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Unsteady Uniform Newtonian Fluid Flow through a Partially Filled Trapezoidal Channel

Mihad Abdelwahab Sinnary*, Jackson Kwanza, David Mwangi Theuri, Edward Richard Onyango

Department of Pure and Applied Mathematics, Jomo Kenyatta University of Agriculture and Technology, Nairobi, Kenya

Abstract

An open channel is a revealed conduit where liquids flow with its top surface limited by the environment. The study of these open channels with unsteady, uniform flows and trapezoidal cross-sections is essential to understand the behavior of fluids and how they interact with the environment. In this study, the effects of various flow parameters on the flow variable i.e. the flow velocity, is investigated, the results are then presented using the modified Saint-Venant equation. The parameters considered in this study include the Manning coefficient, channel bottom slope, channel top width, and channel side slope. To obtain the results, the Saint-Venant partial differential equations of continuity and momentum that govern free surface flow in open channels were solved using the finite difference approximation method. The results were then presented graphically to make them easier to understand. The results show that the velocity of the flow increases as the depth of the channel from the bottom to the free surface increases. Additionally, an increase in the Manning coefficient decreases the flow velocity. Overall, this study provides valuable insights into the behavior of fluids in open channels and how different flow parameters can affect their behavior. These findings can be useful in various applications such as hydraulic engineering, irrigation systems, and wastewater management.
Keywords: Open channel, saint-venant equations, finite difference method.

1. INTRODUCTION

An open channel is a revealed conduit where liquids flow with its top surface limited by the environment. The flow of water in an open channel is a familiar sight, whether it is a natural channel like a river or an artificial channel like an irrigation ditch. Usually, channels have a circular, trapezoidal, rectangular or triangular cross-section.The selection of channel alignment, size, shape of the lateral cross-section, longitudinal slope and the type of lining material are involved in open channel design. The design of an ideal open

*Corresponding Author's email: mihad.asinnary@gmail.com

channel cross-section that satisfies all the desirable aims is practically far from easy. This is due to the interdependent variables involved in the process, and also due to the constraints that are usually encountered in the region of concern.

A major driving force behind the open-channel market is the increasing need to measure water flow due to environmental regulations and population growth. Droughts and dry water beds are becoming increasingly common as a result of growing population and unexpected weather patterns. This is true in Kenya, especially in areas such as Budalangi, Siaya County and Tana River. These floods result in significant loss of life (both human and livestock), destruction of infrastructure, damage of communication networks, and significant economic losses. A mathematical model could be used to examine these areas and hence aid engineers in the construction of open channels to drain the water from the flooded rivers to reservoirs such as dams, which provides for design of drainage ditches, streams and irrigation channels in these areas.

Literature on this topic covers various aspects of open channel flow, including flow dynamics, hydraulic characteristics, and flow measurements. In the late 18th century, French engineer Antoine Chezy created the Chezy equation. It was the first formula to calculate the velocity of uniform channels. It connects the hydraulic radius and the channel slope to the flow velocity in a channel. The Manning equation was proposed in 1889 by the Irish Engineer Robert Manning. He introduced it as an alternative to the Chezy equation. It is an empirical equation that is a function of the channel velocity, flow area, and channel slope and applies to uniform flow in open channels. ([**franz_1982_tabular**]) provides an in-depth look at open channel flow, focusing on flow characteristics and transport phenomena. The research looks at flow dynamics, hydraulic properties, and sediment transport in open channels. It also goes over the significance of the continuity equation, momentum equation, and energy equation in understanding open channel flow, as well as the role of hydraulic properties. ([**mhanifchaudhry_2008_openchannel**]) presents a wide range of applications of efficient solution techniques, statistical processes, and numerical methods appropriate for computer analyses, as well as extensive coverage of steady and unsteady flow techniques in Open-Channel Flow. ([**tsombe_2011_modeling**]) studies the impacts of flow parameters on flow velocity, as well as the depth at which flow velocity is maximum. The finite difference approximation approach is employed to solve the governing equations, followed by a graphical display of the findings. It was found that the velocity of flow increases with increasing depth for a certain flow area, and the velocity is maximum just below the free surface. ([**mbah_2015_open**]) shows the results of a modeled open channel flow across porous material (River). Water was considered to be an incompressible fluid in the model, with a constant and uniform flow, an isothermal system, and a laminar flow pattern. They used analytic methods to solve the Brinkman equation. To illustrate the effect of permeability on flow parameters, the results were analyzed and plotted using MATLAB. They found that the velocity of the open channel decreases as the permeability of the channel increases. ([**marangu_2016_modeling**]) studied the flow of an incompressible fluid of an open trapezoidal channel with one lateral inflow channel. The flow parameters in the lateral inflow trapezoidal channel are investigated to see how each one affects the flow velocity in the main trapezoidal channel. The finite difference method was used to approximate the governing equations. It has been discovered that decreasing the cross-sectional area of the lateral input channel increases flow velocity while increasing the length decreases flow velocity. ([**morfett_2021_hydraulics**]) covers computational modelling, illustrating the application of computational simulation techniques to modern designs. ([**mnassri_2021_on**] investigated the behavior of free-surface waves in an open channel using a shallow-water model and numerical solver. Results from a rectangular channel showed the solver accurately predicted wave propagation and reflection mechanisms. The proposed solver could be extended to other open-channel cross-section shapes. This study contributes to our understanding of wave behavior and provides a tool for predicting and analyzing wave behavior in various settings. [**setyandito_2022_flow**] examined the relationship between Froude number and depth-averaged velocity on a trapezoidal weir, and compared numerical models to experimental results and other weir types. The study found that Froude number increases with average velocity, and that the flow downstream of a trapezoidal weir differs from other weirs. FLOW-3D CFD software was used to model the weir, and Flow Sight was used to display visualization results. From the research presented above, it can be seen that the finite difference method is a numerical tool for solving the modified Saint-Venant equations, which are used to analyze fluid flow in open channels. However, most open channel research has focused on circular, rectangular, or elliptic open channels. The research done on open trapezoidal channels have been carried out experimentally and include lateral inflow. As a result, I focused my research on mathematically modeling a trapezoidal open channel and numerically solving the problem.

2. STATEMENT ON THE PROBLEM

The purpose of this study is to mathematically model an open trapezoidal channel by varying the channel bottom slope, channel top and bottom width, side slopes, and the Manning coefficient to investigate how these parameters affect flow velocity and flow depth in the open trapezoidal channel. The fluid flow is assumed to be incompressible, unsteady and one dimensional, with gravity being the driving force behind the flow.

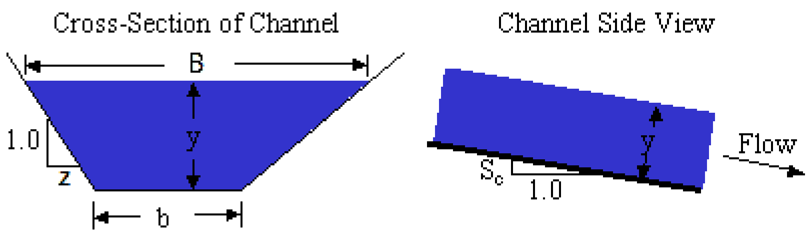

Figure 1: Cross-section of Trapezoidal Channel

The depth of flow (y) is the vertical distance of the lowest point of the channel section (bed of the channel) from the free surface. The top width (B) is the width of the channel segment at the free surface. The wetted perimeter (P) is the length of the channel boundary when it comes into contact with flowing water at any point. It's given by:

$$P = b + 2yw \tag{1}$$

where,

$$w = (1 + z^2)^{0.5} \tag{2}$$

The wetted area (A) is the cross-sectional area of the channel's flow section, given by:

$$A = (b + zy)y \tag{3}$$

The hydraulic radius (R) is the ratio of the cross- sectional area of flow to wetted perimeter, given by:

$$R = \frac{A}{P} \tag{4}$$

3. GOVERNING EQUATIONS

For uniform flow in open channels. Mainly, Chezy's formula and Manning's formula. The Chezy formula is a commonly used formula in fluid mechanics and fluid dynamics that defines mean flow velocity in turbulent open channel flow. It is used for uniform equilibrium and non-uniform gradually varying flows.

$$V = C\sqrt{RS_0}$$

Where C is the Chezy coefficient .Experiments are required to ascertain the value of this coefficient.

The Manning equation was proposed in 1889 by the Irish Engineer Robert Manning. He introduced it as an alternative to the Chezy equation. The Manning equation is an empirical equation that is a function of the channel velocity, flow area, and channel slope and applies to uniform flow in open channels.

$$V = \frac{k}{n} R^{\frac{2}{3}} S^{\frac{1}{2}}$$

Where, n is the coefficient of roughness (or the Manning coefficient), k is a conversion factor (k=1 for SI units).

The Saint-Venant equations (or shallow water equations) are a set of hyperbolic partial differential equations that describe flow in a fluid below a pressure surface. They can be used to explain flow characteristics in one or two planes. The Saint-Venant equations describe the behavior of a thin layer of fluid with a constant density that is bounded on the bottom by the flow bed and on the top by a free surface of water. The Saint-Venant equations are derived from the equation of continuity and equation of momentum.

The continuity equation governing unsteady flow in open channels of arbitrary shape is:

$$\frac{A}{B}\frac{\partial V}{\partial x}+V\frac{\partial y}{\partial x}+\frac{\partial y}{\partial t}=\frac{q}{B} \tag{5}$$

Unsteady flow in open channels of arbitrary form is governed by the momentum equation and is given by:

$$\frac{\partial V}{\partial t}+\alpha V\frac{\partial V}{\partial x}+g\frac{\partial y}{\partial x}=g(S_0-S_f) \tag{6}$$

3.1. Method of solution

The derivatives of the flow variables with respect to time and space are as shown below:

$$\frac{\partial V}{\partial t}=\frac{V_{i,j+1}-0.5(V_{i-1,j}+V_{i+1,j})}{k}+O(k) \tag{7}$$

$$\frac{\partial y}{\partial t}=\frac{y_{i,j+1}-0.5(y_{i-1,j}+y_{i+1,j})}{k}+O(k) \tag{8}$$

$$\frac{\partial V}{\partial x}=\frac{V_{i+1,j}-V_{i-1,j}}{2h}+O(h) \tag{9}$$

$$\frac{\partial y}{\partial x}=\frac{y_{i+1,j}-y_{i-1,j}}{2h}+O(h) \tag{10}$$

It is often considered that the friction slope may be estimated using either the Manning or Chezy resistance equations for computing unsteady flow. We will use the Manning resistance equation, which is written as:

$$S_f=\frac{S_f(i-1,j)+S_f(i+1,j)}{2}=\frac{n^2}{2R^{\frac{4}{3}}}(V_{i-1,j}^2+V_{i+1,j}^2) \tag{11}$$

The finite difference form corresponding to equations 5 and 6 are given as follows.
The continuity equation:

$$y_{i,j+1}=\frac{1}{2}(y_{i-1,j+1}-y_{i+1,j})-k\left\{V_{i,j}\left(\frac{y_{i+1,j}-y_{i-1,j}}{2h}\right)+\frac{A}{B}\left(\frac{V_{i+1,j}-V_{i-1,j}}{2h}\right)\right\} \tag{12}$$

The momentum equation:

$$V_{i,j+1} = \frac{1}{2}(V_{i-1,j} - V_{i+1,j}) - k\left\{ V_{i,j}\left(\frac{V_{i+1,j} - V_{i-1,j}}{2h}\right) + g\left(\frac{y_{i+1,j} - y_{i-1,j}}{2h}\right) - g\left(S_0 - \frac{n^2}{2R^{\frac{4}{3}}}(V_{i-1,j}^2 + V_{i+1,j}^2)\right)\right\} \quad (13)$$

The continuity and momentum equations are represented by equations 12 and 13. The terms $y_{i,j+1}$ and $V_{i,j+1}$ in the continuity and momentum equations are computed using the initial and boundary conditions noted below.

$$V(i,0) = 0 \qquad y(i,0) = 0 \quad (14)$$

$$V(0,j) = 10 \qquad y(0,j) = 0.5$$

$$V(N,j) = 10 \qquad y(N,j) = 0.5 \quad (15)$$

4. RESULTS AND DISCUSSIONS

Matlab was used to solve and plot the graphs for equations 12 and 13 at various points along the channel. Multiple velocity profiles were plotted, and various parameters varied independently while the others remained constant.
The flow parameters were taken as the following values:

g=9.81, So=0.002, y=2.5, b=4, z=1, ni=0.012

5. VELOCITY PROFILES FOR TRAPEZOIDAL OPEN CHANNEL

Figure 2

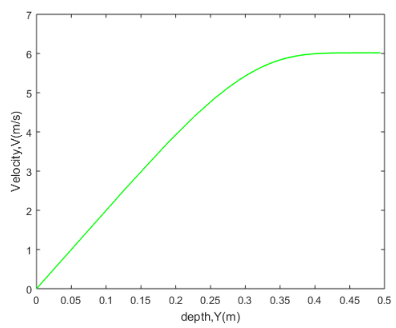

Figure 3

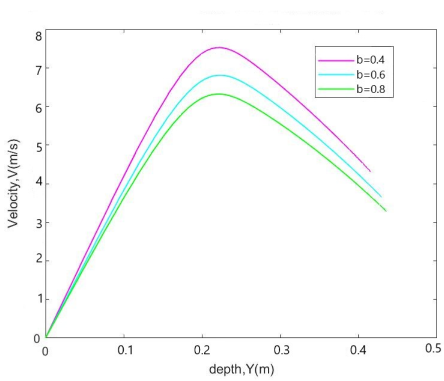

Table 1: Velocity profiles with change in depth and Velocity profiles with change in bottom width

Figure 4

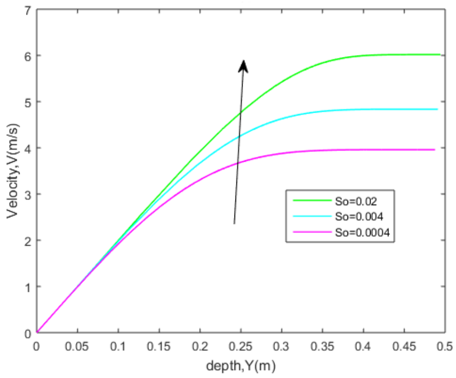

Figure 5

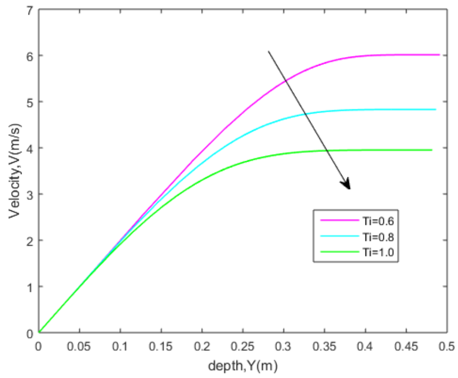

Table 2: Velocity profiles with change in bottom slope and Velocity profiles with change in top width

Figure 6

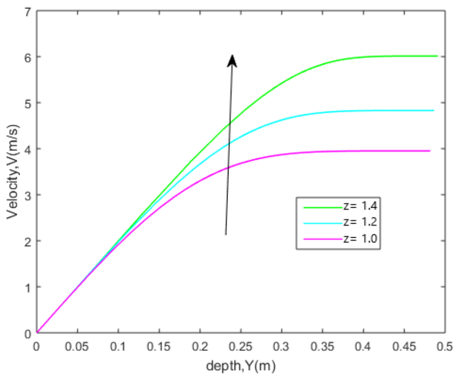

Figure 7

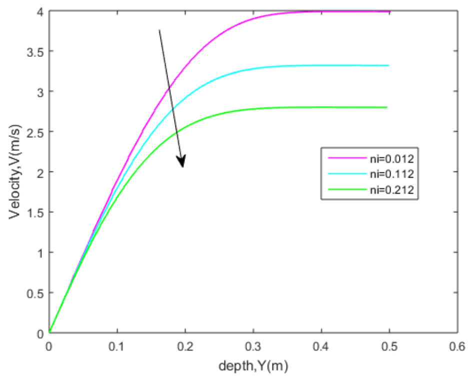

Table 3: Velocity profiles with change in side slopes and Velocity profiles with change in Manning coefficient

From figure 2, the velocity increases as the depth of the channel from the bottom to the free surface increases. The free surface occurs at a depth of 0.4m, and the velocity at this depth is 6m/s. Maximum velocity is observed just below the free surface at a depth of 0.35m. Due to the non-slip condition of fluids, the fluid flow velocity at the channel bottom is zero. It can also be seen from figure 3 that, the velocity from the channel bottom to the free surface decreases as the width of the flow channel increases from 0.4m to 0.8m. This is due to the fact that increasing the width increases the wetted perimeter of the fluid under the conduit. This increase in the wetted perimeter causes an increase in shear stress in the channel bottom, resulting in a decrease in flow velocity. Since discharge is a function of velocity and area, a decrease in velocity results in a decrease in discharge.

It appears at first in figure 4, that increasing the bottom slope had no effect on the flow velocity. However, at depth 0.07m, the flow velocity increases as the bottom slope increases from 0.0004 to 0.02. As the slope increases, the center of gravity shifts upward, causing instability in the fluid molecules and in turn an increase in velocity and hence an increase in discharge. We can see from figure 5 that the velocity from the channel bottom to the free surface decreases as the top width of the flow channel increases from 0.6m to 1m. This is due to the fact that increasing the top width increases the wetted perimeter of the fluid under the conduit.

Based on figure 6, it can be observed that from increasing the side slope from 1(horz./vert.) to 1.4(horz./vert.), there is a decrease in velocity and thus greater discharge. An increase in the channel side slopes results to an increase in the wetted perimeter because of the fluid spreading more in the conduit leading to a larger cross-sectional area. Figure 7 shows that increasing the roughness coefficient from 0.012 to 0.014 causes a decrease in velocity. An increase in the roughness coefficient causes increased shear stress at the channel's sides and bottom, resulting in a decrease in velocity. This is because the motion of the fluid particles near the conduit's surface will be reduced. The velocity of neighboring molecules will be reduced as a result of constant bombardment with flow moving molecules, resulting in a reduction in overall flow velocity.

6. CONCLUSION

The objectives of modeling fluid flow in an open channel with a trapezoidal cross section were met. The results also compare favorably with previous research that varied similar flow parameters. The following are the conclusions and recommendations from investigating the effects of varying the channel bottom slope, channel top width, channel side slope, channel bottom slope and the Manning coefficient on the velocity in the flow channel.

Numerous factors affects a flow channel's velocity. The flow increases in velocity as the channel depth increases from the bottom to the free surface. On the other hand, the velocity decreases as the width of the channel bottom increases. Similar to this, as the slope of the channel side increases, the velocity also decreases. The velocity measured from the channel bottom to the free surface will, however, decrease as the top width of the flow channel widens. Additionally, it has been found that a channel with a steep slope has a higher flow velocity than one with a gentle slope. Lastly, increasing the Manning coefficient will result in a decrease in the flow velocity.

NOMENCLATURE

A	Cross-sectional area (m^2)
b	Bottom channel width (m)
B	Top channel width (m)

C	Factor of flow resistance (chezy coefficient)
F	Force (N)
g	Acceleration due to gravity (m/s^2)
h	Side length of channel
n	The Manning coefficient of roughness ($sm^{-1/3}$)
P	The wetted perimeter (m)
q	Lateral discharge (m^3s^{-1})
S_0	Longitudinal slope of the channel
S_f	Frictional slope
t	Time (s)
v	Fluid velocity (m/s)
x	Length of the channel (m)
y	Depth of the channel (m)
z	Channel side slope
ϕ	Constant in Manning formula
ρ	Fluid density
β	Momentum coefficient
ω	Coefficient of effect of gravity
PDE	Partial differential equations
FDM	Finite difference method
FEM	Finite element method