter and mean velocity are in inversely proportional and flow reversal occurs at the middle line of the channel.

In this present paper, the impact of tiny stenosis on wall shear stress, flow rate and impedance have been analyzed through graphs by considering blood as unsteady, laminar couple stress fluid under the influence of applied magnetic field.

2. Formulation of the problem

The physical problem under consideration is taken in cylindrical polar coordinate system (r,θ,z), where z axis is laying on the central line of the artery and radial axis is perpendicular to z axis. Assume that the stenosis spread symmetrically over a small length of the artery as shown in fig(1). Flow of blood is treated to be laminar, incompressible couple stress fluid of constant density.

The equations governing the pulsatile couple stress fluid flow are

$$\frac{\partial \rho}{\partial t} + \rho(\nabla \cdot \mathbf{q}) = 0 \tag{1}$$

$$\rho\left(\frac{\partial \mathbf{q}}{\partial t} + (\mathbf{q} \cdot \nabla)\mathbf{q}\right) = -\nabla P + \mu \nabla \times \nabla \times \mathbf{q} - \eta \nabla \times \nabla \times \nabla \times \nabla \times \mathbf{q} \tag{2}$$

where p is the fluid pressure, ρ is the density of the couple stress fluid, η is the couple stress viscosity parameter, q is the velocity vector and μ is the blood viscosity.

The walls of stenosed artery is mathematically mentioned by Srinivasacharya and Srikanth [13] as follows

$$R_s(z) = \begin{cases} a - \dfrac{h}{2}\left[1 + \cos\dfrac{2\pi}{L_0}\left(z - d - \dfrac{L_0}{2}\right)\right], & d \le z \le L_0 + d \\ a & otherwise \end{cases}$$

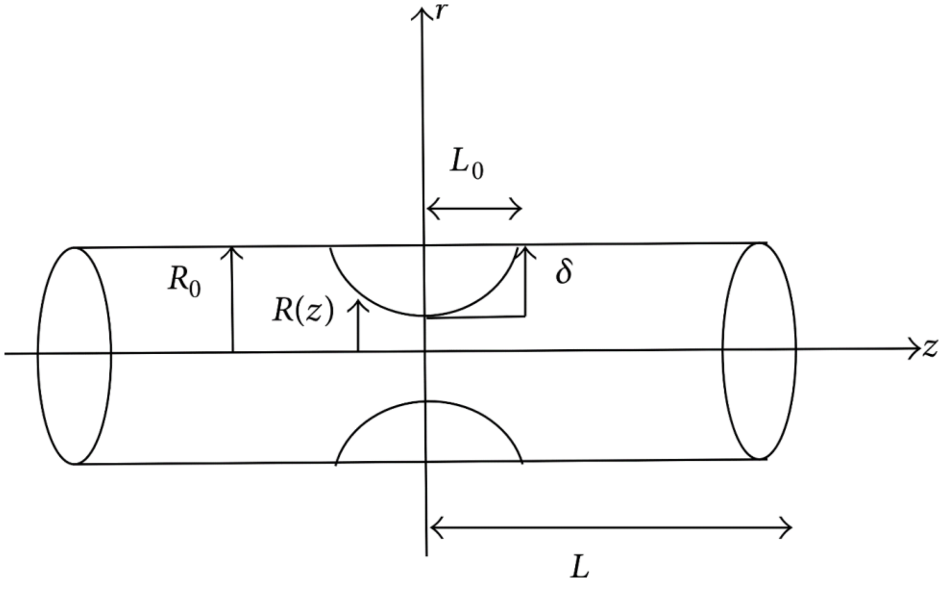

Figure 1: Schematic diagram of stenosed artery

Since the flow is considered to be symmetric, all the variables are independent of θ. Hence, for this flow the velocity is given by **q**=(*u*(*r, z, t*)*, 0, w* (*r, z, t*)). It can be shown that the radial velocity is very small and can be neglected for a low Reynolds number flow in an artery with mild stenosis (Srinivasacharya and Srikanth (2008)) Therefore equations Eq.(2) can be written in non-dimensional form as

$$R_w^2 \frac{\partial w}{\partial t} = -\frac{dp}{dz} + \left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right] w - \frac{1}{\alpha^2}\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right]^2 w \tag{7}$$

where $\alpha^2 = \frac{\mu a^2}{\eta}$ is the couple stress fluid parameter and $R_w^2 = \frac{a^2 \rho w}{\mu}$ is the Womersley number.

The equation of the pulsatile pressure gradient in the above equation in non-dimensional form is

$$-\frac{\partial p}{\partial z} = A_0 + A_1 \cos(t) \tag{8}$$

where A_0 is the amplitude of the pressure gradient (constant), A_1 is the amplitude of the pulsatile component, which is the systolic and diastolic pressure. Where $w=2\pi f_p$, f_p is the frequency of the pulsatile flow.

The boundary conditions in non-dimensional form are

$$\left.\begin{aligned} &\frac{\partial w}{\partial r} = 0,\ \frac{\partial^2 w}{\partial r^2} - \frac{\sigma}{r}\frac{\partial w}{\partial r} = 0, \quad \text{on } r = 0 \text{ for } 0 \le z \le z_{\max} \\ &w = 0,\ \frac{\partial^2 w}{\partial r^2} - \frac{\sigma}{r}\frac{\partial w}{\partial r} = 0, \quad \text{on } r = R_s(z) \text{ for all } z \\ &w = w_0 \quad at \quad t = 0. \end{aligned}\right\} \tag{9}$$

where σ=η/η' is the couple stress fluid parameter which is responsible for the effect of local viscosity of particles apart from the bulk viscosity of the fluid. If η=η' then the effects of couple stresses will be absent in a material, which implies that couple stress tensor is symmetric. In this case the equation (9) shows that, the couple stresses are disappearing on the inner and outer walls of the bifurcated artery.

The influence of the boundary can be conveyed into the governing equations by the radial coordinate transformation given by Shit and Roy (2011), $\xi = \frac{r}{R_s}$. Using this transformation in equations Eq.(7) reduces the form

$$-R_w^2 R^3\left(\xi\frac{\partial R}{\partial t}+\frac{\partial R_2}{\partial t}\right)\frac{\partial w}{\partial \xi}+R^4\frac{dp}{dz}+\frac{1}{\alpha^2}\frac{\partial^4 w}{\partial \xi^4}+\frac{2R}{\alpha^2(\xi R+R_s)}\frac{\partial^3 w}{\partial \xi^3}-$$

$$\left[1+\frac{1}{\alpha^2(\xi R+R_s)^2}\right]R^2\frac{\partial^2 w}{\partial \xi^2}+\left[\frac{1}{\alpha^2(\xi R+R_s)^3}-\frac{1}{(\xi R+R_s)}\right]R^3\frac{\partial w}{\partial \xi}=-R_w^2 R^4\frac{\partial w}{\partial t} \tag{10}$$

In the above equation first term is corresponding to wall motion. The associated boundary conditions in the transformed coordinates are

$$\left.\begin{aligned} &\frac{\partial w}{\partial \xi} = 0,\ \frac{\partial^2 w}{\partial \xi^2} - \frac{\sigma R}{(\xi R + R_s)}\frac{\partial w}{\partial \xi} = 0, \quad \text{on } \xi = 0 \text{ for } 0 \le z \le z_{\max} \\ &w = 0, \quad \frac{\partial^2 w}{\partial \xi^2} - \frac{\sigma R}{(\xi R + R_s)}\frac{\partial w}{\partial \xi} = 0, \quad \text{on} \quad \xi = 1 \text{ for all } z \\ &w = w_0 \quad at \quad t = 0 \end{aligned}\right\} \tag{11}$$

The physical quantities to be analyzed are flow rate, impedance and shear stress for both parent and daughter arteries. The flow rate for both parent and daughter arteries are determined using

$$Q = 2\pi R\left[R\int_0^1 \xi\, w\, d\xi \right] \tag{12}$$

The impedance of the flow in parent and daughter the artery is calculated using

$$(\lambda)_i = \left| \frac{z_3 \dfrac{dp}{dz}}{Q} \right| \tag{13}$$

The mean value of shear stress is calculated by using

$$\tau_{ij} = \frac{1}{R}\frac{\partial w}{\partial \xi} + \frac{1}{4R\alpha^2(\xi R + R_s)^2}\frac{\partial w}{\partial \xi} - \frac{1}{4\alpha^2 R^3}\frac{\partial}{\partial \xi}\left(\frac{\partial^2 w}{\partial \xi^2}\right) - \frac{1}{4R^2\alpha^2(\xi R + R_s)}\frac{\partial^2 w}{\partial \xi^2} \tag{14}$$

3. Solution

The Eq.(10) along with the boundary condition Eq.(11) is solved numerically using finite difference method. First, the equation Eq.(10) is converted into a system of four first order partial differential equations and these equations are replaced with equivalent central finite difference approximations, so that the equations result in a block tridiagonal matrix and then this system is solved using block elimination method. The aim of the current study has been to analyze the flow characteristics of blood through the stenosed bifurcated artery under the consideration that blood is pulsatile couple stress fluid. An appropriate numerical scheme has been chosen to analyze the applicability of the physiological data available in the scientific literatures. The numerical solutions of all these physical parameters are presented graphically for different values of α, β, σ and time on both sides of the bifurcated artery for better understanding of the analysis. We used the following data $a=5$ mm, $d'=10$ mm, $l_0=5$ mm, $\beta=\pi/10$, $r_1=0.51a$, $\tau_{m=2a}$, $\alpha=2.5$, $t=2$ sec and $\sigma=0.5$

4. Results and discussion:

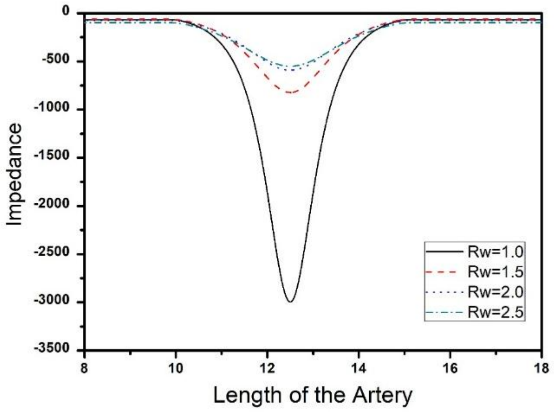

Figure 2: Variations of Impedance with respect to Rw for fixed values of other parameters.

The variations of Impedance with respect to Rw for fixed values of other parameters have been shown in fig.2. It's is noticed that resistive impedance has been decreasing in the normal artery and whereas it's increasing in stenosed region for improved values of Rw. Impedance is increasing with advanced values of σ has depicted in fig. 3 which is useful to control the volume of blood during the surgeries.

The variations of flow rate with respect to Rw and σ have been explored in fig.4 and fig.5 respectively. From fig. 4, it's observed that flow rate has decreased outside the stenotic region and within the stenotic region reverse trend has been noticed. From fig.5, flow rate has improved for advanced values of σ. Fig.6 and fig.7 are exploring about the effect of Rw along the top and bottom layers of the artery in particularly during the stenotic region. It is clear that shear stress is diminishing for higher values of Rw along both the layers of stenosed artery. Influence of σ on shear stress along the bottom and top layers of the artery has been shown respectively in fig.8 and fig.9. From fig.8, shear stress is decreasing in the non-stenotic region and increasing in the stenotic reason for increased values of σ along the bottom layer. From fig.9, it's observed that shear stress is increasing for better values of σ along the top layer of the stenosed artery, which may slowdown the blood supply to the different parts of the body and may leads to heart stroke.

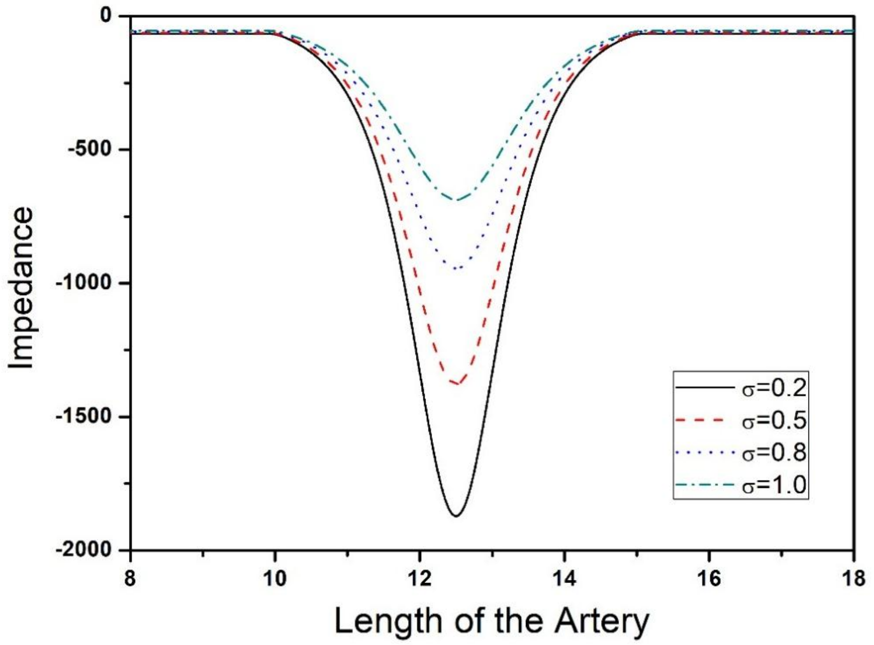

Figure 3: Variations of Impedance with respect to σ for fixed values of other parameters.

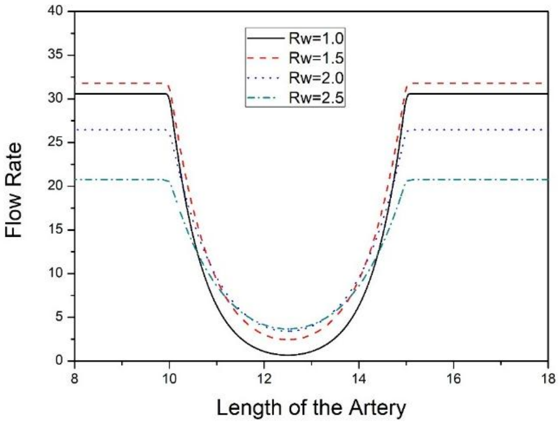

Figure 4: Variations of flowrate with respect to Rw for fixed values of other parameters.

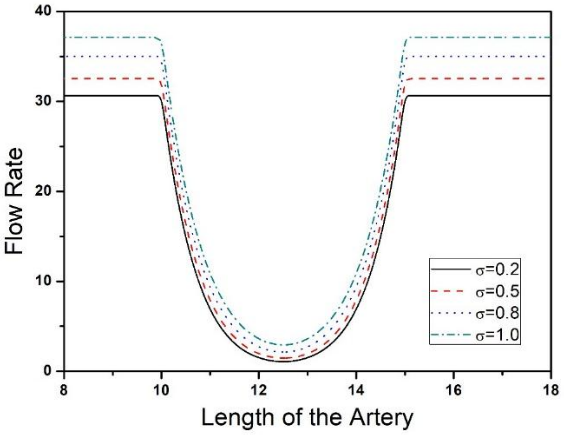

Figure 5: Variations of flow rate with respect to σ for fixed values of other parameters.

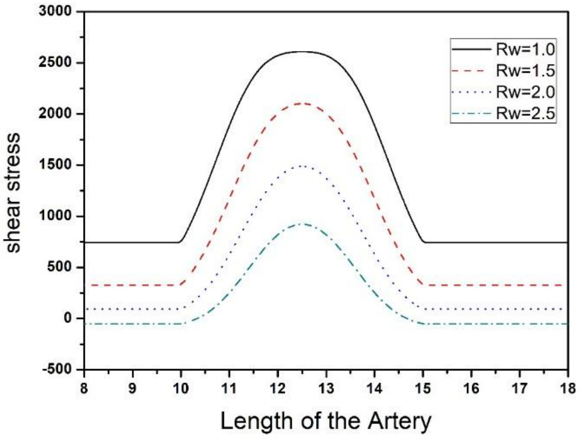

Figure 6: Variations of shear stress along the bottom layer of the artery with respect to Rw for fixed values of other parameters.

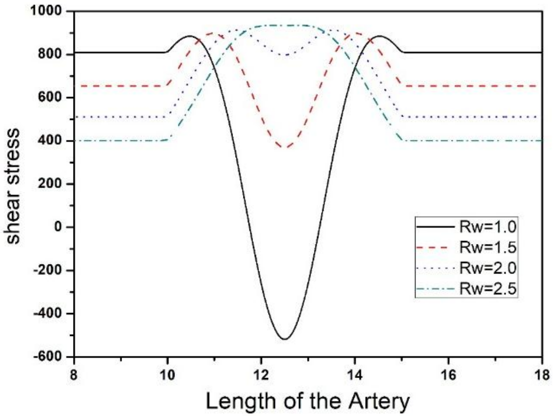

Figure 7: Variations of shear stress along the top layer of the artery with respect to Rw for fixed values of other parameters.

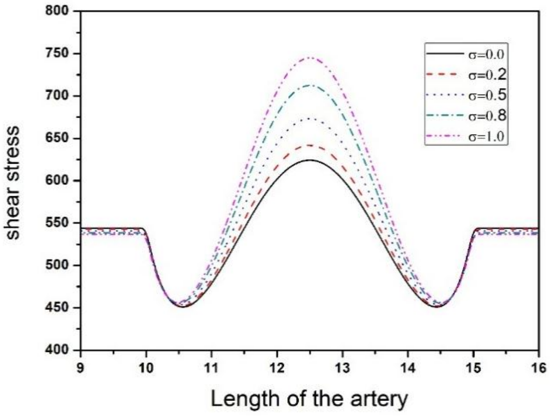

Figure 8: Variations of shear stress along the bottom layer of the artery with respect to σ for fixed values of other parameters.

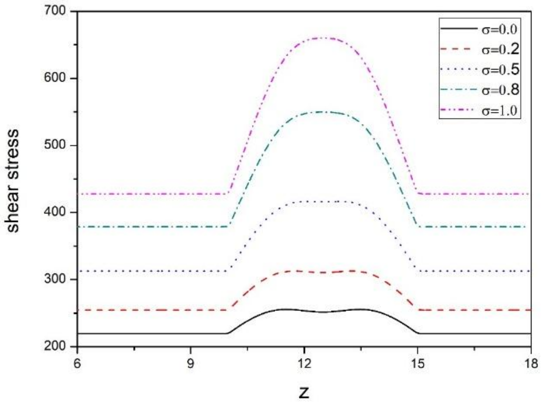

Figure 9: Variations of shear stress along the top layer of the artery with respect to σ for fixed values of other parameters.

5. Conclusions:

Pulsatile nature of blood flow through artery with mild stenosis has been studied in the present article. The following points are concluded.

a) Impedance is increasing for the advanced values of σ and decreased values of R_w.

b) Shear stress is enhancing along the bottom and top layers of the stenosed artery with respect to σ .

These results are very much useful to the doctor’s fraternity to make hypothetical statement about the patient status.

References

[1] D.F. Young, “Effect of a time-dependent stenosis on flow through a tube”, *Journal of Manufacturing Science and Engineering,* Vol. 90, pp. 248–254, (1968).

[2] Mukesh Roy, S. Basant Singh, Mohit Bhandwal and Priya Ranjan, “Modelling of Blood Flow in Stenosed Arteries”, *Procedia Computer Science*, Vol. 115, pp. 821-830, (2017).

[3] D. Srinivasacharya and G. Madhava Rao, “Mathematical model for blood flow through a bifurcated artery using couple stress fluid”, *Mathematical Biosciences*, Vol. 278, pp. 37-47, (2016).

[4] K. N. Asha1 and N. Srivastava, “Geometry of stenosis and its effects on the blood flow through an artery-A theoretical study”, *AIP Conference Proceedings*, 2375, 030006 (2021); doi.org/10.1063/5.0066510

[5] B.K. Sharma, C. Kumawat and O.D. Makinde, "Hemodynamical analysis of MHD two phase blood flow through a curved permeable artery having variable viscosity with heat and mass transfer", Biomech Model Mechanobiol Vol. 21, pp. 797–825 (2022). https://doi.org/10.1007/s10237-022-01561-w

[6] V.K. Stokes, "Couple stesses in fluids", *Physics of Fluids*, Vol. 9, pp. 1709-1715, (1966).

[7] P. Sinha and C. Singh, “ Effects of couple stresses on the blood flow through an artery with mild stenosis”. *Biorheology,* Vol. 21(3), pp.303-315, (1984).

[8] D. Srinivasacharya and G. Madhava Rao, “Pulsatile flow of couple stress fluid through a bifurcated artery”, *Ain Shams Engineering Journal,* Vol. 9 (4), pp. 883-893, (2018).

[9] E. Rahmat, Ahmed Zeeshan, Farooq Hussain and A. Asadollahi, “Peristaltic Blood Flow of Couple Stress Fluid Suspended with Nanoparticles under the Influence of Chemical Reaction and Activation Energy”, *Symmetry*, 11, 276; doi:10.3390/sym11020276, (2019).

[10] M. A. Xenos, “MHD Effects on Blood Flow in a Stenosis”, *Advances in Dynamical Systems and Applications,* Vol. 8(2), pp. 427-437, (2013).

[11] Devendra Kumar, B. Satyanarayana, Rajesh Kumar, Sanjeev Kumar and Narendra Deo, “Application of heat source and chemical reaction in MHD blood

flow through permeable bifurcated arteries with inclined magnetic field in tumor treatments", *Results in Applied Mathematics*, Vol. 10, 100151, (2021).

[12] M. Veera Krishna and Ali J. Chamkha, "MHD peristaltic rotating flow of a couple stress fluid through a porous medium withwall and slip effects", *Special Topics & Reviews in Porous Media: An International Journal*, Vol. 10(3), pp. 245-258, (2019).

[13] D. Srinivasacharya and D. Srikanth, "Effect of couple stresses on the pulsatile flow through a constricted annulus", *C. R. Mecanique*, Vol. 336, pp. 820–827, (2008).