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A General Fixed Point Theorem in Soft S-Metric Space via Implicit Relation

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Abstract

We have demonstrated in article, a general fixed soft point theorem with soft contractive condition for self-mapping by using implicit relation on complete soft S-metric space. These are generalization of the results by Sedghi and Dung [28]. To support our results, we have presented some examples also.

Keywords: S-metric space, fixed soft point**,** soft S-metric space, contractive mappings, implicit relation.

MSC: 54H25, 47H10.

1. Introduction and Preliminaries

The concept of metric space was first introduced by the famous mathematicians Frechet in 1905, whereas definition given by Husdroff in 1914 which was commonly used. One of the first accomplishments of algebraic topology was Brouwer [5] fixed point theorem. It serves as the foundation for more general fixed point theorems that are useful in functional analysis., but this theorem can't tell that the fixed point we obtain is unique. Later on, in 1922 Banach [3] proved the Banach contraction Principle (commonly known as fixed point theorem), which provide us uniqueness and existence of a self-mapping on metric spaces. Thereafter, this theorem was generalized by many others in different metric spaces which can be studied in ([6], [13], [16], [23], [25], [30] and so on. These theorems about fixed points were also been proved by several authors in generalized metric spaces, such as D-metric space was defined by [12]. D* metric space was introduced by Mustafa and Sims [21] that is the alteration of D-metric space. Also, Mustafa and Sims [22] in 2005, established G-metric space. After that, Sedghi *et*

al. [27] introduced the concept of S-metric space. In this space they also proved the theorems on fixed point. More work on S-metric space can be studied in ([28], [29]).
To deal with the problem of uncertainties Molodtsov [20], a Russian scholar, proposed the soft set in theory 1999, which has many applications in various directions. Molodtsov [20] applied soft set theory effectively the in fields like smoothness of functions, operation research, Riemann integration, game theory etc.
In 2012, Das and Samanta [7-9] introduced the concept of soft real number and soft real set.. Further, they explored some of their fundamental characteristics as well as defined soft metric space. Wardowski D. [33] in 2013, established the results on a soft mapping and proved some fixed points therein. In 2013, Yazar *et al.* [34] established results on soft metric spaces and defined some soft contractive mappings. On these soft contractive maps, they also proved various fixed point theorems. Soft set theory and its applications in a variety of domains are gaining popularity. Work on these space can be found in ([1], [2], [4], [14], [17], [19], [24], [26], [32] and so on.).
In this article, we first look at some of the fundamental features of soft S-metrics, which was introduced by Cigdem Aras *et al*. [10-11]. After that, using these features we'll demonstrate a general fixed point theorem on a complete soft S-metric space via implicit relation on soft contractive conditions.

Definition 1.1 [20]: "A pair (F, E) is called a soft set over a given universal set X, if and only if F is a mapping from a set of parameters E (each parameter could be a word or a sentence) into the power set of X denoted by $P(X)$. That is, $F: E \rightarrow P(X)$. Clearly, a soft set over X is a parameterized family of subsets of the given universe X."

Example 1.2: Suppose someone wants to buy a television. So let X denotes the number of televisions which he saw in different showrooms as $X = \{h_1, h_2, h_3, h_4, h_5. h_6\}$. Let the set of parameters E be given by $E = \{e_1, e_2, e_3, e_4, e_5\} = \{\text{branded}, 55\text{ inches}, \text{android}, \text{full HD}, \text{refresh rate}\}$. Suppose that, $F(e_1) = \{h_1, h_3, h_5\}$, $F(e_2) = \{h_1, h_5\}$, $F(e_3) = \{h_1, h_2, h_4, h_5\}, F(e_4) = \{h_1, h_3, h_5\}$, $F(e_5) = \{h_1, h_5, h_6\}$, then the we define soft set (F, E) as a set of approximations, as seen below:

$$(F, E) = \big\{ \text{ branded} = \{h_1, h_3, h_5\}, 55 \text{ inches} = \{h_1, h_5\}, \text{android} \\ = \{h_1, h_2, h_4, h_5\}, \text{full HD} = \{h_1, h_3, h_5\}, \text{refresh rate} = \{h_1, h_5, h_6\}\big\}.$$

Definition 1.3 [18]: "A soft set (F, E) over X is said to be a null soft set denoted by $\widetilde{\Phi}$, if for all $e \epsilon E, F(e) = null\ set\ \phi."$

Definition 1.4 [18]: "A soft set (F, E) over X is said to be an absolute soft set denoted by $\tilde{X}$ if for all $e \epsilon E, F(e) = X."$

Definition 1.5 [7]: "Let $\mathbb{R}$ be the set of real numbers and $\mathcal{B}(\mathbb{R})$ the collection of all non-empty bounded subsets of $\mathbb{R}$ and E be taken as a set of parameters. Then a mapping $F: E \rightarrow \mathcal{B}(\mathbb{R})$ is called a soft real set. If a real soft set is a singleton soft set, it will be called a soft real number and denoted by $\bar{r}, \bar{s}, \bar{t}$ etc $\bar{0}$ *and* $\bar{1}$ are the soft real numbers where $\bar{0}(e) = 0$, $\bar{1}(e) = 1$, for all $e \epsilon E$ respectively."

Example 1.6: Suppose a hospital has 5 wards and there is various number of patients in each ward. If we consider a parameter set E to be number of wards, let it be $E = \{W_1, W_2, W_3, W_4, W_5\}$ and define $F: E \to P(\mathbb{R})$ by $F(e)$ = The number of patients in each ward, $\forall\ e \epsilon E$.
If $F(W_1) = \{34\}$, $F(W_2) = \{40\}$, $F(W_3) = \{20\}$, $F(W_4) = \{54\}$ and $F(W_5) = \{43\}$. Therefore, identifying (F, E) corresponding to the soft element we obtain a soft real number (F, E) such that $(F, E) = \{W_1 = 34,\ W_2 = 40,\ W_3 = 20, W_4 = 54,\ W_5 = 43\}$.

Definition 1.7 [7] "(Properties of Soft Real Numbers): Let $\tilde{r}, \tilde{s}$ be two soft real numbers. Then the following statements hold:
(i) $\tilde{r} \leq \tilde{s}$ if $\tilde{r}(e) \tilde{\leq} \tilde{s}(e)$ for all e ∈ E;
(ii) $\tilde{r} \geq \tilde{s}$ if $\tilde{r}(e) \tilde{\geq} \tilde{s}(e)$ for all e ∈ E;
(iii) $\tilde{r} \tilde{<} \tilde{s}$ if $\tilde{r}(e) \tilde{<} \tilde{s}(e)$ for all e ∈ E;
(iv) $\tilde{r} \tilde{>} \tilde{s}$ if $\tilde{r}(e) \tilde{>} \tilde{s}(e)$ for all e ∈ E."

The idea of soft point is characterized with various approaches. We employ the concept of soft point from [7] in our research.

Definition 1.8 [7]: "A soft set (F, E) over X is said to be a soft point if there is exactly one $e \in E$ such that F(e) ={u}, for some $u \epsilon \tilde{X}$ and F(e′) = ϕ, ∀ e′ ∈ E−{e}. It will be denoted by F_e^u or $\hat{u}_e$."
"The soft point $\hat{u}_e$ is said to be belonging to the soft set (F, E), denoted by $\hat{u}_e \epsilon (F, E)$, if $\hat{u}_e(e) \epsilon F(e)$, i.e., $\{u\} \subseteq F(e)$."
The concept of S-metric space was introduced by Sedghi *et al.* [27], as seen below.

Definition 1.10 [27]: "Let X be a non-empty set. An S-metric on X is a mapping $S: X \times X \times X \to \mathbb{R}^+$ which satisfies the following condition:
(S1) $S(x, y, z) = 0\ if\ and\ only\ if\ x = y = z = 0$;
(S2) $S(x, y, z) \leq S(x, x, a) + S(y, y, a) + S(z, z, a), for\ all\ x, y, z, a \epsilon X$.

Then the pair (X, S) is called an S-metric space."
Aras et al. [10] discussed soft S-metric spaces and their basic characteristics.
"Let $\tilde{X}$ be an absolute soft set, E be a non-empty set of parameters and $SP(\tilde{X})$ be the collection of all soft points of $\tilde{X}$. Let $\mathbb{R}(E)^*$ denotes the set of all non-negative soft real numbers."

Definition 1.11 [10]: "A soft S-metric on $\tilde{X}$ is a mapping $S: SP(\tilde{X}) \times SP(\tilde{X}) \times SP(\tilde{X}) \to \mathbb{R}(E)^*$ which satisfies the following conditions:

$(\bar{S_1})$ $S(\hat{u}_a, \hat{v}_b, \widehat{w}_c) \geq \bar{0}$;
$(\bar{S_2})$ $S(\hat{u}_a, \hat{v}_b, \widehat{w}_c) = \bar{0}$, if and only if $\hat{u}_a = \hat{v}_b = \widehat{w}_c$;
$(\bar{S_3})$ $S(\hat{u}_a, \hat{v}_b, \widehat{w}_c) \leq S(\hat{u}_a, \hat{u}_a, \hat{t}_d) + S(\hat{v}_b, \hat{v}_b, \hat{t}_d) + S(\widehat{w}_c, \widehat{w}_c, \hat{t}_d)$.

For all $\hat{u}_a, \hat{v}_b, \widehat{w}_c, \hat{t}_d \in SP(\tilde{X})$, then the soft set $\tilde{X}$ with a soft S-metric S is called soft S-metric space and denoted by $(\tilde{X}, S, E)$."

Lemma 1.12 [10]: "Let $(\tilde{X}, S, E)$ is a soft S-metric space. Then we have

$$S(\hat{u}_a, \hat{u}_a, \hat{v}_b) = S(\hat{v}_b, \hat{v}_b, \hat{u}_a)."$$

Definition 1.13 [10]: "A soft sequence $\{\hat{u}^n_{a_n}\}$ in $(\tilde{X}, S, E)$ converges to $\hat{v}_b$ if and only if $S(\hat{u}^n_{a_n}, \hat{u}^n_{a_n}, \hat{v}_b) \to \bar{0}\ as\ n \to \infty$ and we denote this by $\lim_{n\to\infty} \hat{u}^n_{a_n} = \hat{v}_b$."

Definition 1.14 [11]: "A soft sequence $\{\hat{u}^n_{a_n}\}$ in $(\tilde{X}, S, E)$ is called a Cauchy sequence if for $\tilde{\varepsilon} > \bar{0}$, there exists $n_0 \in \mathbb{N}$ such that $S(\hat{u}^n_{a_n}, \hat{u}^n_{a_n}, \hat{u}^m_{a_m}) < \tilde{\varepsilon}$ for each $m, n \geq n_0$."

Definition 1.15 [11]: "A soft S-metric space $(\tilde{X}, S, E)$ is said to be complete if every Cauchy sequence is convergent."

Definition 1.16 [11]: "Let $(T, \varphi): (\tilde{X}, S, E) \to (\tilde{Y}, S', E')$ be a soft mapping from soft S-metric space $(\tilde{X}, S, E)$ to a soft S-metric space $(\tilde{Y}, S', E')$. Then (T, φ) is soft continuous at a soft point $\hat{u}_a \in SP(\tilde{X})$ if and only if $(f, \varphi)(\{\hat{u}^n_{a_n}\}) \to (f, \varphi)(\hat{u}_a)$."

Definition 1.17 [11]: "Let $(\tilde{X}, S, E)$ be a soft S-metric space. A map $(T, \varphi): (\tilde{X}, S, E) \to (\tilde{X}, S, E)$ is said to be a soft contraction mapping if there exists a soft real number $\bar{k} \in \mathbb{R}(E), \bar{0} \leq \bar{k} < \bar{1}$ (where $\mathbb{R}(E)$ denotes the soft real number set) such that

$$S((T, \varphi)(\hat{u}_a), (T, \varphi)(\hat{u}_a), (T, \varphi)(\hat{v}_b)) \leq \bar{k}\, S(\hat{u}_a, \hat{u}_a, \hat{v}_b),$$

for all $\hat{u}_a, \hat{v}_b \in SP(\tilde{X})$."

2. Main Result

In this section, we investigate some fixed soft point theorems by introducing an implicit relation on soft S-metric space. Suppose $\widetilde{X_1} = R^5_+\ and\ \widetilde{X_2} = R_+$. Also, let $(M, \varphi): (\widetilde{X_1}, S, E) \to (\widetilde{X_2}, S, E)$ be a mapping of continuous function where $(\mathcal{M}, \varphi)$ denotes family of continuous functions in five variables and $(M, \varphi) \in (\mathcal{M}, \varphi)$. For some soft real number $\bar{k} \in [\bar{0}, \bar{1})$, we consider following conditions:
(A_1) If $\bar{y} \leq (M, \varphi)(\bar{x}, \bar{x}, \bar{0}, \bar{z}, \bar{y})$ with $\bar{z} \leq 2\,\bar{x} + \bar{y}$, then $\bar{y} \leq \bar{k}\,\bar{x}$, for all $\bar{x}, \bar{y}, \bar{z} \in \widetilde{X_2}$
(A_2) If $\bar{y} \leq (M, \varphi)(\bar{y}, \bar{0}, \bar{y}, \bar{y}, \bar{0})$, then $\bar{y} = \bar{0}$, for all $\bar{y} \in \widetilde{X_2}$,

(A_3) If $\bar{x}_i \leq \bar{y}_i + \bar{z}_i$ for all $\bar{x}_i, \bar{y}_i, \bar{z}_i \, \epsilon \, \widetilde{X_2}, i \leq 5$, then

$$(M,\varphi)(\bar{x}_1, \dots, \bar{x}_5) \leq (M,\varphi)(\bar{y}_1, \dots, \bar{y}_5) + (M,\varphi)(\bar{z}_1, \dots, \bar{z}_5).$$

Moreover, for all $\bar{y} \, \epsilon \, \widetilde{X_2}, (M,\varphi)(\bar{0}, \bar{0}, \bar{0}, \bar{y}, 2\,\bar{y}) \leq \bar{k}\,\bar{y}$.

Remark: Note that $\bar{k}$ defined in the condition (A_1) and (A_3) may be different, for e.g., $\bar{k}_1$and $\bar{k}_2$ respectively. Here, we suppose that both values of $\bar{k}_1 \, and \, \bar{k}_2$ are equal by considering them as $\bar{k} = \max\{\bar{k}_1, \bar{k}_2\}$.

Theorem 2.1: Let (T,φ) be a mapping on a complete soft S-metric space $(\tilde{X}, S, E)$ into itself and

$$S\big((T,\varphi)(\hat{u}_a), (T,\varphi)(\hat{u}_a), (T,\varphi)(\hat{v}_b)\big)$$

$$\leq (M,\varphi)\begin{pmatrix} S(\hat{u}_a, \hat{u}_a, \hat{v}_b), S\big((T,\varphi)(\hat{u}_a), (T,\varphi)(\hat{u}_a), \hat{u}_a\big), S\big((T,\varphi)(\hat{u}_a), (T,\varphi)(\hat{u}_a), \hat{v}_b\big), \\ S\big((T,\varphi)(\hat{v}_b), (T,\varphi)(\hat{v}_b), \hat{u}_a\big), S\big((T,\varphi)(\hat{v}_b), (T,\varphi)(\hat{v}_b), \hat{v}_b\big) \end{pmatrix}, \quad (2.1)$$

for all $\hat{u}_a, \hat{v}_b \, \epsilon \, SP(\tilde{X})$ and $(M,\varphi) \, \epsilon \, (\mathcal{M},\varphi)$. Also suppose that (2.2) (T,φ) has a fixed soft point, if the condition (A_1) is satisfied by (M,φ). Furthermore, for any $\hat{u}_a^0 \, \epsilon \, SP(\tilde{X})$ and fixed soft point $\widehat{w}_c$, we have

$$S\big((T,\varphi)(\hat{u}_{a_n}^n), (T,\varphi)(\hat{u}_{a_n}^n), \widehat{w}_c\big) \leq \frac{2\bar{k}^n}{1-\bar{k}} S\big(\hat{u}_a, \hat{u}_a, (T,\varphi)(\hat{u}_a)\big).$$

(2.3) the fixed soft point is unique, if the condition (A_2) is satisfied by (M,φ) and (T,φ) has a fixed soft point.
(2.4) (T,φ) is soft continuous at $\widehat{w}_c$, if the condition (A_3) is satisfied by (M,φ) and (T,φ) has a fixed soft point $\widehat{w}_c$.

Proof: For each $\hat{u}_a \, \epsilon \, SP(\tilde{X})$ and $n \, \epsilon \, \mathbb{N}$, we define a soft sequence $\{\hat{u}_{a_n}^n\}$ as $\hat{u}_{a_1}^1 = (T,\varphi)(\hat{u}_a)$, …, $\hat{u}_{a_{n+1}}^{n+1} = (T,\varphi)(\hat{u}_{a_n}^n)$. It follows from (2.1) that

$$S\big(\hat{u}_{a_{n+1}}^{n+1}, \hat{u}_{a_{n+1}}^{n+1}, \hat{u}_{a_{n+2}}^{n+2}\big) = S\left((T,\varphi)(\hat{u}_{a_n}^n), (T,\varphi)(\hat{u}_{a_n}^n), (T,\varphi)(\hat{u}_{a_{n+1}}^{n+1})\right)$$

$$\leq (M,\varphi)(S\big(\hat{u}_{a_n}^n, \hat{u}_{a_n}^n, \hat{u}_{a_{n+1}}^{n+1}\big), S\big(\hat{u}_{a_{n+1}}^{n+1}, \hat{u}_{a_{n+1}}^{n+1}, \hat{u}_{a_n}^n\big), S\big(\hat{u}_{a_{n+1}}^{n+1}, \hat{u}_{a_{n+1}}^{n+1}, \hat{u}_{a_{n+1}}^{n+1}\big), S\big(\hat{u}_{a_{n+2}}^{n+2}, \hat{u}_{a_{n+2}}^{n+2}, \hat{u}_{a_n}^n\big),$$

$$S\big(\hat{u}_{a_{n+2}}^{n+2}, \hat{u}_{a_{n+2}}^{n+2}, \hat{u}_{a_{n+1}}^{n+1}\big))$$

$$= (M,\varphi)\left(S\big(\hat{u}_{a_n}^n, \hat{u}_{a_n}^n, \hat{u}_{a_{n+1}}^{n+1}\big), S\big(\hat{u}_{a_n}^n, \hat{u}_{a_n}^n, \hat{u}_{a_{n+1}}^{n+1}\big), \bar{0}, S\big(\hat{u}_{a_n}^n, \hat{u}_{a_n}^n, \hat{u}_{a_{n+2}}^{n+2}\big), S\big(\hat{u}_{a_{n+1}}^{n+1}, \hat{u}_{a_{n+1}}^{n+1}, \hat{u}_{a_{n+2}}^{n+2}\big)\right).$$

From $(\bar{S_3})$ and Lemma 1.12 we have

$$S\left(\hat{u}_{a_n}^n, \hat{u}_{a_n}^n, \hat{u}_{a_{n+2}}^{n+2}\right) \leq 2S\left(\hat{u}_{a_n}^n, \hat{u}_{a_n}^n, \hat{u}_{a_{n+1}}^{n+1}\right) + S\left(\hat{u}_{a_{n+2}}^{n+2}, \hat{u}_{a_{n+2}}^{n+2}, \hat{u}_{a_{n+1}}^{n+1}\right)$$

$$= 2S\left(\hat{u}_{a_n}^n, \hat{u}_{a_n}^n, \hat{u}_{a_{n+1}}^{n+1}\right) + S\left(\hat{u}_{a_{n+1}}^{n+1}, \hat{u}_{a_{n+1}}^{n+1}, \hat{u}_{a_{n+2}}^{n+2}\right).$$

Since (M, φ) satisfies the condition (A_1), there exist $\bar{k} \epsilon [\bar{0}, \bar{1})$ such that

$$S\left(\hat{u}_{a_{n+1}}^{n+1}, \hat{u}_{a_{n+1}}^{n+1}, \hat{u}_{a_{n+2}}^{n+2}\right) \leq \bar{k}\, S\left(\hat{u}_{a_n}^n, \hat{u}_{a_n}^n, \hat{u}_{a_{n+1}}^{n+1}\right) \leq \bar{k}^{n+1} S(\hat{u}_a, \hat{u}_a, \hat{u}_{a_1}^1). \quad (2.5)$$

Thus, for all $n < m$, by using $(\bar{S_3})$, Lemma 1.12 and (2.5), we have

$$S\left(\hat{u}_{a_n}^n, \hat{u}_{a_n}^n, \hat{u}_{a_m}^m\right) \leq 2\, S\left(\hat{u}_{a_n}^n, \hat{u}_{a_n}^n, u_{a_{n+1}}^{n+1}\right) + S\left(\hat{u}_{a_m}^m, \hat{u}_{a_m}^m, \hat{u}_{a_{n+1}}^{n+1}\right)$$

$$= 2\, S\left(\hat{u}_{a_n}^n . \hat{u}_{a_n}^n, \hat{u}_{a_{n+1}}^{n+1}\right) + S\left(\hat{u}_{a_{n+1}}^{n+1}, \hat{u}_{a_{n+1}}^{n+1}, \hat{u}_{a_m}^m\right).$$

$$\dots$$

$$\leq 2\left[\bar{k}^{\,n} + \cdots + \bar{k}^{\,m-1}\right] S(\hat{u}_a, \hat{u}_a, \hat{u}_{a_1}^1)$$

$$\leq \frac{2\bar{k}^{\,n}}{1-\bar{k}}\, S\left(\hat{u}_a, \hat{u}_a, \hat{u}_{a_1}^1\right).$$

Thus, we have

$$\left\|S\left(\hat{u}_{a_n}^n, \hat{u}_{a_n}^n, \hat{u}_{a_m}^m\right)\right\| \leq \frac{2\bar{k}^n . \bar{k}}{1-\bar{k}} \left\|S\left(\hat{u}_a, \hat{u}_a, \hat{u}_{a_1}^1\right)\right\|.$$

Now, taking the limit as $n, m \to \infty$ we get $S\left(\hat{u}_{a_n}^n, \hat{u}_{a_n}^n, \hat{u}_{a_m}^m\right) \to \tilde{0}$, as $\bar{0} < \bar{k} < \bar{1}$. This proves that $\{\hat{u}_{a_n}^n\}$ is a Cauchy sequence. Also, as $(\tilde{X}, S, E)$ is complete it follows that there exists soft point $\hat{w}_c \,\epsilon\, SP(\tilde{X})$ such that $\hat{u}_{a_n}^n \to \hat{w}_c$.
Moreover, taking the limit as $m \to \infty$ we get

$$S\left(\hat{u}_{a_n}^n, \hat{u}_{a_n}^n, \hat{w}_c\right) \leq \frac{2\bar{k}^{n+1}}{1-\bar{k}}\, S\left(\hat{u}_a, \hat{u}_a, \hat{u}_{a_1}^1\right).$$

It implies that

$$S\left((T,\varphi)(\hat{u}_{a_n}^n), (T,\varphi)(\hat{u}_{a_n}^n), \hat{w}_c\right) \leq \frac{2\bar{k}^n}{1-\bar{k}}\, S\left(\hat{u}_a, \hat{u}_a, (T,\varphi)(\hat{u}_a)\right).$$

Now we shall prove that (T, φ) has a fixed point $\hat{w}_c$.

From (2.1) we obtain

$$S\left(\hat{u}^{n+1}_{a_{n+1}}, \hat{u}^{n+1}_{a_{n+1}}, (T,\varphi)(\widehat{w}_c)\right)$$

$$= S\big((T,\varphi)(\hat{u}^{n}_{a_n}), (T,\varphi)(\hat{u}^{n}_{a_n}), (T,\varphi)(\widehat{w}_c)\big)$$

$$\leq (M,\varphi)(S\big(\hat{u}^{n}_{a_n}, \hat{u}^{n}_{a_n}, \widehat{w}_c\big), S\big((T,\varphi)(\hat{u}^{n}_{a_n}), (T,\varphi)(\hat{u}^{n}_{a_n}), \widehat{w}_c\big), S\big((T,\varphi)(\hat{u}^{n}_{a_n}), (T,\varphi)(\hat{u}^{n}_{a_n}), \hat{u}^{n}_{a_n}\big),$$

$$S\left((T,\varphi)(\widehat{w}_c), (T,\varphi)(\widehat{w}_c), \hat{u}^{n}_{a_n}\right), S\big((T,\varphi)(\widehat{w}_c), (T,\varphi)(\widehat{w}_c), \widehat{w}_c\big)$$

$$= (M,\varphi)(S\big(\hat{u}^{n}_{a_n}, \hat{u}^{n}_{a_n}, \widehat{w}_c\big), S\big(\hat{u}^{n+1}_{a_{n+1}}, \hat{u}^{n+1}_{a_{n+1}}, \widehat{w}_c\big), S\big(\hat{u}^{n+1}_{a_{n+1}}, \hat{u}^{n+1}_{a_{n+1}}, \widehat{w}_c\big),$$

$$S\big((T,\varphi)(\widehat{w}_c), (T,\varphi)(\widehat{w}_c), \widehat{w}_c\big))$$

Note that $(M,\varphi) \,\epsilon\, (\mathcal{M},\varphi)$, then using Lemma 1.13 and taking the limit as $n \to \infty$ we obtain

$$S(\widehat{w}_c, \widehat{w}_c, (T,\varphi)(\widehat{w}_c)) \leq (M,\varphi)(\bar{0}, \bar{0}, \bar{0}, S\big((T,\varphi)(\widehat{w}_c), (T,\varphi)(\widehat{w}_c), \widehat{w}_c\big),$$

$$S\big((T,\varphi)(\widehat{w}_c), (T,\varphi)(\widehat{w}_c), \widehat{w}_c\big)).$$

Then, from Lemma 1.12 we get

$$S(\widehat{w}_c, \widehat{w}_c, (T,\varphi)(\widehat{w}_c)) \leq (M,\varphi)\big(\bar{0}, \bar{0}, \bar{0}, S(\widehat{w}_c, \widehat{w}_c, (T,\varphi)(\widehat{w}_c)), S(\widehat{w}_c, \widehat{w}_c, (T,\varphi)(\widehat{w}_c))\big).$$

Since (M,φ) satisfies the condition (A_1), then $S(\widehat{w}_c, \widehat{w}_c, (T,\varphi)(\widehat{w}_c)) \leq \bar{k}.\bar{0} = \bar{0}$. This proves that $\widehat{w}_c = (T,\varphi)(\widehat{w}_c)$.

Now we shall prove the uniqueness of fixed point. For that let $\hat{v}_b, \widehat{w}_c \,\epsilon\, SP(\tilde{X})$ be two fixed point of (T,φ). It follows from (2.1) and Lemma 1.12 that

$$S(\hat{v}_b, \hat{v}_b, \widehat{w}_c)$$

$$= S\big((T,\varphi)(\hat{v}_b), (T,\varphi)(\hat{v}_b), (T,\varphi)(\widehat{w}_c)\big)$$

$$\leq (M,\varphi)(S(\hat{v}_b, \hat{v}_b, \widehat{w}_c), S\big((T,\varphi)(\hat{v}_b), (T,\varphi)(\hat{v}_b), \hat{v}_b\big), S\big((T,\varphi)(\hat{v}_b), (T,\varphi)(\hat{v}_b), \widehat{w}_c\big),$$

$$S\big((T,\varphi)(\widehat{w}_c), (T,\varphi)(\widehat{w}_c), \hat{v}_b\big), S\big((T,\varphi)(\widehat{w}_c), (T,\varphi)(\widehat{w}_c), \widehat{w}_c\big))$$

$$= (M,\varphi)(S(\hat{v}_b, \hat{v}_b, \widehat{w}_c), \bar{0}, S(\hat{v}_b, \hat{v}_b, \widehat{w}_c), S(\widehat{w}_c, \widehat{w}_c, \hat{v}_b), \bar{0})$$

$$= (M,\varphi)(S(\hat{v}_b, \hat{v}_b, \widehat{w}_c), \bar{0}, S(\hat{v}_b, \hat{v}_b, \widehat{w}_c), S(\hat{v}_b, \hat{v}_b, \widehat{w}_c), \bar{0}).$$

Since (M,φ) satisfies the condition (A_2), then $S(\hat{v}_b,\hat{v}_b,\widehat{w}_c)=\bar{0}$. This proves that $\hat{v}_b=\widehat{w}_c$.
Next, we show that $(T,\varphi)(\hat{u}^n_{a_n})\to (f,\varphi)(\widehat{w}_c)$.
Let $\widehat{w}_c$ be the fixed point of (T,φ) and $\{\hat{u}^n_{a_n}\}\to\widehat{w}_c\,\epsilon\,SP(\tilde{X})$. It follows from (2.1) that

$$S\left(\widehat{w}_c,\widehat{w}_c,(T,\varphi)(\hat{u}^n_{a_n})\right)$$

$$=S\left((T,\varphi)(\widehat{w}_c),(T,\varphi)(\widehat{w}_c),(T,\varphi)(\hat{u}^n_{a_n})\right)$$

$$\le (M,\varphi)\begin{pmatrix} S(\widehat{w}_c,\widehat{w}_c,\hat{u}^n_{a_n}),S((T,\varphi)(\widehat{w}_c),(T,\varphi)(\widehat{w}_c),\widehat{w}_c),S\left((T,\varphi)(\widehat{w}_c),(T,\varphi)(\widehat{w}_c),\hat{u}^n_{a_n}\right), \\ S((T,\varphi)(\hat{u}^n_{a_n}),(T,\varphi)(\hat{u}^n_{a_n}),\widehat{w}_c),S\left((T,\varphi)(\hat{u}^n_{a_n}),(T,\varphi)(\hat{u}^n_{a_n}),\hat{u}^n_{a_n}\right)\end{pmatrix}$$

$$=(M,\varphi)\begin{pmatrix} S(\widehat{w}_c,\widehat{w}_c,\hat{u}^n_{a_n}),\bar{0},S(\widehat{w}_c,\widehat{w}_c,\hat{u}^n_{a_n}),S((T,\varphi)(\hat{u}^n_{a_n}),(T,\varphi)(\hat{u}^n_{a_n}),\widehat{w}_c), \\ S\left((T,\varphi)(\hat{u}^n_{a_n}),(T,\varphi)(\hat{u}^n_{a_n}),\hat{u}^n_{a_n}\right)\end{pmatrix}.$$

Since (M,φ) satisfies the condition (A_3) and by $(\bar{S_3})$

$$S\left((T,\varphi)(\hat{u}^n_{a_n}),(T,\varphi)(\hat{u}^n_{a_n}),\hat{u}^n_{a_n}\right)\le 2\,S((T,\varphi)(\hat{u}^n_{a_n}),(T,\varphi)(\hat{u}^n_{a_n}),\widehat{w}_c)+S(\hat{u}^n_{a_n},\hat{u}^n_{a_n},\widehat{w}_c).$$

Thus, we get

$$S\left(\widehat{w}_c,\widehat{w}_c,(T,\varphi)(\hat{u}^n_{a_n})\right)$$

$$\le (M,\varphi)\left(S(\widehat{w}_c,\widehat{w}_c,\hat{u}^n_{a_n}),\bar{0},S(\widehat{w}_c,\widehat{w}_c,\hat{u}^n_{a_n}),\bar{0},S(\widehat{w}_c,\widehat{w}_c,\hat{u}^n_{a_n})\right)$$

$$+(M,\varphi)\left(\bar{0},\bar{0},\bar{0},S((T,\varphi)(\hat{u}^n_{a_n}),(T,\varphi)(\hat{u}^n_{a_n}),\widehat{w}_c),2\,S((T,\varphi)(\hat{u}^n_{a_n}),(T,\varphi)(\hat{u}^n_{a_n}),\widehat{w}_c)\right)$$

$$\le (M,\varphi)\left(S(\widehat{w}_c,\widehat{w}_c,\hat{u}^n_{a_n}),\bar{0},S(\widehat{w}_c,\widehat{w}_c,\hat{u}^n_{a_n}),\bar{0},S(\widehat{w}_c,\widehat{w}_c,\hat{u}^n_{a_n})\right)$$

$$+\tilde{k}\,S((T,\varphi)(\hat{u}^n_{a_n}),(T,\varphi)(\hat{u}^n_{a_n}),\widehat{w}_c).$$

Therefore, we have

$$S\left(\widehat{w}_c,\widehat{w}_c,(T,\varphi)(\hat{u}^n_{a_n})\right)\le\frac{1}{1-\tilde{k}}(M,\varphi)\left(S(\widehat{w}_c,\widehat{w}_c,\hat{u}^n_{a_n}),\bar{0},S(\widehat{w}_c,\widehat{w}_c,\hat{u}^n_{a_n}),\bar{0},S(\widehat{w}_c,\widehat{w}_c,\hat{u}^n_{a_n})\right).$$

Note that as $(M,\varphi)\,\epsilon\,(\mathcal{M},\varphi)$, therefore taking the limit as $n\to\infty$ we obtain $S\left(\widehat{w}_c,\widehat{w}_c,(T,\varphi)(\hat{u}^n_{a_n})\right)\to\bar{0}$. This proves that $(T,\varphi)(\hat{u}^n_{a_n})\to\widehat{w}_c=(T,\varphi)(\widehat{w}_c)$.
Here completes the proof. ■

Corollary 2.2: Let $(\tilde{X}, S, E)$ be a complete soft S-metric space and (T,φ) be a self map on $(\tilde{X}, S, E)$ which satisfies the condition

$$S\big((T,\varphi)(\hat{u}_a),(T,\varphi)(\hat{u}_a),(T,\varphi)(\hat{v}_b)\big)$$

$$\leq \bar{k}\,\max\big\{S\big((T,\varphi)(\hat{u}_a),(T,\varphi)(\hat{u}_a),\hat{u}_a\big),S\big((T,\varphi)(\hat{v}_b),(T,\varphi)(\hat{v}_b),\hat{v}_b\big)\big\}, \quad (2.6)$$

for some $\bar{k} \in [\bar{0},\bar{1})$ and all $\hat{u}_a, \hat{v}_b \,\epsilon\, SP(\tilde{X})$. Then (T,φ) possess a unique fixed soft point in $\tilde{X}$. Further, if $\bar{k} \in \left[\bar{0},\frac{\bar{1}}{2}\right)$, then (T,φ) is soft continuous at the fixed point.

Proof: The proof follows from the proof of Theorem 2.1 with $(M,\varphi)(\bar{x},\bar{y},\bar{z},\bar{s},\bar{t}) = \bar{k}\max\{\bar{y},\bar{t}\}$, for all $\bar{x},\bar{y},\bar{z},\bar{s},\bar{t} \,\epsilon\, \widetilde{X_2}$ and $\bar{k} \in [\bar{0},\bar{1})$. Certainly (M,φ) is continuous.
Initially, we have $(M,\varphi)(\bar{x},\bar{x},\bar{0},\bar{z},\bar{y}) = \bar{k}\max\{\bar{x},\bar{y}\}$.
So, if $\bar{y} \leq (M,\varphi)(\bar{x},\bar{x},\bar{0},\bar{z},\bar{y})$ with $\bar{z} \leq 2\,\bar{x}+\bar{y}$, then $\bar{y} \leq \bar{k}\,\bar{x}$ or $\bar{y} \leq \bar{k}\,\bar{y}$. Therefore, $\bar{y} \leq \bar{k}.\bar{x}$. Hence (T,φ) satisfies the condition (A_1).
Next, if $\bar{y} \leq (M,\varphi)(\bar{y},\bar{0},\bar{y},\bar{y},\bar{0}) = \bar{k}\max\{\bar{y},\bar{0}\} = \bar{k}\,\bar{y}$, then $\bar{y} = \bar{0}$ as $\bar{k} < \frac{\bar{1}}{2}$.
Thus, (T,φ) satisfies the condition (A_2).
Finally, if $\bar{x}_i \leq \bar{y}_i + \bar{z}_i$ for $i \leq 5$, then

$$(M,\varphi)(\bar{x}_1,\dots,\bar{x}_5) = \bar{k}\max\{\bar{x}_2,\bar{x}_5\}$$

$$\leq \bar{k}\max\{\bar{y}_2+\bar{z}_2,\bar{y}_5+\bar{z}_5\}$$

$$\leq \bar{k}\max\{\bar{y}_2,\bar{y}_5\} + \tilde{k}\max\{\bar{z}_2,\bar{z}_5\}$$

$$= (M,\varphi)(\bar{y}_1,\dots,\bar{y}_5) + (M,\varphi)(\bar{z}_1,\dots,\bar{z}_5).$$

Also, if $\bar{k} \in \left[\bar{0},\frac{\bar{1}}{2}\right)$, then we have $\bar{2}\,\bar{y} < \bar{1}$ and $(M,\varphi)(\bar{0},\bar{0},\bar{0},\bar{y},\bar{2}\,\bar{y}) \leq \bar{k}\max\{\bar{0},\bar{2}\,\bar{y}\} = \bar{k}\,\bar{2}\,\bar{y}$. Hence, (T,φ) satisfies the condition (A_3).
Example 2.3: Let $\tilde{X} = [\bar{0},\bar{1})$ and $E = \mathbb{N}$ be a parameter set, we define a soft S-metric space by

$$S(\hat{u}_a,\hat{v}_b,\hat{w}_c) = |a-c| + |b-c| + |u-w| + |v-w|.$$

Let (T,φ) be a soft mapping $(T,\varphi)\colon (\tilde{X},S,E) \to (\tilde{X},S,E)$, define as $(T,\varphi)(\hat{u}_a) = \left(\frac{u}{4}\right)_1$, where $T(u) = \frac{u}{4}$ and $\varphi(a) = 1$ are constant functions. We have

$$S\big((T,\varphi)(\hat{u}_a),(T,\varphi)(\hat{u}_a),(T,\varphi)(\hat{v}_b)\big) = S\left(\left(\frac{u}{4}\right)_1,\left(\frac{u}{4}\right)_1,\left(\frac{v}{4}\right)_1\right)$$

$$= \left|\frac{u}{4} - \frac{v}{4}\right| + \left|\frac{u}{4} - \frac{v}{4}\right| = 2\left|\frac{u}{4} - \frac{v}{4}\right|$$

$$= \frac{1}{2}|u - v|.$$

$$S\big((T,\varphi)(\hat{u}_a),(T,\varphi)(\hat{u}_a),\hat{u}_a\big) = S\left(\left(\frac{u}{4}\right)_1, \left(\frac{u}{4}\right)_1, (u)_1\right) = 2\left|\frac{u}{4} - u\right| = \frac{3}{2}|u|.$$

$$S\big((T,\varphi)(\hat{v}_b),(T,\varphi)(\hat{v}_b),\hat{v}_b\big) = S\left(\left(\frac{v}{4}\right)_1, \left(\frac{v}{4}\right)_1, (v)_1\right) = 2\left|\frac{v}{4} - v\right| = \frac{3}{2}|v|.$$

Thus, from condition (2.3) we obtain

$$\frac{1}{2}|u - v| \leq \max\left\{\frac{3}{2}|u|, \frac{3}{2}|v|\right\}.$$

Hence, for $\bar{k} = \frac{\bar{2}}{3}$, which implies that (T,φ) satisfies the condition of Corollary 2.2. Thus $w_c = \bar{0}$ is the unique fixed soft point of (T,φ).

Corollary 2.4: Let (T,φ) is a self mapping on a complete soft S-metric space $(\tilde{X}, S, E)$ which satisfies the following soft contraction condition:

$$S\big((T,\varphi)(\hat{u}_a),(T,\varphi)(\hat{u}_a),(T,\varphi)(\hat{v}_b)\big)$$

$$\leq \bar{k}\max\{S\big((T,\varphi)(\hat{u}_a),(T,\varphi)(\hat{u}_a),\hat{v}_b\big), S\big((T,\varphi)(\hat{v}_b),(T,\varphi)(\hat{v}_b),\hat{u}_a\big)\}, \quad (2.7)$$

for some $\bar{k} \in \left[\bar{0}, \frac{\bar{1}}{3}\right)$ and all $\hat{u}_a, \hat{v}_b \in SP(\tilde{X})$. Then (T,φ) possess unique fixed soft point in $\tilde{X}$. Also, (T,φ) is soft continuous at the fixed soft point.

Proof: Proof follows from proof of Theorem 2.1 with $(M,\varphi)(\bar{x},\bar{y},\bar{z},\bar{s},\bar{t}) = \bar{k}\max\{\bar{z},\bar{s}\}$ for all $\bar{x},\bar{y},\bar{z},\bar{s},\bar{t} \in \widetilde{X_2}$ and $\bar{k} \in \left[\bar{0}, \frac{\bar{1}}{3}\right)$. Indeed (M,φ) is continuous.
Initially, we have $(M,\varphi)(\bar{x},\bar{x},\bar{0},\bar{z},\bar{y}) = \bar{k}\max\{\bar{0},\bar{z}\}$.
$\bar{y} \leq (M,\varphi)(\bar{x},\bar{x},\bar{0},\bar{z},\bar{y})$ with $\bar{z} \leq 2\bar{x} + \bar{y}$, then $\bar{y} \leq \bar{k}\bar{x} + \bar{y}\,\bar{k}$.
which implies that $\bar{y} \leq \frac{2\bar{k}}{1-\bar{k}}\bar{x}$, where $\frac{2\bar{k}}{1-\bar{k}} < \bar{1}$.
Hence, (T,φ) satisfies the condition (A_1).
For next condition, if $\bar{y} \leq (M,\varphi)(\bar{y},\bar{0},\bar{y},\bar{y},\bar{0}) = \bar{k}\,\bar{y}$, then $\bar{y} = \bar{0}$.
Since $\bar{k} < \frac{\bar{1}}{3}$ therefore, (T,φ) satisfies the condition (A_2).
Finally, if $\bar{x}_i \leq \bar{y}_i + \bar{z}_i$ for $i \leq 5$, then

$$(M,\varphi)(\bar{x}_1,\ldots,\bar{x}_5) = \bar{k}\max\{\bar{x}_3,\bar{x}_4\}$$

$$\leq \bar{k} \max\{\bar{y}_3 + \bar{z}_3, \bar{y}_4 + \bar{z}_4\}$$

$$\leq \bar{k} \max\{\bar{y}_3, \bar{y}_4\} + \bar{k} \max\{\bar{z}_3, \bar{z}_4\}$$

$$= (M,\varphi)(\bar{y}_1, \dots, \bar{y}_5) + (M,\varphi)(\bar{z}_1, \dots, \bar{z}_5).$$

Moreover, $(M,\varphi)(\bar{0},\bar{0},\bar{0},\bar{y},\bar{2}\,\bar{y}) \leq \bar{k} \max\{\bar{0}, \bar{2}\bar{y}\} = \bar{k}\,\bar{y}$, where $\bar{k} < \bar{1}$. Thus, (T,φ) satisfies the condition (A_3).

Corollary 2.5: Consider a self mapping (T,φ) on complete soft S-metric space $(\tilde{X}, S, E)$ which satisfies the following condition:

$$S\big((T,\varphi)(\hat{u}_a), (T,\varphi)(\hat{u}_a), (T,\varphi)(\hat{v}_b)\big)$$

$$\leq \bar{h} \max\left\{\begin{matrix} S(\hat{u}_a, \hat{v}_b, \widehat{w}_c), S\big((T,\varphi)(\hat{u}_a), (T,\varphi)(\hat{u}_a), \hat{u}_a\big), S\big((T,\varphi)(\hat{u}_a), (T,\varphi)(\hat{u}_a), \hat{v}_b\big), \\ S\big((T,\varphi)(\hat{v}_b), (T,\varphi)(\hat{v}_b), \hat{u}_a\big), S\big((T,\varphi)(\hat{v}_b), (T,\varphi)(\hat{v}_b), \hat{v}_b\big) \end{matrix}\right\},$$

for some $\bar{h} \in \left[\bar{0}, \frac{\bar{1}}{3}\right)$ and all $\hat{u}_a, \hat{v}_b \,\epsilon\, SP(\tilde{X})$. Then (T,φ) possess a unique fixed point in $(\tilde{X}, S, E)$. Also, (T,φ) is continuous at the fixed point.

Proof: The proof follows using the above Theorem 2.1 with $(M,\varphi)(\bar{x},\bar{y},\bar{z},\bar{s},\bar{t}) = \bar{h} \max\{\bar{x},\bar{y},\bar{z},\bar{s},\bar{t}\}$ $for\ all\ \bar{x},\bar{y},\bar{z},\bar{s},\bar{t} \,\epsilon\, \widetilde{X_2}$ and $\bar{h} \in \left[\bar{0}, \frac{\bar{1}}{3}\right)$.
Actually (M,φ) is continuous. Firstly, we have

$$(M,\varphi)(\bar{x},\bar{x},\bar{0},\bar{z},\bar{y}) = \bar{h} \max\{\bar{x},\bar{x},\bar{0},\bar{z},\bar{y}\}.$$

So, if $\bar{y} \leq (M,\varphi)(\bar{x},\bar{x},\bar{0},\bar{z},\bar{y})$ with $\bar{z} \leq 2\,\bar{x} + \bar{y}$, then $\bar{y} \leq \bar{h}\,\bar{x}$ or $\bar{y} \leq \bar{h}\,\bar{z} \leq \bar{h}\,(2\,\bar{x} + \bar{y})$. Then we get $\bar{y} \leq \bar{k}\,\bar{x}$, with $\bar{k} = \max\left\{\bar{h}, \frac{2\bar{h}}{1-\bar{h}}\right\} < \bar{1}$.
Thus, condition (A_2) is satisfied by (T,φ).
Secondly, if $\bar{y} \leq (M,\varphi)(\bar{y},\bar{0},\bar{y},\bar{y},\bar{0}) = \bar{h}\,\bar{y}$, then $\bar{y} = \bar{0}$ as $\bar{h} < \frac{\bar{1}}{3}$. Therefore, (T,φ) satisfies the condition (A_2).
Lastly, if $\bar{x}_i \leq \bar{y}_i + \bar{z}_i$, for $i \leq 5$, then

$$(M,\varphi)(\bar{x}_1, \dots, \bar{x}_5) = \bar{h} \max\{\bar{x}_1, \dots, \bar{x}_5\}$$

$$\leq \bar{h} \max\{\bar{y}_1 + \bar{z}_1, \dots, \bar{y}_5 + \bar{z}_5\}$$

$$\leq \bar{h} \max\{\bar{y}_1, \dots, \bar{y}_5\} + \leq \bar{h} \max\{\bar{z}_1, \dots, \bar{z}_5\}$$

$$= (M,\varphi)(\bar{y}_1, \dots, \bar{y}_5) + (M,\varphi)(\bar{z}_1, \dots, \bar{z}_5).$$

Moreover,
$(M,\varphi)(\bar{0},\bar{0},\bar{0},\bar{y},2\,\bar{y}) = 2\bar{h}\,\bar{y}$, where $2\,\bar{h} < \bar{1}$.
Thus, condition (A_3) is satisfied.

Corollary 2.6: Let $(\tilde{X},S,E)$ be a complete soft S-metric space and (T,φ) be a mapping on $(\tilde{X},S,E)$ into itself which satisfies

$$S\big((T,\varphi)(\hat{u}_a),(T,\varphi)(\hat{u}_a),(T,\varphi)(\hat{v}_b)\big)$$

$$\leq \bar{\alpha}\,S(\hat{u}_a,\hat{v}_b,\widehat{w}_c) + \bar{\beta}\,S\big((T,\varphi)(\hat{u}_a),(T,\varphi)(\hat{u}_a),\hat{u}_a\big)$$

$$+\bar{\gamma}\,S\big((T,\varphi)(\hat{v}_b),(T,\varphi)(\hat{v}_b),\hat{v}_b\big), \quad (2.8)$$

for some and all $\hat{u}_a,\hat{v}_b \,\epsilon\, SP(\tilde{X})$,. Then (T,φ) has unique fixed point in $(\tilde{X},S,E)$. Also, if $\bar{\gamma} < \frac{\bar{1}}{2}$, then (T,φ) is soft continuous at that fixed point.

Proof: Proof follows using the above Theorem 2.1 along with $(M,\varphi)(\bar{x},\bar{y},\bar{z},\bar{s},\bar{t}) = \bar{\alpha}\,\bar{x} + \bar{\beta}\,\bar{y} + \bar{\gamma}\,\bar{t}$, for some $\bar{\alpha},\bar{\beta},\bar{\gamma} \geq \bar{0}, \bar{\alpha}+\bar{\beta}+\bar{\gamma} < \bar{1}, where\ \bar{x},\bar{y},\bar{z},\bar{s},\bar{t}\,\epsilon\,\widetilde{X_2}$..
Actually (M,φ) is continuous. Now we have $(M,\varphi)(\bar{x},\bar{y},\bar{z},\bar{s},\bar{t}) = \bar{\alpha}\,\bar{x} + \bar{\beta}\,\bar{x} + \bar{\gamma}\,\bar{y}$.
So, if $\bar{y} \leq (M,\varphi)(\bar{x},\bar{x},\bar{0},\bar{z},\bar{y})$ with $\bar{z} \leq 2\,\bar{x} + \bar{y}$, then $\bar{y} \leq \frac{\bar{\alpha}+\bar{\beta}}{1-\bar{\gamma}}\bar{x}$, with $\frac{\bar{\alpha}+\bar{\beta}}{1-\bar{\gamma}} < \bar{1}$.
Thus, condition (A_2) is satisfied by (T,φ).
Secondly, if $\bar{y} \leq (M,\varphi)(\bar{y},\bar{0},\bar{y},\bar{y},\bar{0}) = \bar{\alpha}\,\bar{y}$, then $\bar{y} = \bar{0}$ as $\bar{\alpha} < \bar{1}$. Therefore, (T,φ) satisfies the condition (A_2).
Lastly, if $\bar{x}_i \leq \bar{y}_i + \bar{z}_i$ for $i \leq 5$, then

$$(M,\varphi)(\bar{x}_1,\dots,\bar{x}_5) = \bar{\alpha}\,\bar{x}_1 + \bar{\beta}\,\bar{x}_3 + \bar{\gamma}\,\bar{x}_4$$

$$\leq \bar{\alpha}(\bar{y}_1+\bar{z}_1) + \bar{\beta}\,(\bar{y}_3+\bar{z}_3) + \bar{\gamma}\,(\bar{y}_4+\bar{z}_4)$$

$$= \big(\bar{\alpha}\,\bar{y}_1 + \bar{\beta}\,\bar{y}_3 + \bar{\gamma}\,\bar{y}_4\big) + (\bar{\alpha}\,\bar{z}_1 + \bar{\beta}\,\bar{z}_3 + \bar{\gamma}\,\bar{z}_4)$$

$$= (M,\varphi)(\bar{y}_1,\dots,\bar{y}_5) + (M,\varphi)(\bar{z}_1,\dots,\bar{z}_5).$$

Moreover, $(M,\varphi)(\bar{0},\bar{0},\bar{0},\bar{y},2\,\bar{y}) = \bar{\gamma}\,\bar{y}$, where $\bar{\gamma} < \bar{1}$. Thus, condition (A_3) is satisfied.

Example 2.7: Let $\tilde{X} = [\bar{0},\bar{1})\ and\ E = \mathbb{N}$ be a parameter set. Also let $(T,\varphi)\colon (\tilde{X},S,E) \to (\tilde{X},S,E)$ be a soft self mapping define by $(T,\varphi)(\hat{u}_a) = \left(\frac{u}{2}\right)_1$, where $T(u) = \frac{u}{4}$ and $\varphi(a) = 1$ are constant mappings and $(\tilde{X},S,E)$ is a soft S-metric space define by

$$S(\hat{u}_a, \hat{v}_b, \widehat{w}_c) = |a-c| + |b-c| + |u-w| + |v-w|.$$

We have

$$S\big((T,\varphi)(\hat{u}_a),(T,\varphi)(\hat{u}_a),(T,\varphi)(\hat{v}_b)\big) = S\left(\left(\frac{u}{2}\right)_1, \left(\frac{u}{2}\right)_1, \left(\frac{v}{2}\right)_1\right)$$

$$= \left|\frac{u}{2}-\frac{v}{2}\right| + \left|\frac{u}{2}-\frac{v}{2}\right| = 2\ \left|\frac{u}{2}-\frac{v}{2}\right| = |u-v|.$$

$$S(\hat{u}_a, \hat{v}_b, \widehat{w}_c) = |u-v| + |u-v| = 2\ |u-v|.$$

$$S\big((T,\varphi)(\hat{u}_a),(T,\varphi)(\hat{u}_a),\hat{u}_a\big) = S\left(\left(\frac{u}{2}\right)_1, \left(\frac{u}{2}\right)_1, u_1\right) = 2\left|\frac{u}{2}-u\right| = |u|.$$

$$S\big((T,\varphi)(\hat{v}_b),(T,\varphi)(\hat{v}_b),\hat{v}_b\big) = S\left(\left(\frac{v}{4}\right)_1, \left(\frac{v}{4}\right)_1, v_1\right) = 2\left|\frac{v}{2}-v\right| = |v|.$$

Thus, from condition (2.5) we obtain

$$|u-v| \le 2\ |u-v| + |u| + |v|.$$

which implies that

$$S\big((T,\varphi)(\hat{u}_a),(T,\varphi)(\hat{u}_a),(T,\varphi)(\hat{v}_b)\big)$$

$$\le \frac{1}{2}\ S(\hat{u}_a, \hat{v}_b, \widehat{w}_c) + \frac{1}{3} S\big((T,\varphi)(\hat{u}_a),(T,\varphi)(\hat{u}_a),\hat{u}_a\big) + \frac{1}{3} S\big((T,\varphi)(\hat{v}_b),(T,\varphi)(\hat{v}_b),\hat{v}_b\big),$$

where $\bar{\alpha} = \frac{1}{4}, \bar{\beta} = \frac{1}{3}, \bar{\gamma} = \frac{1}{3}\ with\ \bar{\alpha} + \bar{\beta} + \bar{\gamma} < \bar{1}$.

Hence, (T,φ) satisfies the condition of Corollary 2.6 with $\bar{k} = \frac{\bar{2}}{3}$. Hence (T,φ) has a unique fixed point $\widehat{w}_c = \bar{0}$.

Corollary 2.8: Consider a self mapping (T,φ) on complete soft S-metric space $(\tilde{X}, S, E)$ which satisfies the following condition:

$$S\big((T,\varphi)(\hat{u}_a),(T,\varphi)(\hat{u}_a),(T,\varphi)(\hat{v}_b)\big)$$

$$\le\ \bar{\alpha}\, S(\hat{u}_a, \hat{v}_b, \widehat{w}_c) + \bar{\beta}\, S\big((T,\varphi)(\hat{u}_a),(T,\varphi)(\hat{u}_a),\hat{u}_a\big) + \bar{\gamma}\, S\big((T,\varphi)(\hat{u}_a),(T,\varphi)(\hat{u}_a),\hat{v}_b\big)$$

$$+ \bar{\lambda}\, S\big((T,\varphi)(\hat{v}_b),(T,\varphi)(\hat{v}_b),\hat{u}_a\big) + \bar{\mu}\, S\big((T,\varphi)(\hat{v}_b),(T,\varphi)(\hat{v}_b),\hat{v}_b\big),$$

for some $\bar{\alpha}, \bar{\beta}, \bar{\gamma}, \bar{\lambda}, \bar{\mu} \geq \bar{0}$, with $\max\left\{ \bar{\alpha} + \bar{\beta} + 3\,\bar{\lambda} + \bar{\mu}, \bar{\alpha} + \bar{\gamma} + \bar{\lambda}, \bar{\lambda} + 2\,\bar{\mu} \right\} < \bar{1}$ and all $\hat{u}_a, \hat{v}_b \,\epsilon\, SP(\tilde{X})$. Then (T, φ) has a unique fixed soft point. Furthermore, (T, φ) is soft continuous at that fixed point.

Proof: Proof follows from the above Theorem 2.1 along with $(M, \varphi)(\bar{x}, \bar{y}, \bar{z}, \bar{s}, \bar{t}) = \bar{\alpha}\,\bar{x} + \bar{\beta}\,\bar{y} + \bar{\gamma}\,\bar{z} + \bar{\lambda}\,\bar{s} + \bar{\mu}\,\bar{t}$, for some $\bar{\alpha}, \bar{\beta}, \bar{\gamma}, \bar{\lambda}, \bar{\mu} \geq \bar{0}$, with $\max\left\{ \bar{\alpha} + \bar{\beta} + 3\,\bar{\lambda} + \bar{\mu}, \bar{\alpha} + \bar{\gamma} + \bar{\lambda}, \bar{\lambda} + 2\,\bar{\mu} \right\} < \bar{1}\ , for\ all\ \bar{x}, \bar{y}, \bar{z}, \bar{s}, \bar{t} \,\epsilon\, \widetilde{X_2}$.
Actually (M, φ) is continuous.
Firstly, we have, $(M, \varphi)(\bar{x}, \bar{x}, \bar{0}, \bar{z}, \bar{y}) = \bar{\alpha}\,\bar{x} + \bar{\beta}\,\bar{x} + \bar{\gamma}\,\bar{z} + \bar{\mu}\,\bar{y}$,
So, if $\bar{y} \leq (M, \varphi)(\bar{x}, \bar{x}, \bar{0}, \bar{z}, \bar{y})$, with $\bar{z} \leq 2\,\bar{x} + \bar{y}$,
then, $\bar{y} \leq \bar{\alpha}\,\bar{x} + \bar{\beta}\,\bar{x} + \bar{\gamma}\,\bar{z} + \bar{\mu}\,\bar{y}$
$\leq \bar{\alpha}\,\bar{x} + \bar{\beta}\,\bar{x} + \bar{\gamma}\,(2\,\bar{x} + \bar{y}) + \bar{\mu}\,\bar{y}$,
which implies $\bar{y} \leq \frac{\bar{\alpha} + \bar{\beta} + 2\,\bar{\lambda}}{1 - \bar{\lambda} - \bar{\mu}}\,\bar{x}$ with $\frac{\bar{\alpha} + \bar{\beta} + 2\,\bar{\lambda}}{1 - \bar{\lambda} - \bar{\mu}} < \bar{1}$.
Thus, condition (A_2) is satisfied by (T, φ).
Secondly, if $\bar{y} \leq (M, \varphi)(\bar{y}, \bar{0}, \bar{y}, \bar{y}, \bar{0}) = \bar{\alpha}\bar{y} + \bar{\gamma}\bar{y} + \bar{\lambda}\,\bar{y} = \left(\bar{\alpha} + \bar{\gamma} + \bar{\lambda}\right)\bar{y}$, then $\bar{y} = \bar{0}$, since $\bar{\alpha} + \bar{\gamma} + \bar{\lambda} < \bar{1}$. Therefore, (T, φ) satisfies the condition (A_2).
Lastly, if for if $\bar{x}_i \leq \bar{y}_i + \bar{z}_i, i \leq 5$, then

$$(M, \varphi)(\bar{x}_1, \dots, \bar{x}_5) = \bar{\alpha}\bar{x}_1 + \dots + \bar{\mu}\,\bar{x}_5$$

$$\leq \bar{\alpha}\,(\bar{y}_1 + \bar{z}_1) + \dots + \bar{\mu}\,(\bar{y}_5 + \bar{z}_5)$$

$$= (\bar{\alpha}\,\bar{y}_1 + \dots + \bar{\mu}\,\bar{y}_5) + (\bar{\alpha}\bar{z}_1 + \dots + \bar{\mu}\,\bar{z}_5)$$

$$= (M, \varphi)(\bar{y}_1, \dots, \bar{y}_5) + (M, \varphi)(\bar{z}_1, \dots, \bar{z}_5).$$

Moreover, $(M, \varphi)(\bar{0}, \bar{0}, \bar{0}, \bar{y}, 2\,\bar{y}) = \bar{\lambda}\bar{y} + 2\,\bar{\mu}\,\bar{y} = \left(\bar{\lambda} + 2\,\bar{\mu}\right)\bar{y}$ where $\bar{\lambda} + 2\,\bar{\mu} < \bar{1}$.
Thus, condition (A_3) is also satisfied.

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