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Existence of Random Measures

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Abstract

In this paper, we show that mild consistency conditions on a prospective family of fidis suffice to guarantee the existence of a random measure having those fidis.

Keywords: prospective family of fidis, random measure, random variable, Hausdorff space.

Introduction

The idea of a random variable having values in a set of measurements is a logical extension of the concept of a point process, which may be considered as a random integer-valued measure (Vere-Jones and Daley,1972). Traditionally, these measures existed in Euclidean spaces, but random measurements on relatively compact spaces have lately been studied in more detail. Expository accounts of the topic have been written by both Kallenberg and Jagers (1974). In this section we will focus on two features of this hypothesis - the presence as well as random Radon measures weak convergence; our approaches will differ from those of Jagers and Kallenberg in that we will emphasize the linear functional characteristics of Radon measures.

Probabilities on spaces of measures are more convenient since we'll focus on the distributional features of random measurements. Therefore, the following is a description of a “random Radon” measure.

Suppose S be a "Hausdorff space" that is locally compact. Following Kallenberg & Jagers, we suppose S is second countable, that is its topology has a countable basis. This is the same as saying that S indicates a Polish space of locally compact (Bauer 1972, p. 223). Suppose c^* be the positive cone of *C*, which represents the continuous-time functions space with compact S support. On S, the Radon measure is just a non-negative linear function of the coefficients of the function on C. To explain the notation, we use the symbol x to represent the total space occupied by all Radon measurements. Any $x \in$ may be expressed uniquely as an integral in terms of a Borel measurement that is inner regular in terms of the compact subset of S; the Borel measure can be written as $x(\cdot)$. [Other representation measures are feasible, but they all coincide with $x(\cdot)$ for the class B_0 of moderately compact sets of Borel. These are just the sets of Borel that we will have to consider.] Equip *X* within its ambiguous topology is the poorest topology that allows all of the mapping's $x \rightarrow x(g)$ to be continual for $g \in C$. A random measure is a "Borel probability" measure on the variable *X*. The space of all these random measurements would be represented with Ω.
Jagers (1974, p.198) has revealed that each P∈ Ω is tight. In reality, as S and *X* are both Polish [Bauer (1972, p. 224, 241) and Bourbaki (1952, Chap. III, $2)), therefore any "Borel probability" on *X* should be tight (Billingsley 1968, pp.10). It is a result of the S topology's 2nd countability. Second countability means the Borel σ-algebra of X is created using mappings $x \rightarrow x(g)$ (A), here A-ranges across the class B_0 of a reasonably compact subset of S, for future reference.

Random measures

Let us examine the properties of a family $\{P^1\}$ known to constitute the set of fidis of a random measure *P*. If $\Gamma_2 \subseteq \Gamma_1$ care both finite subsets of c^* write,
$T_{\Gamma_1\Gamma_2}$, for the canonical projection of $[0,\infty)^{\Gamma_1}$ onto $(0,\infty)^{\Gamma_2}$. Then from the relation

$$T_{\Gamma_2} = T_{\Gamma_1\Gamma_2} \circ T_{\Gamma_1\Gamma_2}$$

we get one limitation on the fidis:

$$\text{(i) if } \Gamma_2 = \Gamma_1, \text{ then } P^{\Gamma 2} = P^{\Gamma 1} T^{-1}_{\Gamma_1\Gamma_2}$$

There's also a limitation coming from the Radon measure's positive linear functional aspect. If $x \in X$ then $g_1, g_2 \in C^*$

Therefore,

$$x(g_1+g_2)=x(g_1)+x(g_2)$$

So

$$\text{(ii) if } \Gamma=\{g_1,g_2,g_1+g_2\}$$

Here g_1, $g_2 \in c^*$ then is concentrated on the closed subset of $[0,\infty)^\Gamma$:

$$\{\Psi \epsilon [0,\infty)^\Gamma : \Psi(g_1 + g_2) = \Psi(g_1) + (g_2)\}.$$

We'll show that these two criteria completely define a random measure's fidis. The proof's concept is straightforward, however, a countability issue complicates matters significantly. To solve this issue, we must make use of *C's* separability property.
The topology of uniform convergence on *S* has a counted base; hence, there is a counted subsets of C within the topology that is dense. This subset of rational numbers may be assumed to form a vector lattice across the domain of rational values without losing any generality. Suppose *D* is its positive cone as well as *Y* is the set of $[0,\infty)$-valued functions on *D* that meet the following criterion: $y(g_1+g_2)=y(g_1)+y(g_2)$ for every pair g_1, $g_2 \in D$. For $g \in D$, equip γ with the topology to make all of mapping $y \rightarrow y(g)$ continuously.
Every "randon measure" on *S* may be utilized to describe a γ member. This relationship may be proved to be a one-to-one map of X into Y, establishing a homeomorphism in between the 2 spaces, using basic Riesz space procedures (Bourbaki (1952, Chap. II, $2). For most purposes, *X* and *Y* may be considered the same topological space. In specific, The Borel probabilities on *X* and *Y* have a one-to-one correlation, therefore constructing a Borel probability on Y suffices to generate a random measure. Since the cylinder sets of the type create the topology of *Y*,

$$\{y \in Y : (y(g_1), y(g_2), \ldots\ldots y(g_k)) \in H\},$$

where $\Gamma = \{g_1, g_2 \ldots\ldots g_k) \subseteq D$ and *H* is an open subset of $[0,\infty)^k$, a routine argument can be used to prove that Borel probabilities on *Y* are uniquely specified by the measures of such cylinder sets. It follows that a random measure (= a Borel probability on *X*) is uniquely calculated by its fidis P^Γ, where I range over the finite subsets of *D*.

Theorem 1. Suppose a Borel probability P^Γ on $[0,\infty)^k$ is given for each finite subset Γ of C^*. These are the fidis of a uniquely determined random measure if conditions (i) & (ii) above are satisfied.

Proof. Only the sufficiency needs to be considered. Applying a version of the Kolmogorov extension theorem (Neveu (1965, p. 82)), we deduce from condition (i) (restricted to those $\Gamma \subseteq D$) that there is a probability measure P_0 lies between $[0,\infty)^D$ with the needed finite-dimensional distributions P^Γ, for $\Gamma \subseteq D$. This P_0 is defined on the cylinder σ-algebra, which corresponds with the Borel σ-algebra $[0,\infty)^D$ as *D* is countable.
Now notice that *Y* is a topological subspace of $[0,\infty)^D$. Indeed, it is a closed subset of that space, because it may be represented as the closed cylinder subsets intersection of the form

$$\{\Psi \epsilon [0,\infty)^D : \text{“} \Psi(g_1 + g_2) = \Psi(g_1) + \Psi(g_2)\}\text{”}.$$

where (g_1, g_2) ranges over all pairs of *D* functions. Condition (ii) shows that all of these cylinder sets have a P_0 measure of one; so *Y* also has P_0 measure one. Transferring P0 from *Y* to the homeomorphic space X yields the necessary random measure *P*.
For $\Gamma \subseteq$ D, this *P* contains the desired fidis P^{Γ}; but it remains to prove that this also holds for any $\Gamma \subseteq C^*$. Suppose then that $\Gamma_0 \subseteq C^*$. Carry out the preceding argument again, but this time using the countable dense subset D' of C^* which is obtained from the augmented set D∪ Γ_0. This procedure generates another random measure P' having the desired fidis for each $\Gamma \subseteq D'$. In specific, *P* & P' has the same fidis for each $\Gamma \subseteq$ D; therefore $P = P'$, and $P.T_{\Gamma_0}^{-1} = P'.\ T_{\Gamma_0}^{-1} = P^{\Gamma_0}$ as needed.
Prohorov (1960, 1961) and Le Cam (1961) provided similar evidence for the presence of random measurements on Hausdorff spaces, general compact, σ-compact as well as locally compact spaces, respectively
Starting with a distinct form of fidis, random measurements may be generated. Recall the mapping $x \rightarrow x\ (A)$. where *A* runs via class B_0 produce the Borel σ-algebra on *X*. Therefore, It is simple to show that the sets fidis of a random measure P is unique.

$$P_{A_1}, P_{A_2}, \ldots\ldots P_{A_n}(\cdot)\text{-``}P\{x \in X\ ;(x(A_1), (x(A_2)\ldots\ldots(x(A_n)) \in \cdot\}$$

here $\{A_1, A_2, \ldots A_n,\}$" is any finite Bo subset: On the sets fidis, Jagers (1974, p.193) has presented consistency criteria that guarantee the presence of the random measure, Other writers who have implemented this method contain Jirina (1964,1966, 1972) and Harris (1963, 1968). They use internal regularity on a semi-compact pavement to transform additive measures of random finitely into random countably, while Kallenberg (1974) presented a different kind of existence proof on the basis of some early results about weak convergence.
The sets form of the presence theorem has the benefit of being readily transformed into a point process existence theorem. Our Theorem 1 might also be used for this objective, although the adjustments required will add to the complexity. The term "random measure" refers to a method of determining anything If P is focused on the closed subset X_1 of X·, then it is a point process. Because X_1 is a calculable intersect of closed cylinder subset of *X*, the requirements to assure $P(X_1)$= 1 might be written w.r.t fidis, P^{Γ}; however, in reality, this may be quite messy. However, Theorem 1 isn't completely useless when it comes to dealing with point processes.

Example 1. Assume λ that the Radon measure on *S* is random but fixed. *A* Poisson process having intensity λ indicates point process that has the following property: The number of points that fall into each of the B0 sets *A1, A1,*......... *An* represents pairwise distinct B0 sets, and the means of these sets are independent Poisson variates. We demonstrate the existence of such a process.
It follows that P would have such fidis, for every set of simple functions $f_1, f_2 \ldots\ldots\ldots f_m$ of the type,

$$f_1 = \sum_{k=1}^{n} a_{jk} 1_{A_n}, \text{ with all } a_{jk} \geq 0,$$

The joint C.F. ("characteristic function") of the $x(f_1)$, $x(f_2)$,........ $x(f_m)$ variates can be represented as

$$\int \exp[it_1x(f_1)+...+ it_mx(f_m)]P(dx)=exp\int [-1+exp(it_1f_1+...+it_mf_m)d\lambda \quad (1)$$

It is simple to verify it is a true distribution of C.F lies between $[0, \infty)^m$ Now using such basic functions to approximate members of C^* we may derive the joint C.F. of the $x(g_1)$, $x(g_2)$,... $x(g_m)$variates, here $\Gamma = \{g_l, g_l,..., g_m,\} \subseteq C^*$, would be

$$\Phi(g_1,g_2,......,g_m,;t_1,t_2,....., t_m) = exp \int [-1+\exp(it_1g_1+....+it_mg_m)]d\lambda. \quad (2)$$

Again these represent genuine C.F.'s of distributions on $[0, \infty)^m$. Since

$$\Phi(g_1,g_2,......,g_m,;t_1,t_2,....., t_{m-1},0) = \Phi(g_1,g_2,......,g_{m-1},;t_1,t_2,....., t_{m-1},)$$

and

$$\Phi(g_1,g_2, g_1,+ g_2;t,t,-t) \equiv 1,$$

The related measures on $[0,\infty)^m$ fulfill Theorem 1's consistency criteria; consequently, a random measure with fidis defined by exists (2). Working backward, we may see that (1) fulfill for every simple function $f_1, f_2,...,f_m$, indicating that the random measure is actually the needed Poisson process.

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