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Counting Kinds of Streamlines in Multi-Dimensional Orthogonal Flow in Different Planes

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Abstract

In this research paper, we have tried to find the number of different kinds of stream line flows in different planes in n-dimensional orthogonal fluid. Here n $\in$ N (natural number) and n $\geq$ 2. We have checked by taking a kind of flow like $q = x\hat{i} - y\hat{j}$ which is two dimensional. Here we have obtained one kind of different rectangular hyperbolas in *xy*- plane as $xy = c$ (c is arbitrary constant). We have increased the dimension in such type of flow and counted the number of different kinds of stream lines in different planes one by one. These numbers have been found in an arithmetic sequence. After getting the result we have obtained that total numbers of different kinds of stream line flows in such type of n-dimensional orthogonal flow in n different planes will be n(n-1)/2, n$\geq$2 and n is a natural number.

Key words:-Stream lines, the equation of stream lines, rectangular hyperbola, n-dimensional flow, collection of rectangular hyperbolas in a plane, orthogonal fluid flow.

1. Introduction

In this research paper we have tried to count the number of different kind of stream lines in different planes. As we know that a stream line is a continuous line of flow which is drawn in the fluid so that the tangent at every point on this continuous line and the direction of fluid velocity at that point remains same. If we suppose that *ds* be an element of a tangent and p be the velocity of fluid at a point at given time then, *ds* × *p* = 0, this means *ds*||*p*.

2. How did we check different kinds of streamlines in different planes?

Let us choose $ds = dx\hat{i} + dy\hat{j} + dz\hat{k}$ and $p = u\hat{i} + v\hat{j} + w\hat{k}$ and $ds \times p = 0$ or

$$(dx\hat{i} + dy\hat{j} + dz\hat{k}) \times (u\hat{i} + v\hat{j} + w\hat{k}) = 0$$

or, $\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z}$. This is the basic information about the stream line. We have used this concept of stream line and counted the number of different kinds of stream lines in different planes. In this paper we have chose a special type of fluid velocity which are taken as $p = x\hat{i} - y\hat{j}$, $p = x\hat{i} - y\hat{j} - z\hat{k}$, $p = x\hat{i} - y\hat{j} - z\hat{k}$, ………..,

$$p_n = x_1\hat{i}_1 - x_2\hat{i}_2 - x_3\hat{i}_3 .... - x_n\hat{i}_n$$

Here we have taken x – axis as reference axis.

Let us take a two dimensional orthogonal flow velocity vector $p = x\hat{i} - y\hat{j}$. As we know that $\vec{p} \times d\vec{s} = 0$ in any stream line flow and *ds* is the element of stream line.

or, $(x\hat{i} - y\hat{j}) \times (dx\hat{i} + dy\hat{j}) = 0$

or, $(xdy + ydx)\hat{k} = 0$

or, $\frac{dx}{x} = -\frac{dy}{y}$

Now we integrate the above equation and get *log x + log y = log c log (xy) = log c*
xy = c.

Thus we have found one kind of different stream lines in rectangular hyperbolic form. Here c an arbitrary constant. This means it is a one kind of collection of rectangular hyperbolas in *xy* plane.

We have now chosen a three dimensional orthogonal flow velocity vector as $p = x\hat{i} - y\hat{j} - z\hat{k}$. Again, we calculate the different kinds of stream line flow through this three dimensional velocity vector, $\vec{p} \times d\vec{s} = 0$.

or $(x\hat{i} - y\hat{j} - z\hat{k}) \times (dx\hat{i} + dy\hat{j} + dz\hat{k}) = 0$

or, $$\frac{dx}{x} = -\frac{dy}{y} = -\frac{dz}{z}$$

or, $$\frac{dx}{x} = -\frac{dy}{y}.$$

Now we integrate both sides the above equation as $\int \frac{dx}{x} = -\int \frac{dy}{y}$ and get $xy = c_1$, which is a kind of stream line flows, in *xy* plane. So $xy = c$ is a collection of one kind of different stream lines in *xy* plain. Again we integrate both sides the following equation and get

$$\int \frac{dx}{x} = -\int \frac{dz}{z} \Rightarrow xz = c_2$$

Hence it is another kind of different stream line flows in *xz* plane.

Here c_1 and c_2 are arbitrary constants. Thus we have found two different kinds of the equations of stream line flows, when we have considered upon three dimensional fluid flow.

Now we choose such a four dimensional orthogonal flow. The velocity vector will be written as $\vec{p} = x_1\hat{\tau}_1 - x_2\hat{\tau}_2 - x_3\hat{\tau}_3 - x_4\hat{\tau}_4$ and we take the stream line element on this flow as $ds = dx_1\hat{\tau}_1 + dx_2\hat{\tau}_2 + dx_3\hat{\tau}_3 + dx_4\hat{\tau}_4$. The equation of stream lines will be $\vec{p} \times d\vec{s} = \vec{0}$

or, $(x_1\hat{\tau}_1 - x_2\hat{\tau}_2 - x_3\hat{\tau}_3 - x_4\hat{\tau}_4) \times (dx_1\hat{\tau}_1 + dx_2\hat{\tau}_2 + dx_3\hat{\tau}_3 + dx_4\hat{\tau}_4) = \vec{0}$

or, $$\frac{dx_1}{x_1} = -\frac{dx_2}{x_2} = -\frac{dx_3}{x_3} = -\frac{dx_4}{x_4}$$

Now we integrate the equation $\frac{dx_1}{x_1} = \frac{dx_2}{x_2}$ and get,

$$\int \frac{dx_1}{x_1} = -\int \frac{dx_2}{x_2}$$

or $x_1 x_2 = k_1$, which is a collection of first kind of different stream lines flows.

Again we integrate $\frac{dx_1}{x_1} = -\frac{dx_3}{x_3}$ and get $\int \frac{dx_1}{x_1} = -\int \frac{dx_3}{x_3} \Rightarrow x_1 x_3 = k_2$, which is a collection of second kind of different stream line flows.

But when we equate $\frac{dx_1}{x_1} = -\frac{dx_4}{x_4}$ and after integrating both sides we will find $x_1 x_4 = k_3$. This is a collection of third kind of different stream line flows. Thus when we choose such type of four dimensional orthogonal flow, we will obtain three types of stream line flow.

3. Result and discussion :

As we have found that when we choose a fluid flow of type $\vec{p} = x\hat{i} - y\hat{j}$, the number of different stream lines have been found of one kind that is $xy=t$ in xy plane. When we choose a fluid flow of type $\vec{p} = x\hat{i} - y\hat{j} - z\hat{k}$, then the number of different kinds of stream lines have been found as two, that are $xy=t_1$ and $xz=t_2$, xy plane as well as in xz plane respectively. Similarly, if we take a fluid flow of type $\vec{p} = x_1\hat{\tau}_1 - x_2\hat{\tau}_2 - x_3\hat{\tau}_3 - x_4\hat{\tau}_4$ then we have obtained three different kind of stream lines, these are $x_1 x_2 = t_1$, $x_1 x_3 = t_2$, $x_1 x_4 = t_3$, in x_1x_2 plane, x_1x_3 plane, x_1x_4 plane respectively.

Thus we have observed that as we enhance the number of dimensions of fluid flow the number of different kinds of collection of stream lines also increase. This increase of different kinds of collection of stream lines has been found in an arithmetic sequence.

Thus we have found that, when we choose two dimensional orthogonal flow like $\vec{p} = x\hat{i} - y\hat{j}$, we get a collection of different one kind of stream lines as $xy = c$ in xy

plane. In case of three dimensional orthogonal flow like $\vec{p} = x\hat{i} - y\hat{j} - z\hat{k}$ we have obtained two different collection of stream lines as $xy = c_1$ and $xz=c_2$. Again if we choose four dimensional orthogonal flow such as $\vec{p} = x_1\hat{i}_1 - x_2\hat{i}_2 - x_3\hat{i}_3 - x_4\hat{i}_4$, we find three different collection of stream lines as, $x_1x_2=t_1$, $x_1x_3=t_2$ and $x_1x_4=t_3$, here $c, c_1, c_2, t_1, t_2, t_3$ are all arbitrary constants.

4. Generalization :

We have seen that if we choose a flow like $\vec{p} = x\hat{i} - y\hat{j}$, then the number of different kinds of collection of stream lines has been found of one type that is $xy= t$. In case of fluid flow like $\vec{p} = x\hat{i} - y\hat{j} - z\hat{k}$, we have obtained two different collections of stream lines, which are $xy = t_1$ and $xz = t_2$.

Similarly, if we take a fluid flow like $\vec{p} = x_1\hat{\tau}_1 - x_2\hat{\tau}_2 - x_3\hat{\tau}_3 - x_4\hat{\tau}_4$, we have found three different collections of stream lines as, $x_1\, x_2 = p_1$, $x_1\, x_3 = p_2$, $x_1\, x_4 = p_3$ that are three.

Thus we have observed that two dimensional orthogonal flow type $\vec{p} = x\hat{i} - y\hat{j}$ has a collection of stream lines, such as $xy=t$. In three dimensional orthogonal flow type $\vec{p} = x\hat{i} - y\hat{j} - z\hat{k}$ has two different collections of stream lines such as $xy = t_1$ and $xz = t_2$.

On the basis of these above observations, we can say that in n-dimensional orthogonal flow like,

$$\vec{p} = x_1\hat{\tau}_1 - x_2\hat{\tau}_2 - x_3\hat{\tau}_3 - x_4\hat{\tau}_4 ....x_n\hat{\tau}_n,$$

There will be (n-1) different type of rectangular hyperbolas as stream lines. These will be $x_1\, x_2 = k_1$, $x_1\, x_3 = k_2$, $x_1\, x_4 = k_3$, ……., $x_1\, x_n = k_{n-1}$,

Therefore, total numbers of different kinds of stream lines will be in n-dimensional as follow

$$1 + 2 + 3 + ….. +(n-1) = \frac{(n-1)(n-1+1)}{2}$$

$$= \frac{(n-1)n}{2}$$

Here $t, t_1, t_2, k_1, k_2, k_3, ….., k_{n-1}$ are arbitrary constants.

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