Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 19, Number 1 (2023), pp. 89-99

http://www.ripublication.com/gjpam.htm

(α,γ),(β,δ) Parametric form for Solving Fully Intuitionistic Fuzzy Linear System of Equations

Bidhan Chandra Saw[*] **, Sushanta Man , and Subhendu Bikash Hazra**
Department of Mathematics, Bankura University, Bankura, 722101, West Bengal, India.,

Abstract

This paper presents a method to solve fully intuitionistic fuzzy linear system of equations (FIFLS). Our proposed method is a new method to solve such systems of equations. In this new method $n \times n$ FIFLS is converted to two $n \times n$ crisp systems, which is the only method and is quite efficient.
Keywords: Positive Triangular Intuitionistic Fuzzy Number, Parametric form of Triangular Intuitionistic Fuzzy Number, Fully Intuitionistic Fuzzy Linear Systems (FIFLS).

1. INTRODUCTION

In real-life situations many problems in mathematics, physics, engineering and other science subject ultimately turn into a problem of solving system of linear equations. In this type of experiment, parameters are obtained through some estimation, modelling, or observation. Since all these type of problems involve some uncertainty at some point or the other, the most scientific way to choose the parameters as fuzzy rather than crisp. Fuzzy parameters can handle uncertainty and vagueness. In fuzzy set theory sum of membership and non-membership is always equal to one. But in actual real situation, there may exist some hesitation degree. In intuitionistic fuzzy set, this hesitation degree also considered which makes intuitionistic fuzzy set theory more useful. So, considering intuitionistic fuzzy set (number) seems to fit more suitably to describe uncertainty, therefore it is more useful to take intuitionistic fuzzy set (number) rather than fuzzy set (number).

[*]Corresponding author

The concept of fuzzy set has been first introduced by Zadeh [21]. Since then, there have several generalizations of fuzzy set; intuitionistic fuzzy set (IFS) is one of them. IFS was first introduced by Atanassov [3,4]. Intuitionistic fuzzy linear systems (IFLS) are the linear systems whose parameters are all or partially represented by intuitionistic fuzzy numbers (IFN). Fully intuitionistic fuzzy linear sytems (FIFLS) are the linear systems whose parameters are all represented by intuitionistic fuzzy numbers (IFN) Friedman et al. [15] first proposed a general model for solving $n \times n$ fuzzy linear systems (FLS) with the coefficient matrix consisting of crisp numbers and right hand side column vector consisting of arbitrary fuzzy numbers. By using parametric form of fuzzy number they converted $n \times n$ FLS into $2n \times 2n$ crisp systems. Behera et al. [11] gave a new model for solving FLS. They converted $n \times n$ fuzzy system into $n \times n$ crisp system by using double parametric form. Fuzzy system of linear equations has been studied by several authors [1,2,7–9,13,16–18,20]. Banerjee et al. [6] developed an approach to solve intuitionistic fuzzy linear systems (IFLS). They converted $n \times n$ system of IFLS into two $2n \times 2n$ crisp linear systems. Atti et al. [5] also developed an approach to solve IFLS. They converted $n \times n$ system of equation into four $n \times n$ crisp linear systems. Thee solvability of system of intuitionistic fuzzy linear equations was studied by Pradhan et al. [19]. Bharati. et. al give a method to solve fully intuitionistic fuzzy linear programming problem based on sign distance.

In this paper, we have developed an approach to solve FIFLS following Behera et al. [11], which originally was developed for FLS. The paper is organised as follows: In the next section we review some definitions, in section (3) we discuss in detail the proposed method of solution and in section (4) numerical examples have been included which are solved using the new method. In the section (5) we have given conclusion.

2. PRELIMINARIES

Definition 2.1. Intuitionistic Fuzzy Sets: Let X be a finite set of elements. An IFS $\tilde{A}$ in X is defined as

$$\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)\rangle \mid x \in X\},$$

where the functions $\mu_{\tilde{A}} : X \mapsto [0,1]$ and $\nu_{\tilde{A}} : X \mapsto [0,1]$ respectively represent the degrees of membership and non-membership of the element $x \in X$ to the set $\tilde{A}$ such that $\mu_{\tilde{A}}(x) \in [0,1],\ \nu_{\tilde{A}}(x) \in [0,1],\ 0 \leq \mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \leq 1$.

Definition 2.2. (α, β)**-cuts:** A subset (α, β)-cut of X, generated by IFS $\tilde{A}$, where $\alpha \in [0,1],\ \beta \in [0,1]$ are fixed numbers such that $\alpha + \beta \leq 1$ defined as

$$\tilde{A}_{\alpha,\beta} = \{x \in X : \mu_{\tilde{A}}(x) \geq \alpha,\ \nu_{\tilde{A}}(x) \leq \beta\}$$

$\tilde{A}_{\alpha,\beta}$ is a crisp set of elements which belong to $\tilde{A}$ at least to the degree of α and which does not belong to $\tilde{A}$ at most to the degree of β.

Definition 2.3. Intuitionistic Fuzzy Number (IFN): An IFS $\tilde{A}$ on real line is called IFN if it satisfies the following conditions

1. there exists $x_o \in \mathbb{R}$ such that $\mu_{\tilde{A}}(x_0) = 1 \; and \; \nu_{\tilde{A}}(x_0) = 0$

2. the membership function $\mu_{\tilde{A}}$ is convex
   i.e, $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq min\{\mu_{\tilde{A}}(x_1),\ \mu_{\tilde{A}}(x_2); \forall x_1, x_2 \in \mathbb{R},\ \lambda \in [0,1]\}$

3. the non-membership function $\nu_{\tilde{A}}$ is concave
   i.e, $\nu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \leq max\{\nu_{\tilde{A}}(x_1),\ \nu_{\tilde{A}}(x_2)\}; \forall x_1, x_2 \in \mathbb{R},\ \lambda \in [0,1]\}$.

Definition 2.4. Triangular Intuitionistic Fuzzy Number (TIFN): A triangular intuitionistic fuzzy number $\tilde{A}$ is denoted by $\tilde{A} = \langle (l_1, l_2, l_3), (l_1', l_2, l_3') \rangle$. An IFN $\tilde{A}$ is called TIFN if its membership and non-membership functions follow the following rules:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, \quad x \leq l_1 \\ \frac{x-l_1}{l_2-l_1}, \; l_1 \leq x \leq l_2 \\ \frac{l_3-x}{l_3-l_2}, \; l_2 \leq x \leq l_3 \\ 0, \quad x \geq l_3 \end{cases}$$

and

$$\nu_{\tilde{A}}(x) = \begin{cases} 1, \quad x \leq l_1' \\ \frac{l_2-x}{l_2-l_1'}, \; l_1' \leq x \leq l_2 \\ \frac{x-l_2}{l_3'-l_2}, \; l_2 \leq x \leq l_3' \\ 1, \quad x \geq l_3' \end{cases}$$

where $l_1' \leq l_1 \leq l_2 \leq l_3 \leq l_3'$.
(α, β)-cut of $\tilde{A} = \langle (l_1, l_2, l_3), (l_1', l_2, l_3') \rangle$ may be represented as
$\tilde{A}_{\alpha,\beta} = \langle [\underline{A}(\alpha), \overline{A}(\alpha)], [\underline{A'}(\beta), \overline{A'}(\beta)] \rangle$, where $\underline{A}(\alpha) = l_1 + \alpha(l_2 - l_1)$, $\overline{A}(\alpha) = l_3 - \alpha(l_3 - l_2)$, $\underline{A}'(\beta) = l_2 - \beta(l_2 - l_1')$ and $\overline{A}'(\beta) = l_2 + \beta(l_3' - l_2)$.

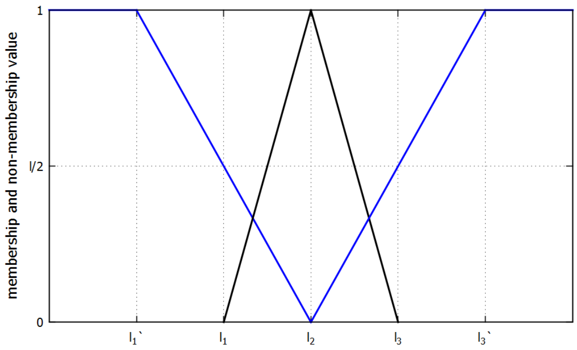

Figure 1: TIFN$\langle(l_1, l_2, l_3), (l'_1, l_2, l'_3)\rangle$

Definition 2.5. Non-Negative TIFN: A triangular intuitionistic fuzzy Number $\tilde{A} = \langle(l_1, l_2, l_3), (l'_1, l_2, l'_3)\rangle$ is said to be non-negative if $l'_1 \geq 0$.

3. FULLY INTUITIONISTIC FUZZY SYSTEM OF LINEAR EQUATIONS:

The $n \times n$ Fully Intuitionistic fuzzy system of linear equation may be written as

$$\begin{aligned}
\tilde{a}_{11}\tilde{x}_1 + \tilde{a}_{12}\tilde{x}_2 + ... + \tilde{a}_{1n}\tilde{x}_n &= \tilde{b}_1 \\
\tilde{a}_{21}\tilde{x}_2 + \tilde{a}_{22}\tilde{x}_2 + ... + \tilde{a}_{2n}\tilde{x}_n &= \tilde{b}_2 \\
&\vdots \\
\tilde{a}_{n1}\tilde{x}_n + \tilde{a}_{n2}\tilde{x}_2 + ... + \tilde{a}_{nn}\tilde{x}_n &= \tilde{b}_n.
\end{aligned}$$

In matrix notation the above system may be written as
$\tilde{A}\tilde{X} = \tilde{B}$, where the coefficient matrix $\tilde{A} = (\tilde{a}_{kj}), 1 \leq k \leq n, 1 \leq j \leq n$ is an intutionistic fuzzy matrix of order $n \times n$,
$\tilde{B} = (\tilde{b}_k), 1 \leq k \leq n$ is a column vector of fuzzy number and $\tilde{X} = (\tilde{x}_j)$ is the vector of fuzzy unknown.
Here we have assumed that $\tilde{A} \geq 0, \tilde{B} \geq 0$ and $\tilde{X} \geq 0$.

3.1. Solution Method Of Fully Intuitionistic Fuzzy System Of Linear Equations:

The system of equations $\tilde{A}\tilde{X} = \tilde{B}$ can be represented as

$$\sum_{j=1}^{n} \tilde{a}_{kj}\tilde{x}_j = \tilde{b}_k, \ for\ k = 1, 2, ..., n. \tag{3.1}$$

Using parametric form of intuitionistic fuzzy number, we may write

$$\tilde{a_{kj}} = \langle[\underline{a_{kj}}(\alpha), \overline{a_{kj}}(\alpha)], [\underline{a'_{kj}}(\beta), \overline{a'_{kj}}(\beta)]\rangle,$$
$$\tilde{x_j} = \langle[\underline{x_j}(\alpha), \overline{x_j}(\alpha)], [\underline{x'_j}(\beta), \overline{x'_j}(\beta)]\rangle,$$
and
$$\tilde{b_k} = \langle[\underline{b_k}(\alpha), \overline{b_k}(\alpha)], [\underline{b'_k}(\beta), \overline{b'_k}(\beta)]\rangle.$$

Substituting the above expression in equation (3.1), we get

$$\sum_{j=1}^{n}\langle[\underline{a_{kj}}(\alpha), \overline{a_{kj}}(\alpha)], [\underline{a'_{kj}}(\beta), \overline{a'_{kj}}(\beta)]\rangle\langle[\underline{x_j}(\alpha), \overline{x_j}(\alpha)], [\underline{x'_j}(\beta), \overline{x'_j}(\beta)]\rangle = \langle[\underline{b_k}(\alpha), \overline{b_k}(\alpha)], [\underline{b'_k}(\beta), \overline{b'_k}(\beta)]\rangle \quad (3.2)$$

for $k = 1, 2, ..., n.$

From equation(3.2), one may get two $n \times n$ system of equations based on α -cut and β-cut respectively

$$\sum_{j=1}^{n}\langle[\underline{a_{kj}}(\alpha), \overline{a_{kj}}(\alpha)]\rangle\langle[\underline{x_j}(\alpha), \overline{x_j}(\alpha)]\rangle = \langle[\underline{b_k}(\alpha), \overline{b_k}(\alpha)]\rangle, \text{ for } k = 1, 2, ..., n. \quad (3.3)$$

and

$$\sum_{j=1}^{n}\langle[\underline{a'_{kj}}(\beta), \overline{a'_{kj}}(\beta)]\rangle\langle[\underline{x'_j}(\beta), \overline{x'_j}(\beta)]\rangle = \langle[\underline{b'_k}(\beta), \overline{b_k}(\beta)]\rangle, \text{ for } k = 1, 2, ..., n. \quad (3.4)$$

Let us define

$$\tilde{a}_{kj}(\alpha, \gamma) = \gamma(\overline{a_{kj}}(\alpha) - \underline{a_{kj}}(\alpha)) + \underline{a_{kj}}(\alpha), \text{ for } j, k = 1, 2, ..., n \quad (3.5)$$
$$\tilde{x}_j(\alpha, \gamma) = \gamma(\overline{x_j}(\alpha) - \underline{x_j}(\alpha)) + \underline{x_j}(\alpha), \text{ for } j = 1, 2, ..., n \quad (3.6)$$
$$\tilde{b}_k(\alpha, \gamma) = \gamma(\overline{b_k}(\alpha) - \underline{b_k}(\alpha)) + \underline{b_k}(\alpha), \text{ for } k = 1, 2, ..., n \quad (3.7)$$
$$\tilde{a}_{kj}(\beta, \delta) = \delta(\overline{a'_{kj}}(\beta) - \underline{a'_{kj}}(\beta)) + \underline{a'_{kj}}(\beta), \text{ for } j, k = 1, 2, ..., n \quad (3.8)$$
$$\tilde{x}_j(\beta, \delta) = \delta(\overline{x'_j}(\beta) - \underline{x'_j}(\beta)) + \underline{x'_j}(\beta), \text{ for } j = 1, 2, ..., n \quad (3.9)$$
$$\tilde{b}_k(\beta, \delta) = \delta(\overline{b_k}(\beta) - \underline{b_k}(\beta)) + \underline{b_k}(\beta), \text{ for } k = 1, 2, ..., n. \quad (3.10)$$

substituting the above expressions in eq.(3.3) and eq.(3.4) we get

$$\sum_{j=1}^{n}\tilde{a}_{kj}(\alpha, \gamma)\tilde{x}_j(\alpha, \gamma) = \tilde{b}_k(\alpha, \gamma), \text{ for } k = 1, 2, ..., n \quad (3.11)$$

and

$$\sum_{j=1}^{n}\tilde{a}_{kj}(\beta, \delta)\tilde{x}_j(\beta, \delta) = \tilde{b}_k(\beta, \delta), \text{ for } k = 1, 2, ..., n. \quad (3.12)$$

The above equation (3.11) is now solved to obtain $\tilde{x}_j(\alpha,\gamma)$. After getting $\tilde{x}_j(\alpha,\gamma)$ one can put $\gamma = 0$ to get $\underline{x_j}(\alpha)\ (= \tilde{x}_j(\alpha,0))$ and $\gamma = 1$ to get $\overline{x_j}(\alpha)\ (= \tilde{x}_j(\alpha,1))$.

Similarly, solving equation (3.12) one can get $\tilde{x}_j(\beta,\delta)$. After getting $\tilde{x}_j(\beta,\delta)$ one can put $\delta = 0$ to get $\underline{x'_j}(\beta)\ (= \tilde{x}_j(\beta,0))$ and $\delta = 1$ to get $\overline{x'_j}(\beta)\ (= \tilde{x}_j(\beta,1))$.

Existence Of Solution : From the non negative fully intuitionistic fuzzy system of linear equations (3.1) one may represent the non negative intuitionistic fuzzy system as two non negative $n \times n$ fuzzy system of equations as
$\tilde{A}(\alpha,\gamma)\tilde{X}(\alpha,\gamma) = \tilde{B}(\alpha,\gamma)$ and
$\tilde{A}(\beta,\delta)\tilde{X}(\beta,\delta) = \tilde{B}(\beta,\delta)$

Theorem 3.1 (Following Behera et al 2015)**.** *Let* $\tilde{A}(\alpha,\gamma) \geq 0$, $\tilde{B}(\alpha,\gamma) \geq 0$ *and* $\tilde{A}(\alpha,\gamma)$ *corresponds to a permutation matrix. Then the non negative fully intuitionstic fuzzy system of linear equations has a non negative consistent fuzzy solution.*

Proof: The hypothesis of De Marr (1972) implies that $[\tilde{A}(\alpha,\gamma)]^{-1}$ exists as non negative matrix. So we have$\tilde{X}(\alpha,\gamma) = [\tilde{A}(\alpha,\gamma)]^{-1}\tilde{B}(\alpha,\gamma) \geq 0$. Hence one may conclude that $\tilde{X}(\alpha,\gamma)$ is a non-negative solution of the system of equation (3.11).

Theorem 3.2 (Following Behera et al 2015)**.** *Let* $\tilde{A}(\beta,\delta) \geq 0$, $\tilde{B}(\beta,\delta) \geq 0$ *and* $\tilde{A}(\beta,\delta)$ *corresponds to a permutation matrix. Then the non negative fully intuitionstic fuzzy system of linear equations has a non negative consistent fuzzy solution .*
Proof: *The hypothesis of De Marr (1972) implies that* $[\tilde{A}(\beta,\delta)]^{-1}$ *exists as non-negative matrix. So we have* $\tilde{X}(\beta,\delta) = [\tilde{A}(\beta,\delta)]^{-1}\tilde{B}(\beta,\delta) \geq 0$. *Hence one may conclude that* $\tilde{X}(\beta,\delta)$ *is a non negative solution of the system of equation* (3.12).

Definition 3.1. If $\tilde{X} = (\tilde{x_1}, \tilde{x_2}, ..., \tilde{x_n})^T$ is a solution set of (12) and for each j, $1 \leq j \leq n$, the inequalities $\underline{x_j}(\alpha) \leq \overline{x_j}(\alpha)$, $\underline{x'_j}(\beta) \leq \overline{x'_j}(\beta)$ hold, then the solution $\tilde{X}$ is called a strong solution of the system (3.1).

Definition 3.2. If $\tilde{X} = (\tilde{x_1}, \tilde{x_2}, ..., \tilde{x_n})^T$ is a solution set of (12) and for some j, $1 \leq j \leq n$, the inequalities $\underline{x_j}(\alpha) > \overline{x_j}(\alpha)$ or $\underline{x'_j}(\beta) > \overline{x'_j}(\beta)$ hold, then the solution $\tilde{X}$ is called a weak solution of the system (3.1).

4. NUMERICAL EXAMPLE

Example 4.1. Let us consider 2×2 fully intuitionistic fuzzy system of linear equations ([12])
$\langle(1,2,3),(0.5,2,4)\rangle\tilde{x}_1 + \langle(2,3,4),(1,3,5)\rangle\tilde{x}_2 = \langle(3,9,25),(1,9,60)\rangle$

$\langle(1,2,3),(0.5,2,4)\rangle\tilde{x}_1 + \langle(1,2,3),(0.5,2,4)\rangle\tilde{x}_2 = \langle(3,8,24),(1,8,55)\rangle.$

Then, following the reduction technique mentioned in section (3.1), we get the 1^{st} 2×2 system as
$(\gamma(2-2\alpha)+1+\alpha)\tilde{x}_1(\alpha,\gamma)+(\gamma(2-2\alpha)+2+\alpha)\tilde{x}_2(\alpha,\gamma) = \gamma(22-22\alpha)+3+6\alpha,$
$(\gamma(2-2\alpha)+1+\alpha)\tilde{x}_1(\alpha,\gamma)+(\gamma(2-2\alpha)+1+\alpha)\tilde{x}_2(\alpha,\gamma) = \gamma(21-21\alpha)+3+5\alpha.$

Writing in matrix-vector form we have,

$$\begin{pmatrix}(\gamma(2-2\alpha)+1+\alpha) & (\gamma(2-2\alpha)+2+\alpha)\\(\gamma(2-2\alpha)+1+\alpha) & (\gamma(2-2\alpha)+1+\alpha)\end{pmatrix}\begin{pmatrix}x_1(\alpha,\gamma)\\x_2(\alpha,\gamma)\end{pmatrix}=\begin{pmatrix}\gamma(22-22\alpha)+3+6\alpha\\\gamma(22-22\alpha)+3+5\alpha\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}x_1(\alpha,\gamma)\\x_2(\alpha,\gamma)\end{pmatrix}=\begin{pmatrix}(\gamma(2-2\alpha)+1+\alpha) & (\gamma(2-2\alpha)+2+\alpha)\\(\gamma(2-2\alpha)+1+\alpha) & (\gamma(2-2\alpha)+1+\alpha)\end{pmatrix}^{-1}\begin{pmatrix}\gamma(22-22\alpha)+3+6\alpha\\\gamma(22-22\alpha)+3+5\alpha\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}x_1(\alpha,\gamma)\\x_2(\alpha,\gamma)\end{pmatrix} = \begin{pmatrix}\dfrac{2\alpha^2\gamma^2-3\alpha^2\gamma+\alpha^2-4\alpha\gamma^2+23\alpha\gamma-4\alpha+2\gamma^2-20\gamma-3}{2\alpha\gamma-\alpha-2\gamma-1}\\ \\ -\alpha\gamma+\alpha+\gamma\end{pmatrix}.$$

Now we get the solution as

$$\tilde{x}_1(\alpha,\gamma) = \frac{2\alpha^2\gamma^2-3\alpha^2\gamma+\alpha^2-4\alpha\gamma^2+23\alpha\gamma-4\alpha+2\gamma^2-20\gamma-3}{2\alpha\gamma-\alpha-2\gamma-1} \text{ and}$$
$$\tilde{x}_2(\alpha,\gamma) = -\alpha\gamma+\alpha+\gamma$$

Putting $\gamma=0$ in $\tilde{x}_1(\alpha,\gamma)$ and $\tilde{x}_2(\alpha,\gamma)$ we get respectively,

$$\tilde{x}_1(\alpha,0) = \underline{x_1}(\alpha) = \frac{\alpha^2-4\alpha-3}{-1-\alpha} \text{ and } \tilde{x}_2(\alpha,0) = \underline{x_2}(\alpha) = \alpha.$$

Putting $\gamma=1$ in $\tilde{x}_1(\alpha,\gamma)$ and $\tilde{x}_2(\alpha,\gamma)$ we get respectively,

$$\tilde{x}_1(\alpha,1) = \overline{x_1}(\alpha) = \frac{15\alpha-21}{\alpha-3}, \text{ and } \tilde{x}_2(\alpha,1) = \overline{x_2}(\alpha) = 1.$$

Then, following the reduction technique mentioned in section (3.1), we get the 2^{nd} 2×2 system as
$(3.5\delta\beta+2-1.5\beta)\tilde{x}_1(\beta,\delta)+(4\delta\beta+3-2\beta)\tilde{x}_2(\beta,\delta)=59\delta\beta+9-8\beta,$
$(3.5\delta\beta+2-1.5\beta)\tilde{x}_2(\beta,\delta)+(3.5\delta\beta+2-1.5\beta)\tilde{x}_2(\beta,\delta)=54\delta\beta+8-7\beta.$

Writing in matrix-vector form we have

$$\begin{pmatrix}(3.5\delta\beta+2-1.5\beta) & (4\delta\beta+3-2\beta)\\ (3.5\delta\beta+2-1.5\beta) & (3.5\delta\beta+2-1.5\beta)\end{pmatrix}\begin{pmatrix}x_1(\beta,\delta)\\ x_2(\beta,\delta)\end{pmatrix}=\begin{pmatrix}59\delta\beta+9-8\beta\\ 54\delta\beta+8-7\beta\end{pmatrix}$$

$$\Rightarrow\begin{pmatrix}x_1(\beta,\delta)\\ x_2(\beta,\delta)\end{pmatrix}=\begin{pmatrix}3.5\delta\beta+2-1.5\beta) & (4\delta\beta+3-2\beta)\\ (3.5\delta\beta+2-1.5\beta) & (3.5\delta\beta+2-1.5\beta)\end{pmatrix}^{-1}\begin{pmatrix}59\delta\beta+9-8\beta\\ 54\delta\beta+8-7\beta\end{pmatrix}$$

$$\Rightarrow\begin{pmatrix}x_1(\beta,\delta)\\ x_2(\beta,\delta)\end{pmatrix}=\begin{pmatrix}\dfrac{38\beta^2\delta^2-78\beta^2\delta+8\beta^2+178\beta\delta-30\beta+24}{(\beta\delta-\beta+2)(7\beta\delta-3\beta+4)}\\ \\ \dfrac{10\beta\delta-2\beta+2}{\beta\delta-\beta+2}\end{pmatrix}.$$

Now we get the solution as

$$\tilde{x}_1(\beta,\delta)=\frac{38\beta^2\delta^2-78\beta^2\delta+8\beta^2+178\beta\delta-30\beta+24}{(\beta\delta-\beta+2)(7\beta\delta-3\beta+4)},\ \text{and}$$
$$\tilde{x}_2(\beta,\delta)=\frac{10\beta\delta-2\beta+2}{\beta\delta-\beta+2}.$$

Putting $\delta=0$ in $\tilde{x}_1(\beta,\delta)$ and $\tilde{x}_2(\beta,\delta)$ we get respectively

$$\tilde{x}_1(\beta,0)=\underline{x_1'}(\beta)=\frac{8\beta^2-30\beta+24}{(2-\beta)(4-3\beta)},\ \text{and}\ \tilde{x}_2(\beta,0)=\underline{x_2'}(\beta)=\frac{2-2\beta}{2-\beta}.$$

Putting $\delta=1$ in $\tilde{x}_1(\beta,\delta)$ and $\tilde{x}_2(\beta,\delta)$ we get respectively

$$\tilde{x}_1(\beta,1)=\overline{x_1'}(\beta)=\frac{-8\beta^2+37\beta+6}{2\beta+2},\ \text{and}\ \tilde{x}_2(\beta,1)=\overline{x_2'}(\beta)=4\beta+1.$$

Here we see that,
$\tilde{x}_{1\alpha,\beta}=\langle[\underline{x_1}(\alpha),\overline{x_1}(\alpha)],[\underline{x_1'}(\beta),\overline{x_1'}(\beta)]\rangle,$
and
$\tilde{x}_{2\alpha,\beta}=\langle[\underline{x_2}(\alpha),\overline{x_2}(\alpha)],[\underline{x_2'}(\beta),\overline{x_2'}(\beta)]\rangle,$

are both triangular shaped Intuitionistic Fuzzy Numbers.
Here we get, $\tilde{x_1} \approx \langle(3, 3, 7), (2, 3, 8.75)\rangle$and $\tilde{x_2} \approx \langle(0, 1, 1), (0, 1, 5)\rangle$.

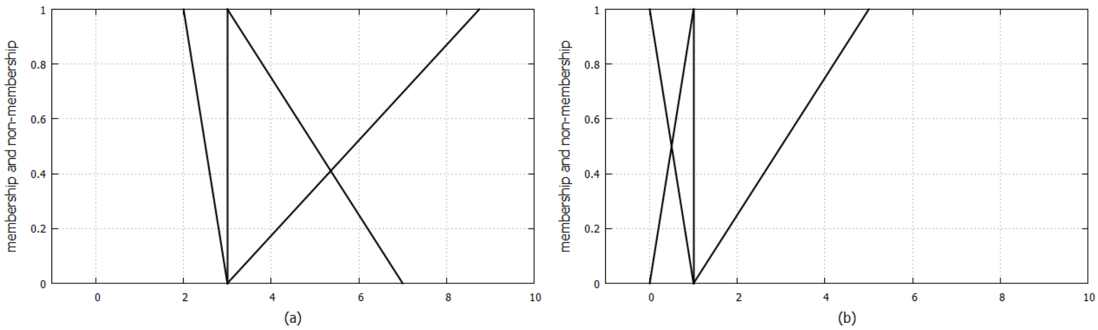

Figure 2: fig(a):$\tilde{x_1}$ fig(b): $\tilde{x_2}$

5. CONCLUSION

This (α,γ),(β,δ) parametric form for solving Fullly Intuitionistic Fuzzy Linear systems is easy and straight forward. The new method gives a lead to handle Fullly Intuitionistic Fuzzy Linear systems, though this method has restriction to applicability, which says that $\tilde{A} \geq 0, \tilde{B} \geq 0$ and $\tilde{X} \geq 0$ must be satisfied. We compared the result of this method with a known fully fuzzy linear programming problem [12] which shows both the results are the same. Future work may focus on removing the restrictions of this method.

REFERENCES

[1] Abbasbandy S., *A method for solving fuzzy linear system*, Iranian Journal Of Fuzzy Systems, **2(2)** (2005), 37-43.

[2] Allahviranloo T., *Numerical methods for fuzzy system of linear equations*, Appl. Math. Comput., **155(2)** (2004), 493-502.

[3] Atanassov K.T., *Intuitionistic fuzzy sets*, Fuzzy sets and systems, **20(1)** (1986), 87-96.

[4] Atanassov K.T., *Ideas for Intuitionistic Fuzzy Sets Equations, Inequations and Optimization.*, Notes on intuitionistic fuzzy sets, **1(1)** (1995), 17-24.

[5] Atti H., Amma B.B., Melliani S., Chadli S., *Intuitionistic Fuzzy Linear Systems*, O. Castillo et al. (eds.), Intuitionistic and Type-2 Fuzzy Logic Enhancements in Neural and Optimization Algorithms: Theory and Applications, Studies in

Computational Intelligence, 862 (2020), 133-144.

[6] Banerjee S., Biswas S., Roy T.K., *Intuitionistic fuzzy linear system* , Advances in Fuzzy Mathematics, **12(3)** (2017), 475-487.

[7] Behera D., Chakraverty S., *A new method for solving real and complex fuzzy system of linear equations*, Computational Mathematics and Modelling, **23(4)** (2012), 507-518.

[8] Behera D.,Chakraverty S., *Solution of fuzzy system of linear equations with crisp coefficients*,Applications and Applied Mathematics, **7** (2012), 648-657.

[9] Behera D., Chakraverty S., *Fuzzy system of linear equations with crisp coefficients*, Journal of Intelligent and Fuzzy Systems, **25(1)** (2013), 201-207.

[10] Behera D., Chakraverty S., *Solution to Fuzzy System of Linear Equations with Crisp Coefficients*, Fuzzy Inf. Eng., **5** (2013), 205-219.

[11] Behera D., Chakraverty S., *New apporach to solve fully fuzzy system of linear equations using single and double parametric form of fuzzy numbers*,Sadhana, **40(1)** (2015), 35-49.

[12] Bharati S.K., Sigh S.R., *Anote on solving a fully intuitionistic fuzzy linear programming problem based on sign distance*, International journal of computer applications, **7** (2015), 648-657.

[13] Buckley J.J., Qu Y., *Solving systems of linear fuzzy equations*,Fuzzy Sets and Systems, **43** (1) (1991), 33-43.

[14] Demarr R., *Non negative matrices with non negative inverses*, Proc. Am. Mathematical Soc., **35(1)**(1972), 307-308.

[15] Friedman M., Ming M.and Kandel A., *Fuzzy Linear Systems*, Fuzzy Sets and Sysytems, **96(2)** (1998), 201-209.

[16] Kumar A., Babbar N., Bansal N., *A new method to solve fully fuzzy linear system with trapezoidal fuzzy numbers*,Canadian Journal on Science and Engnieering Mathematics, **1(3)** (2010), 45-56.

[17] Kumar A., Babbar N., Bansal N., *A method for solving fully fuzzy linear system with trapezoidal fuzzy numbers*, Iranian Journal of Optimization, **4(2)** (2012), 312-323.

[18] Mahapatra G.S., Roy T.K., *Intuitionistic fuzzy number and its arithmetic operation with appilication on system failure* , Journal of Uncertain Systems, **7(2)** (2013), 92-107.

[19] Pradhan R., Pal M., *Solvability of System of intuitionistic fuzzy linear equations*, International journal of fuzzy logic systems, **4(3)** (2014), 13-24.

[20] Seikh M.R., Nayak P.K., Pal M., *Notes on triangular intuitionistic fuzzy numbers*, International Journal Of Mathematics in Operational Research,**5(4)** (2013), 446-465

[21] Zadeh L.A., *Fuzzy Sets*, Information and Control, **8(3)** (1965), 338-353.