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# Ready-to-Utilize RCM and TOCM for Some Selected Challenging and More Challenging Transportation Problems

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**Abstract**

Every researcher in Transportation Problem (TP) tries to publish a direct method to find the optimal solution directly to any given TP. But, no one direct method has been published so far. Through our research study we have recognized and acknowledged a set of 25 'Challenging' and 'More Challenging' TPs for which no single direct method are available in the literature to produce the optimal solution directly to each. Most of the developed algorithms published so far and also to be developed and published in the future to find the initial basic feasible solution (IBFS) or near optimal solution or optimal solution is based on the Reduced Cost Matrix (RCM) or the Total Opportunity Cost Matrix (TOCM) derived from the given TP. Every time the researcher has to derive the RCM or the TOCM for a given TP in order to apply and test his / her developed algorithm. Actually, it is a time consuming process. To make research easy in TP, we have derived and provided the RCM and the TOCM of each of the recognized 25 TPs. which will facilitate the researchers in TP as a ready to utilize database. We hope this article will definitely boost ease of doing research in TP and hence we are proud to be the wheels of your research journey in TP.

**2010 Mathematics Subject Classification:** 90C08

**Key Words:** Transportation Problem, Optimal Solution, MODA method.

## INTRODUCTION

We assume that the reader of this paper knows the basics of Transportation Problems

(TPs) and the key methods developed during various periods to find the initial basic feasible solution (IBFS) and the MODI and Stepping Stone methods of testing the optimality of the obtained IBFS and also improves to optimal solution, if it is not optimal.

In 2016, Abdul Quddoos and et al. [1] introduced a direct method (but actually not direct) called ASM, based on making allocations to zero entry cells of the reduced cost matrix (RCM) for the given TP, to generate an optimal solution directly to a wide range of TPs. In 2019, Murugesan R. and Esakkiammal T. [13] showed through some illustrative TPs that the ASM method is the method to generate best IBFS only and not a direct method to generate the optimal solution. In 2020, by identifying some difficulties in the allocation process when tie occurs among certain 0-entry cells, Murugesan R. and Esakkiammal T. [14] 'improved' the existing ASM method and named it as IASM method and showed that the later produces better IBFS than the best IBFS produced by the ASM method.

In 2021, Esakkiammal T. and Murugesan R. [3] proposed an innovative approach named SOFTMIN, by the way of constructing the RCM for the given TP, which produces optimal solutions to a wide range of TPs. In 2022, Murugesan R. [9] established that the SOFTMIN method performs much better than the IASM method, but not a direct method to produce optimal solution to any given TP. We further analyzed the process of allocation due to the SOFTMIN method on the near optimal solutions obtained for some challenging TPs, and identified that very few changes made in the allocation process have improved the solution. This resulted the 'Improved SOFTMIN' (or briefly I-SOFT) method [6], which performs much better than the SOFTMIN method.

In May 2022, Murugesan R. [8] has studied and assessed the performance of the recently published 12 direct methods during the past one decade (2012-2021) to find the optimal solution directly for TPs under the title NEEYA? NAANA?. Each of the direct methods was tested on a set of 21 challenging TPs for which no one method had produced optimal solution directly to most of the problems. As a result, it is established and confirmed that each direct method is the one for finding an IBFS only and not a direct method for producing the optimal solution.

In August 2022, Murugesan R. [7] proposed an innovative method named MODA (Modified Allocation) which tests the optimality of an obtained solution and also optimizes the solution, if it is not optimal. It is an easy and alternative method to the existing MODI method.

Again by our further research we have observed that Kirca and Satir (1990) [4] first introduced the concept of Total Opportunity Cost Matrix (TOCM) and applied the Least Cost Method with some tie-breaking policies on the TOCM to determine the feasible solution of the TP. Mathirajan and Meenakshi (2004) [5] extended TOCM of Kirca and Satir by using VAM procedure on the TOCM (called the VAM-TOC, also same as the TOCM-VAM). According to the authors, this approach yielded the optimal solution and about 80% of the time it yielded a solution very close to the optimal (0.5% loss of optimality). Md. Amirul Islam et al. (2012) [2] extended TOCM of Kirca Satir by applying Extremum Difference Method (EDM) techniques on the TOCM (called the TOCM-EDM approach) to determine the IBFS of TP. Md. Amirul

Islam et al. (2012) [2] again applied Highest Cost Difference Method (HCDM) on TOCM (called the TOCM-HCDM approach) to find the IBFS of TP. Aminur Rahman Khan et al. (2015) [2] calculates the pointer cost as the sum all entries in the respective row or column of the TOCM (called the TOCM-SUM approach) to find the IBFS of TP. Aminur Rahman Khan et al. (2015) [2] also applied Russel's Appoximation Method (RAM) on TOCM (called the TOCM-RAM approach) to compute the IBFS of TP. Murugesan and Esakkiammal (2020) [12] compared TOCM-VAM method with the ASM method by testing 50 benchmark TPs and established that the ASM method has produced optimal solution directly to 40 TPs, whereas TOCM-VAM method has produced optimal solution directly to only 27 TPs. Though the ASM method produces optimal solution to a good number of TPs, Murugesan and Esakkiammal (2020) [11] have identified some challenging TPs for which the ASM method produces only near optimal solution.

For getting IBFS to a given TP, various new methods such as EDM, HCDM, SUM and RAM (2015) [2] have been applied on the TOCM of the given TP. But so far, the more efficient ASM method has not been applied on the TOCM of a TP. Hence in 2020 [10], we have published the new approach called TOCM-ASM which applies the ASM method on the TOCM of the given TP. The performance of this approach is compared only with the TOCM-VAM method. Murugesan and Esakkiammal [12] have showed that the TOCM-VAM method produces better IBFS to good number of TPs next to the ASM method. That is why the TOCM-VAM method only is considered for comparison with the published TOCM-ASM approach.

The paper is organized as follows: Section 1 – Briefs the introduction. Section 2 – Lists a set of 11 'Challenging TPs'. Section 3 – Lists a set of 14 'More Challenging TPs'. Section 4 – Provides the RCM and TOCM tables for the 'Challenging TPs'. Section 5 – Provides the RCM and TOCM tables for the 'More Challenging TPs'. Section 6 – Draws the conclusion.

**Derive the Reduced Cost Matrix (RCM)**

a. **Carry out the Row Minimum Subtraction (RMS) Process**. From each of the costs of every row of the balanced TP, subtract the minimum cost of the corresponding row. This will effect in a resultant matrix with at least one 0-entry in every row.
b. **Carry out the Column Minimum Subtraction (CMS) Process**. From each of the costs of every column of the resultant matrix obtained in Step (a), subtract the minimum cost of the corresponding column. This will effect in a further reduced resultant matrix with at least one 0-entry in each column. This obtained reduced matrix is called the Reduced Cost Matrix (RCM). It is noted that each row and each column of an RCM will have at least one 0-entry.

**Derive the Total Opportunity Cost Matrix (TOCM)**

a. **Row Opportunity Cost Matrix (ROCM).** For each row of the given balanced TP, the smallest cost of that row is subtracted from each element of the same row. The resulting matrix is called the ROCM.

b. **Column Opportunity Cost Matrix (COCM).** For each column of the given balanced TP, the smallest cost of that column is subtracted from each element of the same column. The resulting matrix is called the COCM.
c. **Total Opportunity Cost Matrix (TOCM).** The TOCM is obtained by adding the COCM with the ROCM.

## LIST OF CHALLENGING TPs

Through our research we have tested a large numbers of balanced and unbalanced TPs on various direct and other efficient methods and have identified a set of 11 TPs (6 BTPs and 5 UTPs) for which the various methods have produced near optimal solution to most of the problems and rarely produced optimal solution to very few problems. No one direct method has produced optimal solution to each of the problems. Thereby, the listed problems are termed as "Challenging TPs' to the transportation methods. The list of identified 'Challenging TPs' is shown in Table 1 and their minimal Total Transportation Costs (TTCs) are shown in Table 2. The starred ones have alternative optimal solutions.

**Table 1:** A list of 'Challenging TPs'

| **BTP Problem No.** |
|---|
| **Problem 1**<br>$[C_{ij}]$ 3×3= [16 20 12; 14 818; 26 24 16]<br>$[S_i]$ 4×1= [20, 16, 9] $[D_j]$ 1×4= [18, 12, 15] |
| **Problem 2**<br>$[C_{ij}]$ 4×4= [7 5 9 11; 4 3 8 6; 3 8 10 5; 2 6 7 3]<br>$[S_i]$ 4×1= [30, 25, 20, 15] $[D_j]$ 1×4= [30, 30, 20, 10] |
| **Problem 3**<br>$[C_{ij}]$ 4×5= [25 14 34 46 45; 10 47 14 20 4; 22 42 38 21 46; 36 20 41 38 44]<br>$[S_i]$ 4×1= [27, 35, 37, 45] $[D_j]$ 1×5= [22, 27, 28, 33, 34] |
| **Problem 4**<br>$[C_{ij}]$ 5×4= [10 20 5 7; 13 9 12 8; 4 15 7 9; 14 7 1 1; 3 12 5 19] $[S_i]$ 5×1= [200, 300, 200, 400, 400]<br>$[D_j]$ 1×5= [500, 600, 200, 200] |
| **Problem 5**<br>$[C_{ij}]$ 5×6= [5, 3, 7, 3, 8, 5;5, 6, 11, 5, 7, 12;2, 7, 3, 4, 8, 2;9, 7, 10, 5, 10, 9; 5, 3, 7, 3, 7, 5]<br>$[S_i]$ 5×1= [30, 40, 20, 40, 30]<br>$[D_j]$ 1×6= [10, 40, 40, 20, 10, 40] |
| **Problem 6**<br>$[C_{ij}]$ 6×6= [5 1 2 3 4 7; 7 2 3 1 5 6; 9 1 9 5 2 3; 6 5 8 4 1 4; 8 7 11 6 4 5; 2 5 7 5 2 1]<br>$[S_i]$ 6×1= [400, 500, 300, 150, 600, 350]<br>$[D_j]$ 1×6= [300, 500, 700, 300, 250, 250] |

| **UTP Problem No.** |
|---|
| **Problem 1**<br>$[C_{ij}]$ 3×3= [11 21 16; 7 17 13; 11 23 21]<br>$[S_i]$ 3×1= [14, 26, 36] $[D_j]$ 1×3= [18, 28, 25] |
| **Problem 2**<br>$[C_{ij}]$ 3×3 = [15, 22, 17; 11, 17, 16; 20, 25, 21]<br>$[S_i]$ 3×1= [20, 25, 40] $[D_j]$ 1×3= [35, 45, 30] |
| **Problem 3**<br>$[C_{ij}]$ 3×5= [10 8 12 9 3; 4 4 6 6 7; 15 7 11 13 8]<br>$[S_i]$ 3×1= [15, 12, 16] $[D_j]$ 1×5= [8, 8, 4, 7,6] |
| **Problem 4**<br>$[C_{ij}]$ 4×6 = [9, 12, 9, 6, 9, 10; 7, 3, 7, 7, 5, 5;6, 4, 9, 11, 3, 11;6, 8, 11, 2, 2, 10]<br>$[S_i]$ 4×1= [5, 6, 2, 2] $[D_j]$ 1×6= [4, 4, 6, 2, 4, 2] |
| **Problem 5**<br>$[C_{ij}]$ 5×4 = [60 120 75 180; 58 100 60 165; 62 110 65 170; 65 115 80 175; 70 135 85 195]<br>$[S_i]$ 5×1= [8000, 9200, 6250, 4900, 6100]<br>$[D_j]$ 1×4= [5000, 2000, 10000, 6000] |
| -------- |

**Table 2:** Minimal TTC of ‘Challenging TPs’ shown in Table 1

| **BTP #** | **Minimal TTC in $** | **UTP #** | **Minimal TTC in $** |
|---|---|---|---|
| 1. | 592 | 1. | 1133 |
| 2. | 410 | 2. * | 1515 |
| 3. | 3458 | 3. | 193 |
| 4. | 8200 | 4. * | 71 |
| 5. * | 8600 | 5. | 214675 |
| 6. | 6400 | --- | --- |

**LIST OF MORE CHALLENGING TPs**

Through our further research, we have also recognized a set of another 14 TPs (7 BTPs and 7 UTPs) for which the various direct methods have produced near optimal solutions only. Consequently, the identified problems are termed as ‘More Challenging TPs’ to the available transportation methods. The recognized set of ‘More Challenging TPs’ is shown in Table 3 and their minimal TTCs is shown in Table 4. The starred ones have alternative optimal solutions.

**Table 3:** A set of 'More Challenging TPs'

| **BTP Problem No.** |
|---|
| **Problem 1**<br>$[C_{ij}]$ 3×4= [6 1 9 3; 11 5 2 8; 10 12 4 7]<br>$[S_i]$ 3×1= [70, 55, 90]<br>$[D_j]$ 1×4= [85, 35, 50, 45] |
| **Problem 2**<br>$[C_{ij}]$ 3×5= [5 7 10 5 3; 8 69 12 14; 10 9 8 10 15]<br>$[S_i]$ 3×1= [5, 10, 10]<br>$[D_j]$ 1×5= [3, 3, 10, 5, 4] |
| **Problem3**<br>$[C_{ij}]$ 3×5= [1 9 13 36 51; 24 1216 20 1; 14 33 1 23 26]<br>$[S_i]$ 3×1= [50, 100, 150]<br>$[D_j]$ 1×5= [100, 70, 50, 40,40] |
| **Problem 4**<br>$[C_{ij}]$ 4×5= 4 9 810 12; 6 10 3 2 3; 3 2 7 10 3; 3 5 5 4 8]<br>$[S_i]$ 4×1= [24, 18, 20, 16]$[D_j]$ 1×5= [10, 20, 10, 18, 20] |
| **Problem 5**<br>$[C_{ij}]$ 4×6= [1 2 1 4 5 2;3 3 2 1 4 3;4 2 5 9 6 2;3 1 7 3 4 6]<br>$[S_i]$ 4×1= [30, 50, 75, 20]<br>$[D_j]$ 1×6= [20, 40, 30, 10, 50, 25] |
| **Problem 6**<br>$[C_{ij}]$ 5×5= [73 40 9 79 20; 62 93 96 8 13; 96 65 80 50 65; 57 58 29 12 87; 56 23 87 18 12]<br>$[S_i]$ 5×1= [8, 7, 9, 3, 5]<br>$[D_j]$ 1×5= [6, 8, 10, 4, 4] |
| **Problem 7**<br>$[C_{ij}]$ 5×5= [8 8 2 10 2; 11 4 10 9 4; 5 2 2 11 10; 10 6 6 5 2; 8 11 8 6 4]<br>$[S_i]$ 5×1= [40, 70, 35, 90, 85]<br>$[D_j]$ 1×5= [80, 55, 60, 80, 45] |

| **UTP Problem No.** |
|---|
| **Problem 1**<br>$[C_{ij}]$ 3×3= [6 10 14; 12 19 21; 15 14 17]<br>$[S_i]$ 3×1= [50, 50, 50]<br>$[D_j]$ 1×3= [30, 40, 55] |
| **Problem 2**<br>$[C_{ij}]$ 3×3= [4 8 8; 16 24 16; 8 16 24]<br>$[S_i]$ 3×1= [76, 82, 77]<br>$[D_j]$ 1×3= [72, 102, 41] |
| **Problem 3**<br>$[C_{ij}]$ 3×4= [19 30 50 10; 70 30 40 60; 40 8 70 20]<br>$[S_i]$ 3×1= [7, 9, 18]$[D_j]$ 1×4= [40, 8, 7, 14] |

| |
|---|
| **Problem 4**<br>$[C_{ij}]$ 3×4= [10 15 12 12; 8 10 11 9; 11 12 13 10]<br>$[S_i]$ 3×1= [20, 15, 12]<br>$[D_j]$ 1×4= [14, 12, 8, 22] |
| **Problem 5**<br>$[C_{ij}]$ 3×4= [42 48 38 37; 40 49 52 51; 39 38 40 43]<br>$[S_i]$ 3×1= [160, 150, 190]<br>$[D_j]$ 1×4= [80, 90, 110, 160] |
| **Problem 6**<br>$[C_{ij}]$ 3×4= [3 48 14 2; 4 230 10; 36 8 12 12]<br>$[S_i]$ 3×1= [24, 24, 2]<br>$[D_j]$ 1×4= [6, 12, 3, 44] |
| **Problem 7**<br>$[C_{ij}]$ 4×3= [2 7 14; 3 3 1; 5 4 7; 1 6 2]<br>$[S_i]$ 4×1= [5, 8, 7, 15]<br>$[D_j]$ 1×3= [7, 9, 18] |

**Table 4:** Minimal TTC of 'More Challenging TPs' shown in Table 3

| BTP # | Minimal TTC in $ | UTP # | Minimal TTC in $ |
|---|---|---|---|
| 1.* | 1160 | 1. | 1650 |
| 2. | 183 | 2.* | 2424 |
| 3. | 2700 | 3. | 743 |
| 4. | 316 | 4. | 4720 |
| 5. | 430 | 5. | 17050 |
| 6. | 1102 | 6.* | 180 |
| 7. | 1475 | 7. | 75 |

## THE RCM AND TOCM TABLES TO THE CHALLENGING TPs

Most of the algorithms developed to solve the TPs are based on constructing the Reduced Cost Matrix (RCM) or the Total Opportunity Cost Matrix (TOCM). Every time the researcher has to construct the RCM or TOCM of the given TP in order to test the proposed algorithm. In order to ease the works of the researcher, we have constructed and provided the RCM and TOCM of some selected 'Challenging and More Challenging TPs'. Any proposed algorithm tested on the selected TPs will definitely improve the performance level of the proposed algorithm. The RCM and TOCM of the 'Challenging BTPs' are provided under Section 4a and that of for the 'Challenging UTPs' are provided under Section 4b.

### RCM and TOCM Tables to the 'Challenging BTPs'

In this section, the RCM and TOCM tables of each of the 'Challenging BTPs' is shown one by one. In each of the table titles, the term CB refers to 'Challenging Balanced TPs'.

**Table CB-1.1:** Challenging BTP-1, Minimal TTC = $592

| | **Destinations** | | | |
|---|---|---|---|---|
| **Sources** | D1 | D2 | D3 | **Supply** |
| S1 | 16 | 20 | 12 | **20** |
| S2 | 14 | 8 | 18 | **16** |
| S3 | 26 | 24 | 16 | **09** |
| Demand | **18** | **12** | **15** | **45** |

**Table CB-1.2:** The RCM of BTP-1

| | **Destinations** | | | |
|---|---|---|---|---|
| **Sources** | D1 | D2 | D3 | **Supply** |
| S1 | 0 | 8 | 0 | **20** |
| S2 | 2 | 0 | 10 | **16** |
| S3 | 6 | 8 | 0 | **09** |
| Demand | **18** | **12** | **15** | **45** |

**Table CB-1.3:** The TOCM of BTP-1

| | **Destinations** | | | |
|---|---|---|---|---|
| **Sources** | D1 | D2 | D3 | **Supply** |
| S1 | 6 | 20 | 0 | **20** |
| S2 | 6 | 0 | 16 | **16** |
| S3 | 22 | 24 | 4 | **09** |
| Demand | **18** | **12** | **15** | **45** |

**Table CB-2.1:** Challenging BTP-2, Minimal TTC = $410

| | **Destinations** | | | | |
|---|---|---|---|---|---|
| **Sources** | D1 | D2 | D3 | D4 | **Supply** |
| S1 | 7 | 5 | 9 | 11 | **30** |
| S2 | 4 | 3 | 8 | 6 | **25** |
| S3 | 3 | 8 | 10 | 5 | **20** |
| S4 | 2 | 6 | 7 | 3 | **15** |
| Demand | **30** | **30** | **20** | **10** | **90** |

**Table CB-2.2:** The RCM of BTP-2

| Sources | Destinations | | | | Supply |
|---|---|---|---|---|---|
| | D1 | D2 | D3 | D4 | |
| S1 | 2 | 0 | 0 | 5 | **30** |
| S2 | 1 | 0 | 1 | 2 | **25** |
| S3 | 0 | 5 | 3 | 1 | **20** |
| S4 | 0 | 4 | 1 | 0 | **15** |
| Demand | **30** | **30** | **20** | **10** | **90** |

**Table CB-2.3:** The TOCM of BTP-2

| Sources | Destinations | | | | Supply |
|---|---|---|---|---|---|
| | D1 | D2 | D3 | D4 | |
| S1 | 7 | 2 | 6 | 14 | **30** |
| S2 | 3 | 0 | 6 | 6 | **25** |
| S3 | 1 | 10 | 10 | 4 | **20** |
| S4 | 0 | 7 | 5 | 1 | **15** |
| Demand | **30** | **30** | **20** | **10** | **90** |

**Table CB-3.1:** Challenging BTP-3, Minimal TTC = $3458

| Sources | Destinations | | | | | Supply |
|---|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | **D5** | |
| **S1** | 25 | 14 | 34 | 46 | 45 | **27** |
| **S2** | 10 | 47 | 14 | 20 | 41 | **35** |
| **S3** | 22 | 42 | 38 | 21 | 46 | **37** |
| **S4** | 36 | 20 | 41 | 38 | 44 | **45** |
| Demand | **22** | **27** | **28** | **33** | **34** | **144** |

**Table CB-3.2:** The RCM of BTP-3

| Sources | Destinations | | | | | Supply |
|---|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | **D5** | |
| **S1** | 10 | 0 | 10 | 32 | 31 | **27** |
| **S2** | 5 | 43 | 0 | 16 | 0 | **35** |
| **S3** | 0 | 21 | 7 | 0 | 25 | **37** |
| **S4** | 15 | 0 | 11 | 18 | 24 | **45** |
| Demand | **22** | **27** | **28** | **33** | **34** | **144** |

**Table CB-3.3:** The TOCM of BTP-3

| Sources | Destinations | | | | | Supply |
|---|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | **D5** | |
| **S1** | 26 | 0 | 40 | 58 | 72 | **27** |
| **S2** | 6 | 76 | 10 | 16 | 0 | **35** |
| **S3** | 13 | 49 | 41 | 1 | 67 | **37** |
| **S4** | 42 | 6 | 48 | 36 | 64 | **45** |
| Demand | **22** | **27** | **28** | **33** | **34** | **144** |

**Table CB-4.1:** Challenging BTP-4, Minimal TTC = $8200

| Source | Destinations | | | | Supply |
|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | |
| **S1** | 10 | 20 | 5 | 7 | **200** |
| **S2** | 13 | 9 | 12 | 8 | **300** |
| **S3** | 4 | 15 | 7 | 9 | **200** |
| **S4** | 14 | 7 | 1 | 1 | **400** |
| **S5** | 3 | 12 | 5 | 19 | **400** |
| Demand | **500** | **600** | **200** | **200** | **1500** |

**Table CB-4.2:** The RCM of BTP-4

| Source | Destinations | | | | Supply |
|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | |
| **S1** | 5 | 14 | 0 | 2 | **200** |
| **S2** | 5 | 0 | 4 | 0 | **300** |
| **S3** | 0 | 10 | 3 | 5 | **200** |
| **S4** | 13 | 5 | 0 | 0 | **400** |
| **S5** | 0 | 8 | 2 | 16 | **400** |
| Demand | **500** | **600** | **200** | **200** | **1500** |

**Table CB-4.3:** The TOCM of BTP-4

| Source | Destinations | | | | Supply |
|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | |
| **S1** | 12 | 28 | 4 | 8 | **200** |
| **S2** | 15 | 3 | 15 | 7 | **300** |
| **S3** | 1 | 19 | 9 | 13 | **200** |
| **S4** | 24 | 6 | 0 | 0 | **400** |
| **S5** | 0 | 14 | 6 | 34 | **400** |
| Demand | **500** | **600** | **200** | **200** | **1500** |

**Table CB-5.1:** Challenging BTP-5, Minimal TTC = $8600

| | **Destinations** | | | | | | |
|---|---|---|---|---|---|---|---|
| **Sources** | **D1** | **D2** | **D3** | **D4** | **D5** | **D6** | **Supply** |
| **S1** | 5 | 3 | 7 | 3 | 8 | 5 | **300** |
| **S2** | 5 | 6 | 11 | 5 | 7 | 12 | **400** |
| **S3** | 2 | 7 | 3 | 4 | 8 | 2 | **200** |
| **S4** | 9 | 7 | 10 | 5 | 10 | 9 | **400** |
| **S5** | 5 | 3 | 7 | 3 | 7 | 5 | **300** |
| Demand | **100** | **400** | **400** | **200** | **100** | **400** | **1600** |

**Table CB-5.2:** The RCM of BTP-5

| | **Destinations** | | | | | | |
|---|---|---|---|---|---|---|---|
| **Sources** | **D1** | **D2** | **D3** | **D4** | **D5** | **D6** | **Supply** |
| **S1** | 2 | 0 | 3 | 0 | 3 | 2 | **300** |
| **S2** | 0 | 1 | 5 | 0 | 0 | 7 | **400** |
| **S3** | 0 | 5 | 0 | 2 | 4 | 0 | **200** |
| **S4** | 4 | 2 | 4 | 0 | 3 | 4 | **400** |
| **S5** | 2 | 0 | 3 | 0 | 2 | 2 | **300** |
| Demand | **100** | **400** | **400** | **200** | **100** | **400** | **1600** |

**Table CB-5.3:** The TOCM of BTP-5

| | **Destinations** | | | | | | |
|---|---|---|---|---|---|---|---|
| **Sources** | **D1** | **D2** | **D3** | **D4** | **D5** | **D6** | **Supply** |
| **S1** | 5 | 0 | 8 | 0 | 6 | 5 | **300** |
| **S2** | 3 | 4 | 14 | 2 | 2 | 17 | **400** |
| **S3** | 0 | 9 | 1 | 3 | 7 | 0 | **200** |
| **S4** | 11 | 6 | 12 | 2 | 8 | 11 | **400** |
| **S5** | 5 | 0 | 8 | 0 | 4 | 5 | **300** |
| Demand | **100** | **400** | **400** | **200** | **100** | **400** | **1600** |

**Table CB-6.1:** Challenging BTP-6, Minimal TTC = $6400

| | **Destinations** | | | | | | |
|---|---|---|---|---|---|---|---|
| **Sources** | **D1** | **D2** | **D3** | **D4** | **D5** | **D6** | **Supply** |
| **S1** | 5 | 1 | 2 | 3 | 4 | 7 | **400** |
| **S2** | 7 | 2 | 3 | 1 | 5 | 6 | **500** |
| **S3** | 9 | 1 | 9 | 5 | 2 | 3 | **300** |
| **S4** | 6 | 5 | 8 | 4 | 1 | 4 | **150** |
| **S5** | 8 | 7 | 11 | 6 | 4 | 5 | **600** |
| **S6** | 2 | 5 | 7 | 5 | 2 | 1 | **350** |
| Demand | **300** | **500** | **700** | **300** | **250** | **250** | **2300** |

**Table CB-6.2:** The RCM of BTP-6

| Sources | Destinations | | | | | | Supply |
|---|---|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | **D5** | **D6** | |
| **S1** | 3 | 0 | 0 | 2 | 3 | 6 | **400** |
| **S2** | 5 | 1 | 1 | 0 | 4 | 5 | **500** |
| **S3** | 7 | 0 | 7 | 4 | 1 | 2 | **300** |
| **S4** | 4 | 4 | 6 | 3 | 0 | 3 | **150** |
| **S5** | 3 | 3 | 6 | 2 | 0 | 1 | **600** |
| **S6** | 0 | 4 | 5 | 4 | 1 | 0 | **350** |
| Demand | **300** | **500** | **700** | **300** | **250** | **250** | **2300** |

**Table CB-6.3:** The TOCM of BTP-6

| Sources | Destinations | | | | | | Supply |
|---|---|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | **D5** | **D6** | |
| **S1** | 7 | 0 | 1 | 4 | 6 | 12 | **400** |
| **S2** | 11 | 2 | 3 | 0 | 8 | 10 | **500** |
| **S3** | 15 | 0 | 15 | 4 | 2 | 4 | **300** |
| **S4** | 9 | 8 | 13 | 6 | 0 | 6 | **150** |
| **S5** | 10 | 9 | 16 | 7 | 3 | 5 | **600** |
| **S6** | 1 | 8 | 11 | 8 | 2 | 0 | **350** |
| Demand | **300** | **500** | **700** | **300** | **250** | **250** | **2300** |

**The RCM and TOCM Tables to the 'Challenging UTPs'**

In this section, in each of the tables the term CU refers to 'Challenging Unbalanced TPs'.

**Table CU-1.1:** Challenging UTP-1, Minimal TTC = $1133

| Sources | Destinations | | | | Supply |
|---|---|---|---|---|---|
| | D1 | D2 | D3 | D4 | |
| S1 | 11 | 21 | 16 | 0 | **14** |
| S2 | 7 | 17 | 13 | 0 | **26** |
| S3 | 11 | 23 | 21 | 0 | **36** |
| Demand | **18** | **28** | **25** | **5** | **76** |

**Table CU-1.2:** The RCM of UTP-1

| **Sources** | **Destinations** | | | | **Supply** |
|---|---|---|---|---|---|
| | D1 | D2 | D3 | D4 | |
| S1 | 4 | 4 | 3 | 0 | **14** |
| S2 | 0 | 0 | 0 | 0 | **26** |
| S3 | 4 | 6 | 8 | 0 | **36** |
| Demand | **18** | **28** | **25** | **5** | **76** |

**Table CU-1.3:** The TOCM of UTP-1

| **Sources** | **Destinations** | | | | **Supply** |
|---|---|---|---|---|---|
| | D1 | D2 | D3 | D4 | |
| S1 | 15 | 25 | 19 | 0 | **14** |
| S2 | 7 | 17 | 13 | 0 | **26** |
| S3 | 15 | 29 | 29 | 0 | **36** |
| Demand | **18** | **28** | **25** | **5** | **76** |

**Table CU-2.1:** Challenging UTP-2, Minimal TTC = $1515

| **Sources** | **Destinations** | | | **Supply** |
|---|---|---|---|---|
| | **D1** | **D2** | **D3** | |
| **S1** | 15 | 22 | 17 | **20** |
| **S2** | 11 | 17 | 16 | **25** |
| **S3** | 20 | 25 | 21 | **40** |
| **S4** | 0 | 0 | 0 | **25** |
| Demand | **35** | **45** | **30** | **110** |

**Table CU-2.2:** The RCM of UTP-2

| **Sources** | **Destinations** | | | **Supply** |
|---|---|---|---|---|
| | **D1** | **D2** | **D3** | |
| **S1** | 0 | 7 | 2 | **20** |
| **S2** | 0 | 6 | 5 | **25** |
| **S3** | 0 | 5 | 1 | **40** |
| **S4** | 0 | 0 | 0 | **25** |
| Demand | **35** | **45** | **30** | **110** |

**Table CU-2.3:** The TOCM of UTP-2

| Sources | Destinations | | | Supply |
|---|---|---|---|---|
| | **D1** | **D2** | **D3** | |
| **S1** | 15 | 29 | 19 | **20** |
| **S2** | 11 | 23 | 21 | **25** |
| **S3** | 20 | 30 | 22 | **40** |
| **S4** | 0 | 0 | 0 | **25** |
| Demand | **35** | **45** | **30** | **110** |

**Table CU-3.1:** Challenging UTP-3, Minimal TTC = $193

| Sources | Destinations | | | | | | Supply |
|---|---|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | **D5** | **D6** | |
| **S1** | 10 | 8 | 12 | 9 | 3 | 0 | **15** |
| **S2** | 4 | 4 | 6 | 6 | 7 | 0 | **12** |
| **S3** | 15 | 7 | 11 | 13 | 8 | 0 | **16** |
| Demand | **8** | **8** | **4** | **7** | **6** | **10** | **43** |

**Table CB-3.2:** The RCM of UTP-5

| Sources | Destinations | | | | | | Supply |
|---|---|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | **D5** | **D6** | |
| **S1** | 6 | 4 | 6 | 3 | 0 | 0 | **15** |
| **S2** | 0 | 0 | 0 | 0 | 4 | 0 | **12** |
| **S3** | 11 | 3 | 5 | 7 | 5 | 0 | **16** |
| Demand | **8** | **8** | **4** | **7** | **6** | **10** | **43** |

**Table CB-3.3:** The TOCM of UTP-5

| Sources | Destinations | | | | | | Supply |
|---|---|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | **D5** | **D6** | |
| **S1** | 16 | 12 | 18 | 12 | 3 | 0 | **15** |
| **S2** | 4 | 4 | 6 | 6 | 11 | 0 | **12** |
| **S3** | 26 | 10 | 16 | 20 | 13 | 0 | **16** |
| Demand | **8** | **8** | **4** | **7** | **6** | **10** | **43** |

**Table CU-4.1:** Challenging UTP-4, Minimal TTC = $71

| Sources | Destinations | | | | | | Supply |
|---|---|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | **D5** | **D6** | |
| **S1** | 9 | 12 | 9 | 6 | 9 | 10 | **5** |
| **S2** | 7 | 3 | 7 | 7 | 5 | 5 | **6** |
| **S3** | 6 | 4 | 9 | 11 | 3 | 11 | **2** |
| **S4** | 6 | 8 | 11 | 2 | 2 | 10 | **2** |
| **S5** | 0 | 0 | 0 | 0 | 0 | 0 | **7** |
| Demand | **4** | **4** | **6** | **2** | **4** | **2** | **22** |

**Table CU-4.2:** The RCM of UTP-4

| **Sources** | **Destinations** | | | | | | **Supply** |
|---|---|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | **D5** | **D6** | |
| **S1** | 3 | 6 | 3 | 0 | 3 | 4 | **5** |
| **S2** | 4 | 0 | 4 | 4 | 2 | 2 | **6** |
| **S3** | 3 | 1 | 6 | 8 | 0 | 8 | **2** |
| **S4** | 4 | 6 | 9 | 0 | 0 | 8 | **2** |
| **S5** | 0 | 0 | 0 | 0 | 0 | 0 | **7** |
| Demand | **4** | **4** | **6** | **2** | **4** | **2** | **22** |

**Table CU-4.3:** The TOCM of UTP-4

| **Sources** | **Destinations** | | | | | | **Supply** |
|---|---|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | **D5** | **D6** | |
| **S1** | 12 | 18 | 12 | 6 | 12 | 14 | **5** |
| **S2** | 11 | 3 | 11 | 11 | 7 | 7 | **6** |
| **S3** | 9 | 5 | 15 | 19 | 3 | 19 | **2** |
| **S4** | 10 | 14 | 20 | 2 | 2 | 18 | **2** |
| **S5** | 0 | 0 | 0 | 0 | 0 | 0 | **7** |
| Demand | **4** | **4** | **6** | **2** | **4** | **2** | **22** |

**Table CB-5.1:** Challenging UTP-5, Minimal TTC = $214675

| **Sources** | **Destinations** | | | | | **Supply** |
|---|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | **D5** | |
| **S1** | 60 | 120 | 75 | 180 | 0 | **800** |
| **S2** | 58 | 100 | 60 | 165 | 0 | **920** |
| **S3** | 62 | 110 | 65 | 170 | 0 | **625** |
| **S4** | 65 | 115 | 80 | 175 | 0 | **490** |
| **S5** | 70 | 135 | 85 | 195 | 0 | **610** |
| Demand | **500** | **200** | **1000** | **600** | **1145** | **3445** |

**Table CB-5.2:** The RCM of UTP-5

| Sources | Destinations | | | | | Supply |
|---|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | **D5** | |
| **S1** | 2 | 20 | 15 | 15 | 0 | **800** |
| **S2** | 0 | 0 | 0 | 0 | 0 | **920** |
| **S3** | 4 | 10 | 5 | 5 | 0 | **625** |
| **S4** | 7 | 15 | 20 | 10 | 0 | **490** |
| **S5** | 12 | 35 | 25 | 30 | 0 | **610** |
| Demand | **500** | **200** | **1000** | **600** | **1145** | **3445** |

**Table CB-5.3:** The TOCM of UTP-5

| Sources | Destinations | | | | | Supply |
|---|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | **D5** | |
| **S1** | 62 | 140 | 90 | 195 | 0 | **800** |
| **S2** | 58 | 100 | 60 | 165 | 0 | **920** |
| **S3** | 66 | 120 | 70 | 175 | 0 | **625** |
| **S4** | 72 | 130 | 100 | 185 | 0 | **490** |
| **S5** | 82 | 170 | 110 | 225 | 0 | **610** |
| Demand | **500** | **200** | **1000** | **600** | **1145** | **3445** |

**THE RCM AND TOCM TABLES TO THE MORE CHALLENGING TPs**

The RCM and TOCM tables of the 'More Challenging BTPs' are shown under 5a and that of by the 'More Challenging UTPs' are shown under 5b.

**The RCM and TOCM Tables to the More Challenging BTPs**

In this section, the RCM and TOCM tables of each of the 'More Challenging BTPs' are shown one by one. In each of the table titles the term MB refers to 'More challenging Balanced TPs'.

**Table MB-1.1:** Challenging BTP-1, Minimal TTC = $1160

| Sources | Destinations | | | | Supply |
|---|---|---|---|---|---|
| | D1 | D2 | D3 | D4 | |
| S1 | 6 | 1 | 9 | 3 | **70** |
| S2 | 11 | 5 | 2 | 8 | **55** |
| S3 | 10 | 12 | 4 | 7 | **90** |
| Demand | **85** | **35** | **50** | **45** | **215** |

**Table MB-1.2:** The RCM of BTP-1

| | **Destinations** | | | | |
|---|---|---|---|---|---|
| **Sources** | D1 | D2 | D3 | D4 | **Supply** |
| S1 | 0 | 0 | 8 | 0 | **70** |
| S2 | 4 | 3 | 0 | 4 | **55** |
| S3 | 1 | 8 | 0 | 1 | **90** |
| Demand | **85** | **35** | **50** | **45** | **215** |

**Table MB-1.3:** The TOCM of BTP-1

| | **Destinations** | | | | |
|---|---|---|---|---|---|
| **Sources** | D1 | D2 | D3 | D4 | **Supply** |
| S1 | 5 | 0 | 15 | 2 | **70** |
| S2 | 14 | 7 | 0 | 11 | **55** |
| S3 | 10 | 19 | 2 | 7 | **90** |
| Demand | **85** | **35** | **50** | **45** | **215** |

**Table MB-2.1:** More Challenging BTP-2, Minimal TTC = $183

| | **Destinations** | | | | | |
|---|---|---|---|---|---|---|
| **Sources** | **D1** | **D2** | **D3** | **D4** | **D5** | **Supply** |
| **S1** | 5 | 7 | 10 | 5 | 3 | **5** |
| **S2** | 8 | 6 | 9 | 12 | 14 | **10** |
| **S3** | 10 | 9 | 8 | 10 | 15 | **10** |
| Demand | **3** | **3** | **10** | **5** | **4** | **25** |

**Table MB-2.2:** The RCM of BTP-2

| | **Destinations** | | | | | |
|---|---|---|---|---|---|---|
| **Sources** | **D1** | **D2** | **D3** | **D4** | **D5** | **Supply** |
| **S1** | 0 | 4 | 7 | 0 | 0 | **5** |
| **S2** | 0 | 0 | 3 | 4 | 8 | **10** |
| **S3** | 0 | 1 | 0 | 0 | 7 | **10** |
| Demand | **3** | **3** | **10** | **5** | **4** | **25** |

**Table MB-2.3:** The TOCM of BTP-2

| | **Destinations** | | | | | |
|---|---|---|---|---|---|---|
| **Sources** | **D1** | **D2** | **D3** | **D4** | **D5** | **Supply** |
| **S1** | 2 | 5 | 9 | 2 | 0 | **5** |
| **S2** | 5 | 0 | 4 | 13 | 19 | **10** |
| **S3** | 7 | 4 | 0 | 7 | 19 | **10** |
| Demand | **3** | **3** | **10** | **5** | **4** | **25** |

**Table MB-3.1:** More Challenging BTP-3, Minimal TTC = $2700

| | **Destinations** | | | | | |
|---|---|---|---|---|---|---|
| **Sources** | **D1** | **D2** | **D3** | **D4** | **D5** | **Supply** |
| **S1** | 1 | 9 | 13 | 36 | 51 | **50** |
| **S2** | 24 | 12 | 16 | 20 | 1 | **100** |
| **S3** | 14 | 33 | 1 | 23 | 26 | **150** |
| Demand | **100** | **70** | **50** | **40** | **40** | **300** |

**Table MB-3.2:** The RCM of BTP-3

| | **Destinations** | | | | | |
|---|---|---|---|---|---|---|
| **Sources** | **D1** | **D2** | **D3** | **D4** | **D5** | **Supply** |
| **S1** | 0 | 0 | 12 | 16 | 50 | **50** |
| **S2** | 23 | 3 | 15 | 0 | 0 | **100** |
| **S3** | 13 | 24 | 0 | 3 | 25 | **150** |
| Demand | **100** | **70** | **50** | **40** | **40** | **300** |

**Table MB-3.3:** The TOCM of BTP-3

| | **Destinations** | | | | | |
|---|---|---|---|---|---|---|
| **Sources** | **D1** | **D2** | **D3** | **D4** | **D5** | **Supply** |
| **S1** | 0 | 8 | 24 | 51 | 100 | **50** |
| **S2** | 46 | 14 | 30 | 19 | 0 | **100** |
| **S3** | 26 | 56 | 0 | 25 | 50 | **150** |
| Demand | **100** | **70** | **50** | **40** | **40** | **300** |

**Table MB-4.1:** More Challenging BTP-4, Minimal TTC = $316

| | **Destinations** | | | | | |
|---|---|---|---|---|---|---|
| **Sources** | **D1** | **D2** | **D3** | **D4** | **D5** | **Supply** |
| **S1** | 4 | 9 | 8 | 10 | 12 | **24** |
| **S2** | 6 | 10 | 3 | 2 | 3 | **18** |
| **S3** | 3 | 2 | 7 | 10 | 3 | **20** |
| **S4** | 3 | 5 | 5 | 4 | 8 | **16** |
| Demand | **10** | **20** | **10** | **18** | **20** | **78** |

**Table MB-4.2:** The RCM of BTP-4

| Sources | Destinations | | | | | Supply |
|---|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | **D5** | |
| **S1** | 0 | 5 | 3 | 6 | 7 | **24** |
| **S2** | 4 | 8 | 0 | 0 | 0 | **18** |
| **S3** | 1 | 0 | 4 | 8 | 0 | **20** |
| **S4** | 0 | 2 | 1 | 1 | 4 | **16** |
| Demand | **10** | **20** | **10** | **18** | **20** | **78** |

**Table MB-4.3:** The TOCM of BTP-4

| Sources | Destinations | | | | | Supply |
|---|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | **D5** | |
| **S1** | 1 | 12 | 9 | 14 | 17 | **24** |
| **S2** | 7 | 16 | 1 | 0 | 1 | **18** |
| **S3** | 1 | 0 | 9 | 16 | 1 | **20** |
| **S4** | 0 | 5 | 4 | 3 | 10 | **16** |
| Demand | **10** | **20** | **10** | **18** | **20** | **78** |

**Table MB-5.1:** More Challenging BTP-5, Minimal TTC = $430

| Sources | Destinations | | | | | | Supply |
|---|---|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | **D5** | **D6** | |
| **S1** | 1 | 2 | 1 | 4 | 5 | 2 | **30** |
| **S2** | 3 | 3 | 2 | 1 | 4 | 3 | **50** |
| **S3** | 4 | 2 | 5 | 9 | 6 | 2 | **75** |
| **S4** | 3 | 1 | 7 | 3 | 4 | 6 | **20** |
| Demand | **20** | **40** | **30** | **10** | **50** | **25** | **175** |

**Table MB-5.2:** The RCM of BTP-5

| Sources | Destinations | | | | | | Supply |
|---|---|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | **D5** | **D6** | |
| **S1** | 0 | 1 | 0 | 3 | 1 | 1 | **30** |
| **S2** | 2 | 2 | 1 | 0 | 0 | 2 | **50** |
| **S3** | 2 | 0 | 3 | 7 | 1 | 0 | **75** |
| **S4** | 2 | 0 | 6 | 2 | 0 | 5 | **20** |
| Demand | **20** | **40** | **30** | **10** | **50** | **25** | **175** |

**Table MB-5.3:** The TOCM of BTP-5

| Sources | Destinations | | | | | | Supply |
|---|---|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | **D5** | **D6** | |
| **S1** | 0 | 2 | 0 | 6 | 5 | 1 | **30** |
| **S2** | 4 | 4 | 2 | 0 | 3 | 3 | **50** |
| **S3** | 5 | 1 | 7 | 15 | 6 | 0 | **75** |
| **S4** | 4 | 0 | 12 | 4 | 3 | 9 | **20** |
| Demand | **20** | **40** | **30** | **10** | **50** | **25** | **175** |

**Table MB-6.1:** More Challenging BTP-6, Minimal TTC = $1102

| Sources | Destinations | | | | | Supply |
|---|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | **D5** | |
| **S1** | 73 | 40 | 9 | 79 | 20 | **8** |
| **S2** | 62 | 93 | 96 | 8 | 13 | **7** |
| **S3** | 96 | 65 | 80 | 50 | 65 | **9** |
| **S4** | 57 | 58 | 29 | 12 | 87 | **3** |
| **S5** | 56 | 23 | 87 | 18 | 12 | **5** |
| Demand | **6** | **8** | **10** | **4** | **4** | **32** |

**Table MB-6.2:** The RCM of BTP-6

| Sources | Destinations | | | | | Supply |
|---|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | **D5** | |
| **S1** | 20 | 20 | 0 | 70 | 11 | **8** |
| **S2** | 10 | 74 | 88 | 0 | 5 | **7** |
| **S3** | 2 | 4 | 30 | 0 | 15 | **9** |
| **S4** | 1 | 35 | 17 | 0 | 75 | **3** |
| **S5** | 0 | 0 | 75 | 6 | 0 | **5** |
| Demand | **6** | **8** | **10** | **4** | **4** | **32** |

**Table MB-6.3:** The TOCM of BTP-6

| Sources | Destinations | | | | | Supply |
|---|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | **D5** | |
| **S1** | 81 | 48 | 0 | 141 | 19 | **8** |
| **S2** | 60 | 155 | 175 | 0 | 6 | **7** |
| **S3** | 86 | 57 | 101 | 42 | 68 | **9** |
| **S4** | 46 | 81 | 37 | 4 | 150 | **3** |
| **S5** | 44 | 11 | 153 | 16 | 0 | **5** |
| Demand | **6** | **8** | **10** | **4** | **4** | **32** |

**Table MB-7.1:** More Challenging BTP-7, Minimal TTC = $1475

| Sources | Destinations | | | | | Supply |
|---|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | **D5** | |
| **S1** | 8 | 8 | 2 | 10 | 2 | **40** |
| **S2** | 11 | 4 | 10 | 9 | 4 | **70** |
| **S3** | 5 | 2 | 2 | 11 | 10 | **35** |
| **S4** | 10 | 6 | 6 | 5 | 2 | **90** |
| **S5** | 8 | 11 | 8 | 6 | 4 | **85** |
| Demand | **80** | **55** | **60** | **80** | **45** | **320** |

**Table MB-7.2:** The RCM of BTP-7

| Sources | Destinations | | | | | Supply |
|---|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | **D5** | |
| **S1** | 3 | 6 | 0 | 6 | 0 | **40** |
| **S2** | 4 | 0 | 6 | 3 | 0 | **70** |
| **S3** | 0 | 0 | 0 | 7 | 8 | **35** |
| **S4** | 5 | 4 | 4 | 1 | 0 | **90** |
| **S5** | 1 | 7 | 4 | 0 | 0 | **85** |
| Demand | **80** | **55** | **60** | **80** | **45** | **320** |

**Table MB-7.3:** The TOCM of BTP-7

| Sources | Destinations | | | | | Supply |
|---|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | **D5** | |
| **S1** | 9 | 12 | 0 | 13 | 0 | **40** |
| **S2** | 13 | 2 | 14 | 9 | 2 | **70** |
| **S3** | 3 | 0 | 0 | 15 | 16 | **35** |
| **S4** | 13 | 8 | 8 | 3 | 0 | **90** |
| **S5** | 7 | 16 | 10 | 3 | 2 | **85** |
| Demand | **80** | **55** | **60** | **80** | **45** | **320** |

**The RCM and TOCM Tables to the 'More Challenging UTPs'**

In this section, the RCM and TOCM tables of each of the 'More Challenging UTPs' are shown one by one. In each of the table titles the term MU refers to 'More challenging Unbalanced TPs'.

**Table MU-1.1:** More Challenging UTP-1, Minimal TTC = $1650

| Source | Destinations | | | | Supply |
|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | |
| **S1** | 6 | 10 | 14 | 0 | **50** |
| **S2** | 12 | 19 | 21 | 0 | **50** |
| **S3** | 15 | 14 | 17 | 0 | **50** |
| Demand | **30** | **40** | **55** | **25** | **150** |

**Table MU-1.2:** The RCM of UTP-1

| Source | Destinations | | | | Supply |
|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | |
| **S1** | 6 | 0 | 0 | 0 | **50** |
| **S2** | 6 | 9 | 7 | 0 | **50** |
| **S3** | 9 | 4 | 3 | 0 | **50** |
| Demand | **30** | **40** | **55** | **25** | **150** |

**Table MU-1.3:** The TOCM of UTP-1

| Source | Destinations | | | | Supply |
|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | |
| **S1** | 6 | 10 | 14 | 0 | **50** |
| **S2** | 18 | 28 | 28 | 0 | **50** |
| **S3** | 24 | 18 | 20 | 0 | **50** |
| Demand | **30** | **40** | **55** | **25** | **150** |

**Table MU-2.1:** Challenging UTP-2, Minimal TTC = $2424

| Sources | Destinations | | | | Supply |
|---|---|---|---|---|---|
| | D1 | D2 | D3 | D4 | |
| S1 | 4 | 8 | 8 | 0 | **76** |
| S2 | 16 | 24 | 16 | 0 | **82** |
| S3 | 8 | 16 | 24 | 0 | **77** |
| Demand | **72** | **102** | **41** | **20** | **235** |

**Table MU-2.2:** The RCM of UTP-2

| Sources | Destinations | | | | Supply |
|---|---|---|---|---|---|
| | D1 | D2 | D3 | D4 | |
| S1 | 0 | 0 | 0 | 0 | **76** |
| S2 | 12 | 16 | 8 | 0 | **82** |
| S3 | 4 | 8 | 16 | 0 | **77** |
| Demand | **72** | **102** | **41** | **20** | **235** |

**Table MU-2.3:** The TOCM of UTP-2

| Sources | Destinations | | | | Supply |
|---|---|---|---|---|---|
| | D1 | D2 | D3 | D4 | |
| S1 | 4 | 8 | 8 | 0 | **76** |
| S2 | 28 | 40 | 24 | 0 | **82** |
| S3 | 12 | 24 | 40 | 0 | **77** |
| Demand | **72** | **102** | **41** | **20** | **235** |

**Table MU-3.1:** More Challenging UTP-3, Minimal TTC = $743

| Sources | Destinations | | | | Supply |
|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | |
| **S1** | 19 | 30 | 50 | 10 | **7** |
| **S2** | 70 | 30 | 40 | 60 | **9** |
| **S3** | 40 | 8 | 70 | 20 | **18** |
| **S4** | 0 | 0 | 0 | 0 | **35** |
| Demand | **40** | **8** | **7** | **14** | **69** |

**Table MU-3.2:** The RCM of UTP-3

| Sources | Destinations | | | | Supply |
|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | |
| **S1** | 9 | 20 | 40 | 0 | **7** |
| **S2** | 40 | 0 | 10 | 30 | **9** |
| **S3** | 32 | 0 | 62 | 12 | **18** |
| **S4** | 0 | 0 | 0 | 0 | **35** |
| Demand | **40** | **8** | **7** | **14** | **69** |

**Table MU-3.3:** The TOCM of UTP-3

| Sources | Destinations | | | | Supply |
|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | |
| **S1** | 28 | 50 | 90 | 10 | **7** |
| **S2** | 110 | 30 | 50 | 90 | **9** |
| **S3** | 72 | 8 | 132 | 32 | **18** |
| **S4** | 0 | 0 | 0 | 0 | **35** |
| Demand | **40** | **8** | **7** | **14** | **69** |

**Table MU-4.1:** More Challenging UTP-4, Minimal TTC = $4720

| Sources | Destinations | | | | Supply |
|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | |
| **S1** | 10 | 15 | 12 | 12 | **20** |
| **S2** | 8 | 10 | 11 | 9 | **15** |
| **S3** | 11 | 12 | 13 | 10 | **12** |
| **S4** | 0 | 0 | 0 | 0 | **09** |
| Demand | **14** | **12** | **08** | **22** | **56** |

**Table MU-4.2:** The RCM of UTP-4

| Sources | Destinations | | | | Supply |
|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | |
| **S1** | 0 | 5 | 2 | 2 | **20** |
| **S2** | 0 | 2 | 3 | 1 | **15** |
| **S3** | 1 | 2 | 3 | 0 | **12** |
| **S4** | 0 | 0 | 0 | 0 | **09** |
| Demand | **14** | **12** | **08** | **22** | **56** |

**Table MU-4.3:** The TOCM of UTP-4

| Sources | Destinations | | | | Supply |
|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | |
| **S1** | 10 | 20 | 14 | 14 | **20** |
| **S2** | 8 | 12 | 14 | 10 | **15** |
| **S3** | 12 | 14 | 16 | 10 | **12** |
| **S4** | 0 | 0 | 0 | 0 | **09** |
| Demand | **14** | **12** | **08** | **22** | **56** |

**Table MU-5.1:** More Challenging UTP-5, Minimal TTC = $17050

| Sources | Destinations | | | | | Supply |
|---|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | **D5** | |
| **S1** | 42 | 48 | 38 | 37 | 0 | **160** |
| **S2** | 40 | 49 | 52 | 51 | 0 | **150** |
| **S3** | 39 | 38 | 40 | 43 | 0 | **190** |
| Demand | **80** | **90** | **110** | **160** | **60** | **500** |

**Table MU-5.2:** The RCM of UTP-5

| Sources | Destinations | | | | | Supply |
|---|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | **D5** | |
| **S1** | 3 | 10 | 0 | 0 | 0 | **160** |
| **S2** | 1 | 11 | 14 | 14 | 0 | **150** |
| **S3** | 0 | 0 | 2 | 6 | 0 | **190** |
| Demand | **80** | **90** | **110** | **160** | **60** | **500** |

**Table MU-5.3:** The TOCM of UTP-5

| Sources | Destinations | | | | | Supply |
|---|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | **D5** | |
| **S1** | 45 | 58 | 38 | 37 | 0 | **160** |
| **S2** | 41 | 60 | 66 | 65 | 0 | **150** |
| **S3** | 39 | 38 | 42 | 49 | 0 | **190** |
| Demand | **80** | **90** | **110** | **160** | **60** | **500** |

**Table MU-6.1:** More Challenging UTP-6, Minimal TTC = $180

| Sources | Destinations | | | | Supply |
|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | |
| **S1** | 3 | 48 | 14 | 2 | **24** |
| **S2** | 4 | 2 | 30 | 10 | **24** |
| **S3** | 36 | 8 | 12 | 12 | **02** |
| **S4** | 0 | 0 | 0 | 0 | **15** |
| Demand | **6** | **12** | **3** | **44** | **65** |

**Table MU-6.2:** The RCM of UTP-6

| Sources | Destinations | | | | Supply |
|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | |
| **S1** | 1 | 46 | 12 | 0 | **24** |
| **S2** | 2 | 0 | 28 | 8 | **24** |
| **S3** | 28 | 0 | 4 | 4 | **02** |
| **S4** | 0 | 0 | 0 | 0 | **15** |
| Demand | **6** | **12** | **3** | **44** | **65** |

**Table MU-6.3:** The TOCM of UTP-6

| Sources | Destinations | | | | Supply |
|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | |
| **S1** | 4 | 94 | 26 | 2 | **24** |
| **S2** | 6 | 2 | 58 | 18 | **24** |
| **S3** | 64 | 8 | 16 | 16 | **02** |
| **S4** | 0 | 0 | 0 | 0 | **15** |
| Demand | **6** | **12** | **3** | **44** | **65** |

**Table MU-7.1:** More Challenging UTP-7, Minimal TTC = $75

| Sources | Destinations | | | | Supply |
|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | |
| **S1** | 2 | 7 | 14 | 0 | **5** |
| **S2** | 3 | 3 | 1 | 0 | **8** |
| **S3** | 5 | 4 | 7 | 0 | **7** |
| **S4** | 1 | 6 | 2 | 0 | **15** |
| Demand | **7** | **9** | **18** | **1** | **35** |

**Table MU-7.2:** The RCM of UTP-7

| Sources | Destinations | | | | Supply |
|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | |
| **S1** | 1 | 4 | 13 | 0 | **5** |
| **S2** | 2 | 0 | 0 | 0 | **8** |
| **S3** | 4 | 1 | 6 | 0 | **7** |
| **S4** | 0 | 3 | 1 | 0 | **15** |
| Demand | **7** | **9** | **18** | **1** | **35** |

**Table MU-7.3:** The TOCM of UTP-7

| Sources | Destinations | | | | Supply |
|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | |
| **S1** | 3 | 11 | 27 | 0 | **5** |
| **S2** | 5 | 3 | 1 | 0 | **8** |
| **S3** | 9 | 5 | 13 | 0 | **7** |
| **S4** | 1 | 9 | 3 | 0 | **15** |
| Demand | **7** | **9** | **18** | **1** | **35** |

## CONCLUSION

In this paper, we have provided a set of 25 recognized and acknowledged ‘challenging’ and ‘more challenging’ TPs along with their derived RCM and TOCM

tables to each of the problem as a database. This database will be very useful to the researchers in TPs to make use of readily the provided database to apply and test any to be developed algorithms in TPs. In order to ease the works of the researcher, we have constructed and provided the RCM and TOCM of some selected 'Challenging and More Challenging TPs' as a database. Any proposed algorithm tested on the recognized TPs will definitely improve the efficiency of the proposed algorithm. This is the added advantage of this article. Also, a new chapter in TP research will embrace you all via this article.

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