Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 18, Number 2 (2022), pp. 735-752
©Research India Publications
http://www.ripublication.com/gjpam.htm

An Analogue of Lagarias' Inequality Pertaining to the Riemann Hypothesis

Darrell Cox[*1] **and Sourangshu Ghosh**[2]
[1]*Department of Mathematics, Grayson County College, United States*
[2]*Department of Civil and Environmental Engineering, University of Illinois Urbana Champaign, United States*

Abstract

An inequality analogous to Lagarias' inequality is introduced. A function having some properties similar to those of harmonic numbers is used. Another function related to this function by a generalized Dirichlet product is also investigated.

1. INTRODUCTION

Lagarias' [1] Theorem 1.1 is

Theorem 1 *The inequality* $\sum_{d|n} d \leq H_n + \exp(H_n)\log(H_n)$ *(with equality only for* $n = 1$*) is equivalent to the Riemann hypothesis.*

The function $\sigma(n) = \sum_{d|n} d$ is the sum of divisors function and $H_n = \sum_{j=1}^{n} \frac{1}{j}$ is called the n-th harmonic number.

Let $\varphi(n)$ denote Euler's totient function.

Theorem 2 *4 does not divide* $\varphi(n)$, $n > 4$, *if and only if* $n = p^\alpha$ *where* p *is a prime of the form* $4k + 3$, $\alpha \geq 1$, *or twice a power of a prime of the form* $4k + 3$.

Proof: Apostol's [2] Theorem 2.5 is

Theorem 3 *(a)* $\varphi(p^\alpha) = p^\alpha - p^{\alpha-1}$ *for prime* p *and* $\alpha \geq 1$.
(b) $\varphi(mn) = \varphi(m)\varphi(n)(d/\varphi(d))$, *where* $d = (m, n)$.
(c) $\varphi(mn) = \varphi(m)\varphi(n)$ *if* $(m, n) = 1$.
(d) $a|b$ *implies* $\varphi(a)|\varphi(b)$.
(e) $\varphi(n)$ *is even for* $n \geq 3$. *Moreover, if* n *has* r *distinct odd prime factors, then* $2^r|\varphi(n)$.

[*]darkencox273@gmail.com

By part (a), 4 does not divide $\varphi(p^\alpha)$ if p is a prime of the form $4k+3$. By part(c), 4 does not divide $\varphi(mn)$ if m is a power of a prime of the form $4k+3$ and $n=2$. By part (e), 4 divides $\varphi(n)$ if $n \geq 3$ and n has an odd prime factor. By part (a), 4 divides $\varphi(p^\alpha)$ if $p=2$ and $\alpha > 2$.

Let $h(n)$ denote $\sum_{k=1,\varphi(k)\neq[\varphi(k)/4]4}^{n} 1/\varphi(k)$. The inequality to be considered here is

Conjecture 1 *$\sigma(n) < \sum_{d|n}(\exp(h(d)))^2$ if n is a superabundant number. For sufficiently large n, $\sigma(n) < \sum_{d|n}(\exp(h(d)))^2$ only if n is a superabundant number.*

2. A PROPERTY OF $h(n)$

A plot of $\log(H_n)$ versus $h(n)$ for $n \leq 1500000$ is

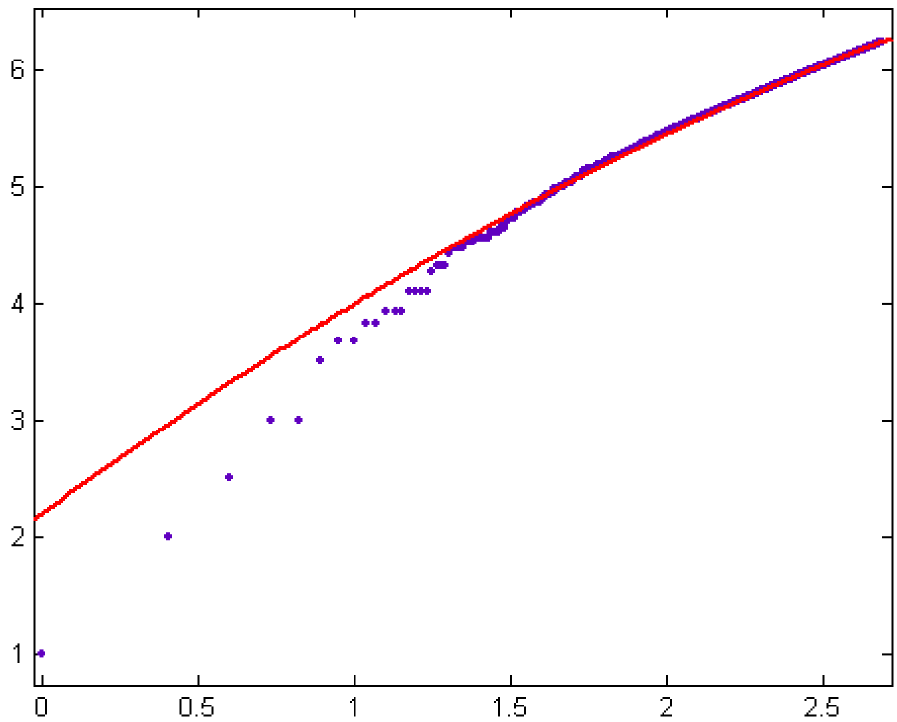

Figure 1: $\log(H_n)$ versus $h(n)$.

For a quadratic least-squares fit of the curve, $p_1 = -0.173$ with a 95% confidence interval of $(-0.1731, -0.1728)$, $p_2 = 1.964$ with a 95% confidence interval of (1.963, 1.965), $p_3 = 2.2$ with a 95% confidence interval of (2.199, 2.201), SSE=7.455, R-squared=0.9994, and RMSE=0.002229. This is the origin of Conjecture 1.

Let $\rho(n)$ denote $\sum_{d|n}(\exp(h(d)))^2$. For relatively small n values, $\rho(n)$ is much larger than $\sigma(n)$. A plot of $130\sigma(n)$ versus $\rho(n)$ for $n \leq 100$ is

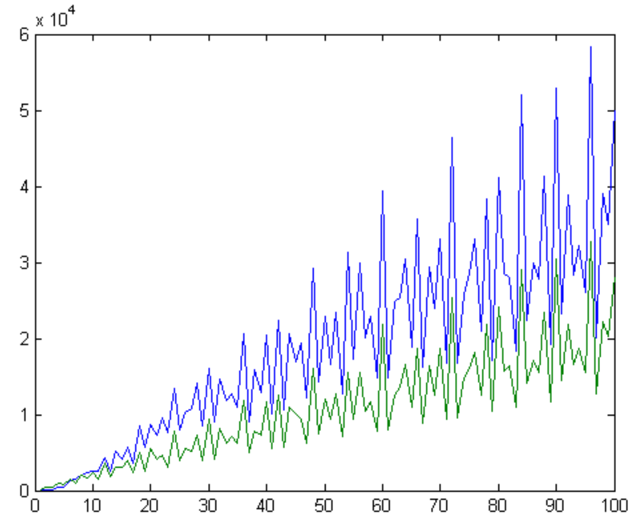

Figure 2: $\rho(n)$ versus $130\sigma(n)$.

The peaks and valleys of the two curves occur at roughly the same locations. The factor of 130 is arbitrary.

3. MORE PROPERTIES OF $h(n)$

A plot of $h(n)$ versus $\log\log n$ for $n =$ 2, 3, 4,. . ., 10000 is

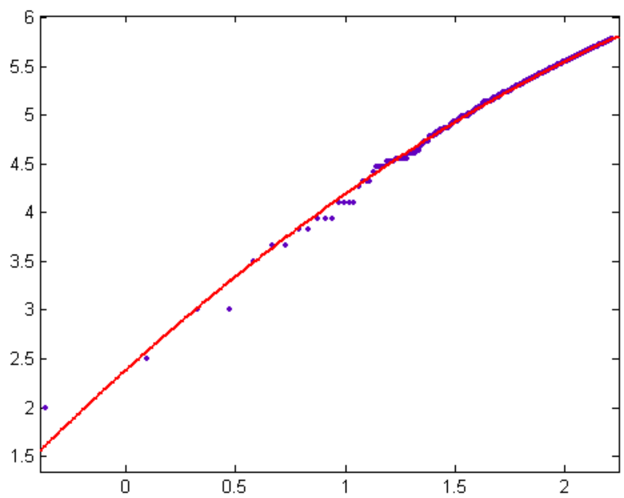

Figure 3: $h(n)$ versus $\log\log n$.

For a quadratic least-squares fit of the curve, $p_1 = -0.2253$ with a 95% confidence interval of (-0.2268, -0.2237), $p_2 = 2.03$ with a 95% confidence interval of (2.024, 2.036), $p_3 = 2.388$ with a 95% confidence interval of (2.382, 2.393), SSE=0.4531, R-squared=0.9986, and RMSE=0.006732.

A plot of $\sum_{k=1}^{x}[x/k]h(k)$ versus $x \log x$ for $x \leq 10000$ is

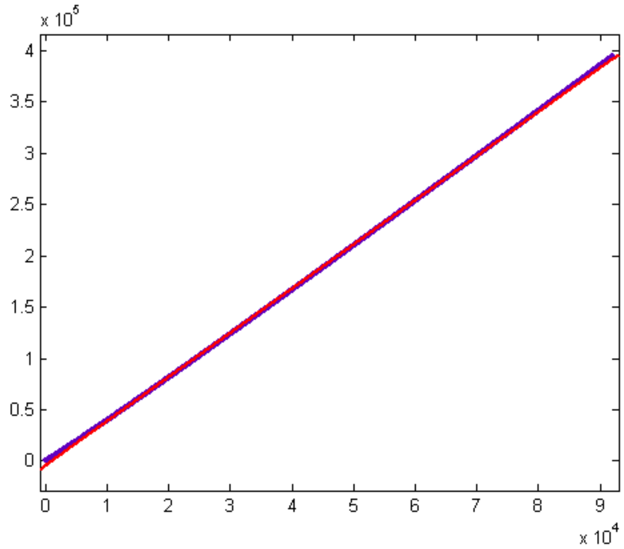

Figure 4: $\sum_{k=1}^{x}[x/k]h(k)$ versus $x \log x$.

For a linear least-squares fit of the curve, $p_1 = 4.299$ with a 95% confidence interval of (4.298, 4.3), $p_2 = -4435$ with a 95% confidence interval of $(-4486, -4385)$, SSE=$1.85 \cdot 10^{10}$, R-squared=0.9999, and RMSE=1360.

A plot of $\sum_{k=1}^{x}[x/k]h(k)/(x \log x)$ for $x \leq 10000$ is

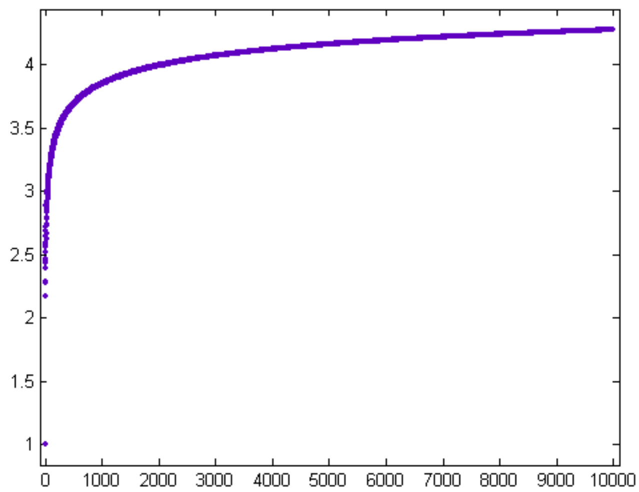

Figure 5: $\sum_{k=1}^{x}[x/k]h(x)/(x \log x)$.

This is the basis of the following conjecture

Conjecture 2 $\sum_{k=1}^{x}[x/k]h(k) = x\log x + O(x)$

Shapiro's theorem (Theorem 4.8 in Apostol's' book) is

Theorem 4 *Let* $\{\alpha(n)\}$ *be a nonnegative sequence such that* $\sum_{n\leq x}\alpha(n)[\frac{x}{n}] = x\log x + O(x)$ *for all* $x \geq 1$. *Then: (a) For* $x \geq 1$ *we have* $\sum_{n\leq x}\frac{\alpha(n)}{n} = \log x + O(1)$. *(b) There is a constant* $B > 0$ *such that* $\sum_{n\leq x}\alpha(n) \leq Bx$ *for all* $x \geq 1$. *(c) There is a constant* $A > 0$ *and an* $x_0 > 0$ *such that* $\sum_{n\leq x}\alpha(n) \geq Ax$ *for all* $x \geq x_0$.

A plot of $\sum_{n\leq x} h(n)$ for $x \leq 1000$ is

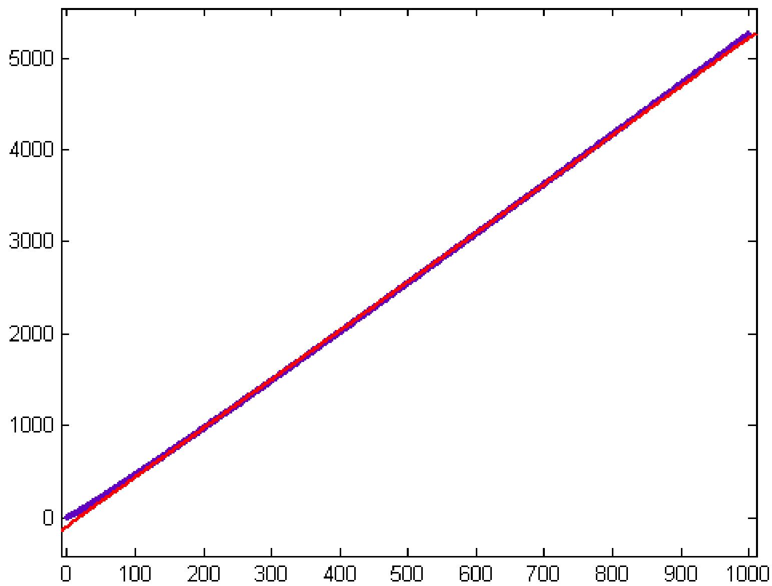

Figure 6: $\sum_{n\leq x} h(n)$.

For a linear least-squares fit of the curve, $p_1 = 5.312$ with a 95% confidence interval of (5.308, 5.317), $p_2 = -96.05$ with a 95% confidence interval of $(-98.7, -93.41)$, SSE=$4.54 \cdot 10^5$, R-squared=0.9998, and RMSE=21.33. Parts (b) and (c) of Shapiro's theorem then appear to be applicable.

A plot of $\sum_{n\leq x}\frac{h(n)}{n}$ versus $\log x$ for $x \leq 1000$ is

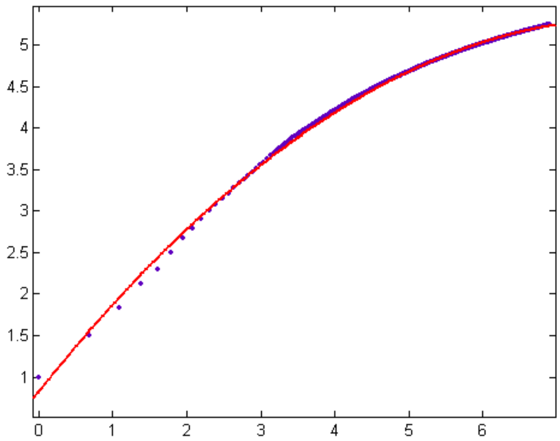

Figure 7: $\sum_{n\leq x}\frac{h(n)}{n}$ versus $\log x$.

For a quadratic least-squares fit of the curve, $p_1 = -0.06959$ with a 95% confidence interval of $(-0.0701, -0.6898)$, $p_2 = 1.118$ with a 95% confidence interval of (1.113, 1.124), $p_3 = 0.8258$ with a 95% confidence interval of (0.8119, 0.84), SSE=0.2343, R-squared=0.9988, and RMSE=0.01533. Part (a) of Shapiro's theorem then appears to be applicable.

4. A COMPANION FUNCTION OF $h(n)$

Let
$M(n)$ denote Mertens' function and $g(n)$ denote $\sum_{k=1,\varphi(k)\neq[\varphi(k)/4]4}^{n} M([n/k])/\varphi(k)$.
A plot of $g(n)$ and $M(n)$ versus n for $n \leq 1000$ is

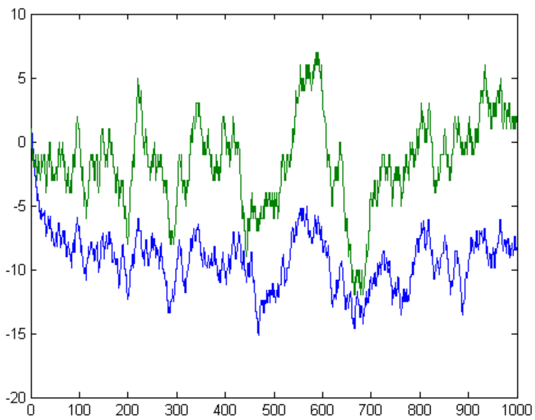

Figure 8: $g(n)$ and $M(n)$.

The upper curve is Mertens' function. The peaks and valleys of $g(n)$ are at roughly the same locations.

Let $\tau(n)$ denote $\sum_{d|n}(\exp(g(d)))^2$. A plot of $\sigma(n)$ versus $\tau(n)$ for $n \leq 1400$ is

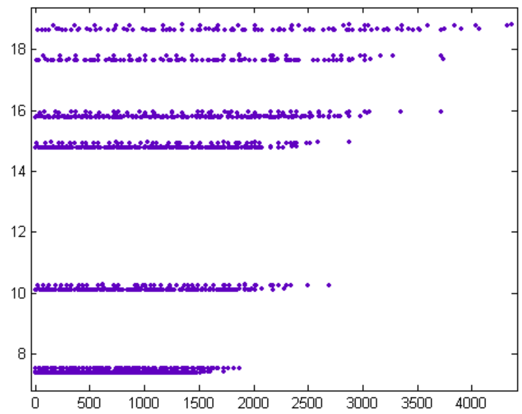

Figure 9: $\sigma(n)$ versus $\tau(n)$

The n values for the top "line" are abundant numbers divisible by 12 but not 5. The n values of the bottom group of "lines" consist of the primes greater than 3, squares of primes greater than 3, products of two distinct primes greater than 3, etc.

In general, these are number systems similar to the natural numbers except that certain primes are missing and there are gaps of n values corresponding to the spikes of positive $g(n)$ values. A detailed plot of n versus $\tau(n)$ for $10.28 > \tau(n) > 10.1$ (the next-to-bottom "line") is

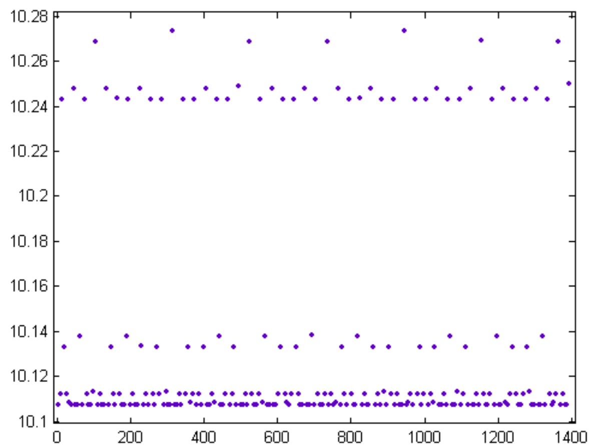

Figure 10: n versus $\tau(n)$

The seven n values in the top "line" are divisible by $3 \cdot 5 \cdot 7$. The successive prime factorizations are $3 \cdot 5 \cdot 7$, $3^2 \cdot 5 \cdot 7$, $3 \cdot 5^2 \cdot 7$, $3 \cdot 5 \cdot 7^2$, $3^3 \cdot 5 \cdot 7$, $3 \cdot 5 \cdot 7 \cdot 11$, and $3 \cdot 5 \cdot 7 \cdot 13$. The τ values for $3^2 \cdot 5 \cdot 7$ and $3^3 \cdot 5 \cdot 7$ are slightly larger than the other values. The n values in the second-to-top and third-to-top "lines" are divisible by 3 and 5 but not 7. The fourteen n values in the second-to-top "line" are divisible by 3^2. Among these values are $3^2 \cdot 5$ times the primes 11, 13, 17, 19, 23, 29, and 31. The remaining values are products of powers of 3 and powers of 5. The twenty-six n values in the third-to-top "line" are not divisible by 3^2. Among these values are $3 \cdot 5$ times the primes 11, 13, 17,. . ., 89 and $3 \cdot 5^2$ times the primes 11, 13, and 17. The remaining values are $3 \cdot 5$, $3 \cdot 5^2$, and $3 \cdot 5^3$. The n values in the fourth-to-top and fifth-to-top "lines" are divisible by 3 and 7 but not 5. The nine n values in the fourth-to-top "line" are divisible by 3^2. Among these values are $3^2 \cdot 7$ times 11, 13, 17 and 19. The remaining values are products of powers of 3 and powers of 7. The seventeen n values in the fifth-to-top "line" are not divisible by 3^2. Among these values are $3 \cdot 7$ times the primes 11, 13, 17,. . ., 61. The remaining values are $3 \cdot 7$, $3 \cdot 7^2$, and $3 \cdot 7^3$. The n values of the sixth-to-top and seventh-to-top "lines" are divisible by 3 but not 5 or 7. The fifty-three n values in the sixth-to-top "line" are divisible by 3^2. Among these values are 3^2 times the primes 11, 13, 17,. . ., 151, 3^3 times the primes 11, 13, 17,. . . , 47, and 3^4 times the primes 11, 13, 17. Two more values are $3^2 \cdot 11^2$ and $3^2 \cdot 11 \cdot 13$. The remaining values are 3^2, 3^3, 3^4, 3^5, and 3^6. The n values in the seventh-to-top "line" are 3, 3 times the primes 11, 13, 17,. . ., 463, 3 times 11^2, 13^2, 17^2, and 19^2, $3 \cdot 11$ times the primes 13, 17, 19,. . ., 41, $3 \cdot 17$ times 19 and 23, and $3 \cdot 19 \cdot 23$.

A plot of $\sigma(n)$ versus $\tau(n)$, $\tau(n) < 30$, for $n \leq 40000$ is

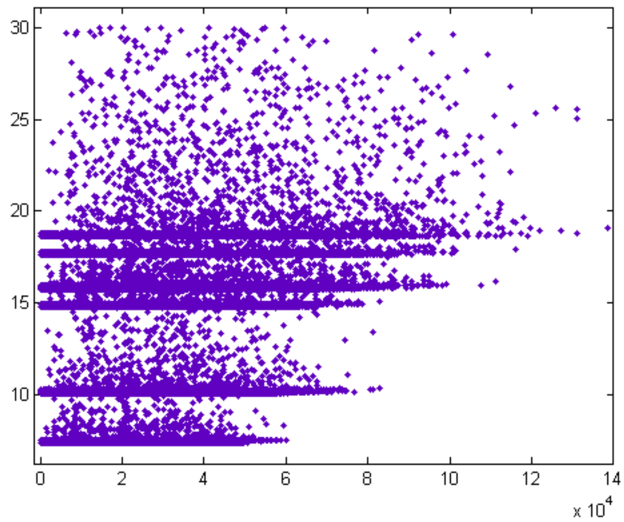

Figure 11: $\sigma(n)$ versus $\tau(n)$

There are 9939 $\tau(n)$ values greater than or equal to 30. This is mainly due to the 6886 $g(n)$ values greater than 0, the largest being 27.299133 for $n = 31989$. A plot of the positive $g(n)$ values is

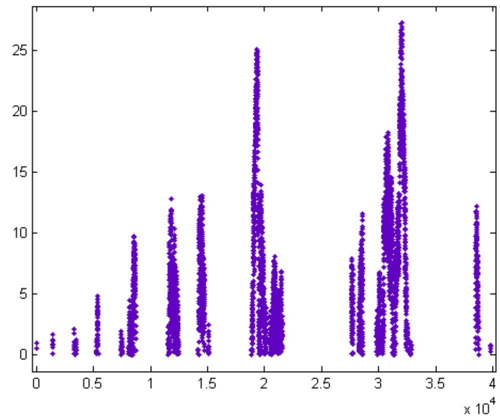

Figure 12: positive $g(n)$ values

The top "line" in the previous plot is defined to be the $\tau(n)$ values greater than 18.625 and less than 18.68. The n values for $21.0 > \tau(n) \geq 18.68$ are usually abundant numbers, including some that are divisible by 5. Of the 1049 $\tau(n)$ values in this range, 175 of the n values are not abundant numbers. Of these n values, 132 are even and $g(n/d) > 0$ for some divisor d (possibly 1) of n. Besides 1, the most common d value is 2, but there are d values of 4, 5, 8, 10, and 22. The g/d values range from 0.537686 to 0.909781. For $n = 34809$, 36357, 36795, 36813, 36933, 36987, 37023, and 37257, the $g(n/3)$ values are 1.106033, 1.146869, 1.095903, 1.095577, 1.145570, 1.125707, 1.099118, and 1.138946 respectively. For $n = 8369$, 27701, and 32479 (primes), the $g(n)$ values are 1.219868, 1.217757, and 1.294848 respectively.

A plot of the n values where $14.7795 > \tau(n) > 14.77$ for $n \leq 40000$ is

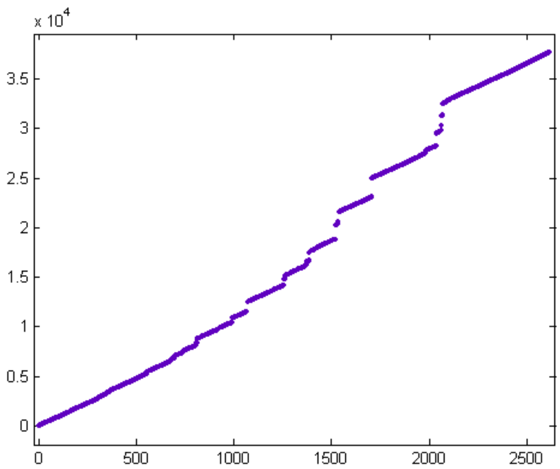

Figure 13: n values

The discontinuities are due to the spikes of the positive $g(n)$ values. The n values are divisible by 2 but not by 2^2, 3, 5, or 7.

5. A GENERALIZED DIRICHLET PRODUCT OF $g(n)$ AND $h(n)$

A plot of $\sum_{k\leq n} h(k)g([\frac{n}{k}])$ versus $\sqrt{n}$ for $n \leq 10000$ is

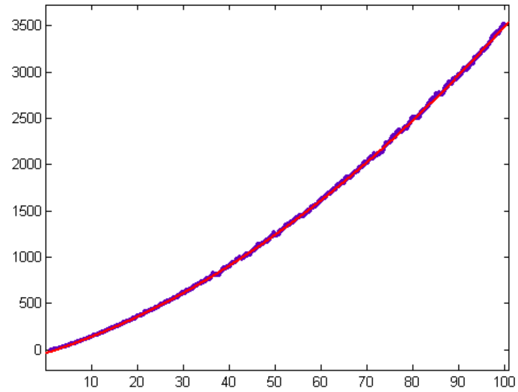

Figure 14: $\sum_{k\leq n} h(k)g([\frac{n}{k}])$ versus $\sqrt{n}$.

For a quadratic least-squares fit of the curve, $p_1 = 0.195$ with a 95% confidence interval of (0.1945, 0.1954), $p_2 = 15.79$ with a 95% confidence interval of (15.73, 15.84), $p_3 = -35.79$ with a 95% confidence interval of $(-37.34, -34.24)$, SSE=$1.729 \cdot 10^6$, R-squared=0.9998, and RMSE=13.15.

6. DIRICHLET INVERSES OF $g(n)$ AND $h(n)$ AND GENERALIZED MÖBIUS INVERSION

Theorem 2.8 of Apostol's book is

Theorem 5 *If f is an arithmetical function with $f(1) \neq 0$ there is a unique arithmetical function f^{-1}, called the Dirichlet inverse of f, such that $f * f^{-1} = f^{-1} * f = I$. Moreover, f^{-1} is given by the recursion formulas $f^{-1}(1) = 1/f(1)$, $f^{-1}(n) = \frac{-1}{f(1)} \sum_{d|n, d<n} f(\frac{n}{d})f^{-1}(d)$ for $n > 1$.*

A plot of $\sum_{k\leq n} g^{-1}(k)h([\frac{n}{k}])$ (denoted by $D(n)$) for $n \leq 10000$ is

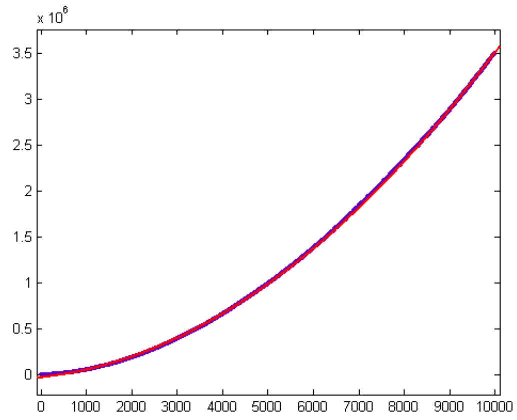

Figure 15: $D(n)$.

For a quadratic least-squares fit of the curve, $p_1 = 0.03022$ with a 95% confidence interval of (0.03019, 0.03024), $p_2 = 52.79$ with a 95% confidence interval of (52.52, 53.05), $p_3 = -30750$ with a 95% confidence interval of $(-31330, -30180)$, SSE=$9.529 \cdot 10^{11}$, R-squared=0.9999, and RMSE=9763.
Theorem 2.22 (generalized inversion formula) in Apostol's book is

Theorem 6 *If α has a Dirichlet inverse α^{-1}, then the equation (10) $G(x) = \sum_{n\le x} \alpha(n)F(\frac{x}{n})$ implies (11) $F(x) = \sum_{n\le x} \alpha^{-1}G(\frac{x}{n})$. Conversely, (11) implies (10).*

$\sum_{k\le n} g(k)D(\frac{n}{k})$ then equals $h(n)$.
A plot of $\sum_{k\le n} h^{-1}(k)g(\frac{n}{k})$ (denoted by $E(n)$) for $n \le 10000$ is

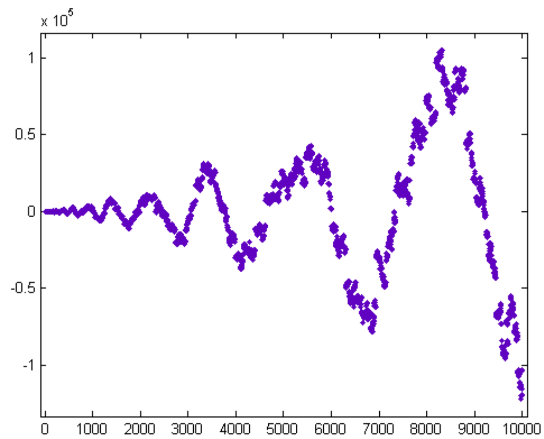

Figure 16: $E(n)$.

$\sum_{k\le n} h(k)E(\frac{n}{k})$ then equals $g(n)$.

7. INSTANCES WHERE $\sigma(n)$ IS GREATER THAN $\rho(n)$

For $n \leq 1500000$, there are 906116 instances (60.41%) where $\sigma(n)$ is greater than $\rho(n)$. For $n \leq 4500000$, there are 3874169 instances (86.09%) where $\sigma(n)$ is greater than $\rho(n)$.

For $n \leq 300000$, there are 11918 instances where $\sigma(n)$ is greater than $\rho(n)$. Let p, q, and r denote distinct primes. There are 8903 instances where n is a prime (out of the 25997), 8 instances where n is a prime squared, and 3007 instances where n is of the form pq. These n values account for all the instances where $\sigma(n)$ is greater than $\rho(n)$. A plot of $\sigma(n)$ and $\rho(n)$ versus n is

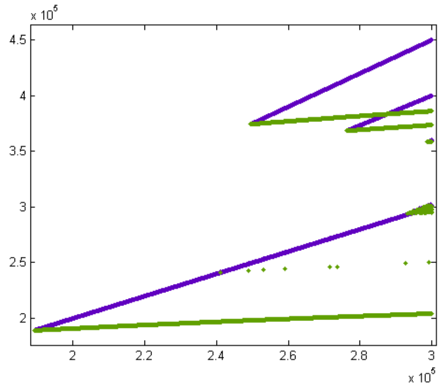

Figure 17: $\rho(n)$ and $\sigma(n)$.

The bottom blue/green vertex corresponds to the prime n. The next higher vertex corresponds to the n values of the form p^2. The remaining vertices or beginnings of vertices correspond to the n values of the form pq. In this case, p equals 2 in 2134 instances, 3 in 698 instances, and 5 in 20 instances. In the remaining 155 instances, p and q are relatively large. A plot of $\log p$ versus $\log q$ for increasing n values (293141, 293627, 293719,. . ., 299939) is

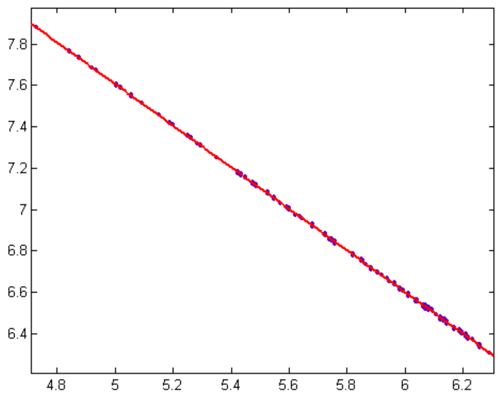

Figure 18: $\log p$ versus $\log q$.

For a linear least-squares fit of the curve, $p_1 = -1.005$ with a 95% confidence interval of $(-1.007, -1.003)$, $p_2 = 12.63$ with a 95% confidence interval of (12.63, 12.64), SSE=0.003916, R-squared=0.9998, and RMSE=0.005059. These n values account for the green smudge superimposed on the $\sigma(n)$ curve for prime n in Figure 17.

For $n \leq 400000$, there are 42993 instances where $\sigma(n) > \rho(n)$. In 16766 of these instances, n is a prime. In 22 of these instances, $n = p^2$. In 23219 of these instances, $n = pq$. In 2207 of these instances, $n = p^2q$. In 2 of these instances, $n = p^3$. In 187 of these instances, $n = p^3q$. In the remaining 590 instances, $n = pqr$. A plot of $\sigma(n)$ and $\rho(n)$ versus n is

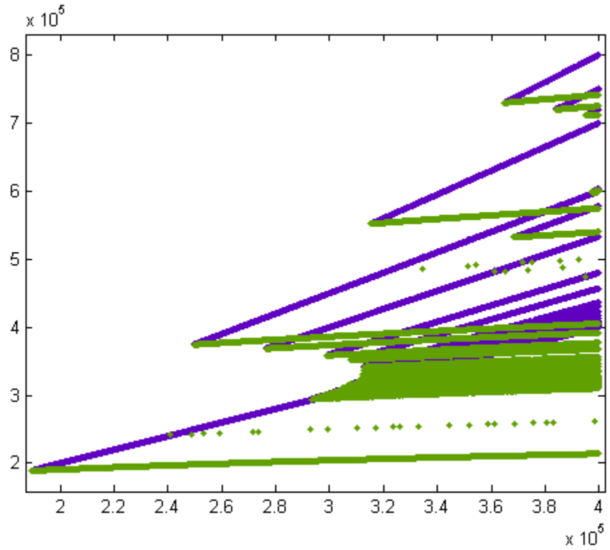

Figure 19: $\rho(n)$ and $\sigma(n)$.

The top and third-to-top blue/green vertices correspond to the n values of the form pqr. The 516 n values for the first of these vertices are $2 \cdot 3$ times the primes 60811, 60821, 60859,. . ., 66653. The 49 n values for the second of these vertices are $2 \cdot 5$ times the primes 39499, 39503, 39509,. . ., 39989. In both cases, all the primes between the beginning and ending primes are included. The beginning of a vertex at the approximate coordinates $(4 \cdot 10^5, 6 \cdot 10^5)$ also corresponds to n values of the form pqr. The 25 n values are 397382, 397478, 397498,. . ., 399974. The p values are 2. For a linear least-squares fit of $\log q$ versus $\log r$, $p_1 = -1.002$ with a 95% confidence interval of $(-1.005, -0.9984)$, $p_2 = 12.21$ with a 95% confidence interval of (12.19, 12.23), SSE=$9.78 \cdot 10^{-5}$, R-squared=0.9999, and RMSE=0.002062. The second-to-top vertex corresponds to the n values of the form p^3q. The n values for this vertex are 2^3 times the primes 48017, 48023, 48029,. . ., 49999. All the primes between the beginning and ending primes are included. The fourth-to-top and fifth-to-top vertices correspond to the n values of the form p^2q. The n values for the first of these vertices are 3^2 times the primes 40927, 40933, 40339,. . ., 44417. The n values for the second of these vertices are 2^2 times the primes 78839, 78853, 78857,..., 99991. In both cases, these are all the primes between the beginning and ending primes. The scattered points at the approximate y-coordinate of $5 \cdot 10^5$ also correspond to n values of the form p^2q. The 25 n values are $2 \cdot 409^2$, $2 \cdot 419^2$, $2 \cdot 421^2$, $2 \cdot 431^2$, $2 \cdot 433^2$, $2 \cdot 439^2$, $2 \cdot 443^2$, $3 \cdot 347^2$, $3 \cdot 349^2$, $3 \cdot 353^2$, $3 \cdot 359^2$, and $5 \cdot 281^3$. For each of the three vertices, all the primes between the beginning and ending primes are included. The n values where $n = p^3$ are 71^3 and 73^3.

For $n \leq 4500000$, there are 845079 instances where $n = pqr$ and $\sigma(n) > \rho(n)$. The numbers of instances where $p =$ 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, and 157 are 332653, 185374, 97279, 61953, 35980, 27891, 19678, 16279, 12429, 9098, 7950, 6148, 5150, 4575, 3866, 3138, 2573, 2303, 1895, 1623, 1436, 1172, 989, 801, 624, 516, 440, 357, 296, 231, 138, 99, 65, 46, 22, 10, and 2 respectively. A plot of the logarithms of these counts is

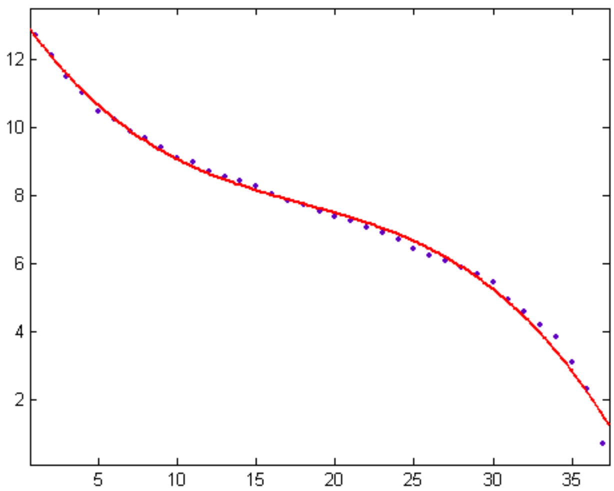

Figure 20: Logarithm of counts

For a cubic least-squares fit of the curve, $p_1 = -0.0005526$ with a 95% confidence interval of $(-0.0006268, -0.0004856)$, $p_2 = 0.03008$ with a 95% confidence interval of (0.026, 0.03416), $p_3 = -0.6713$ with a 95% confidence interval of $(-0.7385, -0.6041)$, $p_4 = 13.33$ with a 95% confidence interval of (13.03, 13.63), SSE=1.334, R-squared=0.995, and RMSE=0.204.

8. SUPERABUNDANT NUMBERS

The first few superabundant numbers are 2, 4, 6, 12, 24, 36, 48, 60, 120, 180, 240, 360, 720, 840, 1260, 1680, 2520, 5040, 10080, 15120, 25200, 27720, 55440, 110880, 166320, 277200, 332640, 554400, 665280, 720720, 1441440, 2162160, 3603600, 4324320,.... A plot of $\rho(n)$ versus $\sigma(n)$ at these n values is

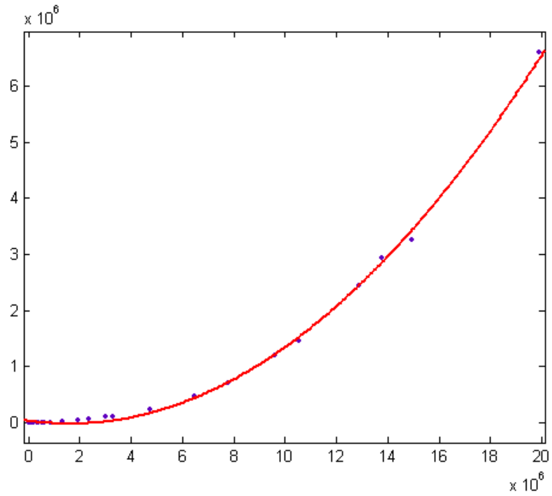

Figure 21: $\rho(n)$ versus $\sigma(n)$.

For a quadratic least-squares fit of the curve, $p_1 = 2.568 \cdot 10^{-8}$ with a 95% confidence interval of $(2.457 \cdot 10^{-8}, 2.678 \cdot 10^{-8})$, $p_2 = -0.1542$ with a 95% confidence interval of $(-0.1839, -0.1246)$, $p_3 = 104200$ with a 95% confidence interval of $(-3090, 211400)$, SSE=$1.632 \cdot 10^{12}$, R-squared=0.9978, and RMSE=229400.

9. CONCLUSION

The above plot is evidence that the first part of the conjecture is not likely to fail. The instances where $\sigma(n) > \rho(n)$ can be analyzed to some extent using primes and logarithms.

10. METHODS

The C program for computing the inequality analogue assumes 1500000 h_n values have been computed and stored in the look-up table "out1bz1.h". The prime look-up table consisting of the first 114155 primes (those less than 1500000) is not included. Theorem 2.3 in Apostol's book is

Theorem 7 *If* $n \geq 1$ *we have* $\varphi(n) = \sum_{d|n} \mu(d)\frac{n}{d}$.

The Möbius function is denoted by μ. This formula is used to compute Euler's totient function.

```
//
// compute Mobius function
//
#include <math.h>
#include <stdio.h>
#include "table5.h" // prime look-up table
int mobius(unsigned int a, unsigned int *table, unsigned int tsize) {
unsigned int i,count,p;
```

```
if (a==1)
    return(1);
count=0;
for (i=0; i<tsize; i++) {
    p=table[i];
    if (p>a)
        break;
    if (a==(a/p)*p) {
        a=a/p;
        if (a==(a/p)*p)
            return(0);
        count=count+1;
        if (a==1)
            break;
        }
    }
if ((count&1)==0)
    return(1);
else
    return(-1);
}
//
// compute Euler's phi function
//
int mobius(unsigned int a, unsigned int *t, unsigned int tsize);
unsigned int nueuler(unsigned int n, unsigned int *table, unsigned int tsize) {
unsigned int d;
int sum;
if (n==1)
    return(1);
sum=0;
for (d=1; d<=n; d++) {
    if (n==(n/d)*d)
        sum=sum+(n/d)*mobius(d, table, tsize);
    }
return((unsigned int)sum);
}
//
// compute h(x)
//
unsigned int nueuler(unsigned int a, unsigned int *table, unsigned int tsize);
unsigned int max=1500000;
unsigned int tsize=114155; // size of prime look-up table
```

```
void main() {
unsigned int temp,x;
double sum;
FILE *Outfp;
Outfp = fopen(”out1bz1.dat”,”w”);
sum=0.0;
for (x=1; x<=max; x++) {
    temp=nueuler(x,table,tsize);
    if (temp!=(temp/4)*4)
        sum=sum+1.0/(double)temp;
    fprintf(Outfp,” %llf, \n”,sum);
    printf(” %d %llf \n”,x,sum);
    }
fclose(Outfp);
return;
}
//
// inequality analogue
//
#include <stdio.h>
#include <math.h>
#include ”out1bz1.h”
unsigned int max=1500000;
void main() {
unsigned int x,d,sum1;
double sum;
FILE *Outfp;
Outfp = fopen(”out1bz2.dat”,”w”);
for (x=1; x<=max; x++) {
    sum=0.0;
    sum1=0;
    for (d=1; d<=x; d++) {
        if (x==(x/d)*d) {
            sum=sum+exp(h[d-1])*exp(h[d-1]);
            sum1=sum1+d;
            }
        }
    fprintf(Outfp,” %llf, %d, \n”,sum,sum1);
    printf(” %llf, %d, \n”,sum,sum1);
    }
fclose(Outfp);
return;
}
```

REFERENCES

[1] J. Lagarias, *An Elementary Problem Equivalent to the Riemann Hypothesis*, arXiv:math/0008177v2 [math.NT], 6 May 2001

[2] T. M. Apostol, *Introduction to Analytic Number Theory*, Springer, (1976)