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SVS – A Two Phase Method For Solving Assignment Problems

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Abstract

In this paper, we have proposed a new method entitled 'Prof. SVS' (or simply 'SVS') for generating the optimal solutions to a wide range of assignment problems (APs). The SVS algorithm consists of two phases. In the first phase, a solution which is either optimal or very close to the optimal is found out for a given AP. In the second phase, testing the optimality of the obtained solution and improving it to optimal, if it's not optimal, is done. The SVS method has been tested on set of 42 benchmark APs of balanced and unbalanced categories for which it has produced optimal solutions directly to 36 problems by first phase itself and only for 6 problems we have to apply the second phase for improving them towards optimal. Thereby, the SVS method is the best and easiest one to solve a wide range of APs. Further, the proposed SVS method will originate all possible numbers of optimal solutions to a given AP, provided they exist.

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1. INTRODUCTION

The readers of this article recognize the fundamentals of the assignment problems (APs) and the popular 'Hungarian' method of solving them. In 1955, KuhnH.W. [1] developed the 'Hungarian' method based on zeros assignment approach and it has been the efficient method for solving the balanced and unbalanced APs. Howsoever, in the recent years several methods, based on zeros assignment approach as well as on one's assignment approach, have been developed to solve the balanced and unbalanced TPs distinctly. Here, we present a brief review of literature about the methods developed based on the zeros assignment approach during the very recent years.

In 2020, Murugesan R. and Esakkiammal T. [3] introduced a new zeros assignment approach entitled ‘TERM’ for solving a wide range of APs with least effort of mathematical calculations. Simulation results authenticate that the ‘TERM’ method is an efficient method which produces optimal solution directly to 80% cases.

In 2021, Murugesan R. and Esakkiammal T. [2] introduced a very simple zeros assignment technique, called ‘MANTRA’, which generates optimal solution directly to a given unbalanced AP without converting it into a balanced one. This technique takes less time to solve an unbalanced AP in comparison to the existing ‘Hungarian’ method.

In 2022 Murugesan R. [4] introduced a new method named ‘CASSI’ which tests the optimality of an assignment solution obtained by an assignment method and improves it towards optimal, if it is not optimal. This was necessitated because certain methods, such as ‘TERM’ [3], have not produced the optimal solutions directly to certain problems. In the literature of assignment problems, the ‘CASSI’ method is of its first kind, which can be used for optimality testing and optimizing a solution obtained through any method based on the zeros assignment approach.

In the same year 2022, Murugesan R. [5] introduced a novel and innovative method entitled ‘E-SOFT’ for determining the optimal solutions to the unbalanced assignment problems (UAPs) and viewed its performance with the existing ‘Hungarian’ method and the ‘Mantra’ technique.

In the same year 2022, Murugesan R. [6] introduced a simple and new iterative technique named ‘NILA’ for optimality testing and optimizing of a solution obtained by any method in assignment problems.

The paper is totally organized as follows: Section 1 briefs an introduction about the very recent methods developed for solving the APs. Section 2 presents the algorithm of the proposed SVS method. Section 3 illustrates two benchmark APs from the literature. Section 4 lists a set of 42 benchmark APs from the literatures and textbooks, Finally, Section 5 draws the conclusion.

2. ALGORITHM FOR THE PROPOSED ‘PROF SVS’ METHOD

‘Professor SVS’ (or simply ‘Prof. SVS’ or in short ‘SVS’) method is a two phased method which generates tested optimal solution to a given assignment problem, whether it is balanced or unbalanced, in an easy way. The first phase finds a solution which is either optimal or very close to the optimal. The second phase tests the optimality and improves to optimal, if not optimal. This method has been dedicated to our favorite college mathematics teacher Professor S. Venkata Subramanian, simply known as ‘Prof. SVS’, who had taught the subject ‘Operations Research’ to me and my classmates in the final year when we were all studying our under-graduate mathematics course during the period 1978-1981 in Sri Paramakalyani College, Alwarkurichi, Tenkasi District, Tamil Nadu State, India. We are very happy to dedicate this article at the time (May 2022) of receiving A+ grade by NAAC in third cycle by this 60 years old prestigious and reputed arts and sciences college. Prof. SVS is very simple, sincere and dedicated teacher and also a loving and caring personality. He had loved and cared every student under him as a brother. We all loved his way of teaching. He only made me to love the subject 'Operations Research'. Now I am able to introduce number of new methods and techniques in this subject because of the strong foundation he had developed in me. We are very grateful for having such a noble teacher in our college life.

Phase #1 (Obtaining a solution to the given AP by applying the existing E-SOFT algorithm)

As the existing E-SOFT method generates the solution to an AP based on the *reduced cost matrix,* first we explain its derivation from the given assignment cost matrix.

1. **Row Minimum Subtraction (RMS) operation**.

Select the minimum element from each row and subtract it from each element in the corresponding row so that each row will contain at least one 0-entry.

2. **Column Minimum Subtraction (CMS) operation**.

Select the minimum element from each column and subtract it from each element in the corresponding column so that each column will contain at least one 0-entry.

3. **Reduced Cost Matrix (RCM)**.

The matrix derived by applying the RMS and / CMS operations on the assignment cost matrix of the given AP is called the reduced cost matrix (RCM). It is obvious that there will be at least one 0-entry in each row and in each column of an RCM. In an RCM, the cells with only 0-entries are called *0-entry cells*. Now the algorithm follows:

Step 1: Conversion into Minimization AP

If the given AP with size $m \times n$ is of maximization type, then convert it into a minimization one.

Step 2: Check the Balanced condition

If $m = n$, then the given AP is balanced. Perform the RMS operation followed by the CMS operation to obtain an RCM. Go to Step 5 for making individual assignments; else the given AP is unbalanced and go to Step 3.

Step 3: Conversion into Balanced AP.

Add required numbers of dummy row(s) only if $m < n$ or add required numbers of dummy column(s) only if $n < m$. The assignment cost in each of the cells in the dummy row(s) or dummy column(s) is assumed to be zero. Let N be the size of the balanced AP.

Step 4: Derivation of an RCM.

On the balanced AP with size N, perform the RMS operation only if (m < n) or perform the CMS operation only if (n < m) to obtain an RCM.

Step 5: Build the assignments one by one in the RCM by computing 'Soft Min' elements

(i) For each row, find the ***s**um **o**f **f**irst **t**hree* (soft) ***min**imum* (min) elements. Write the resulting sum under the Soft Min elements column by enclosing it in parentheses against the respective row. Similarly, do the same computation for each column.

(ii) Mark by *, the maximum among the Soft Min elements computed for rows and columns, along the corresponding row(s) and/ column(s).

(iii) Making the assignments

a. Select the row or column which is marked by * and assign at the cell having a unique 0-entry in that row or column.

b. If tie occurs among certain 0-enty cells in that selected row or column, then select the 0-entry cell which has the least original assignment cost figure for assignment.

c. If tie occurs among the least original assignment cost figure, then consider each such 0-entry cell for assignment as a separate case and finally choose the best assignment plan among them. Such a situation may produce an alternative assignment plan to the given UAP.

PRIORITY RULES

1. In Step 5(iii-b or c), while selecting the 0-entry cell for assignment corresponding to the least original assignment cost figure, *give priority to the dummy cell* when all the non-dummy cells have NON-ZERO assignment costs.
2. While selecting a 0-entry cell for assignment corresponding to the least original assignment cost figure, *give priority to the non-dummy cell* when at least one non-dummy cell has ZERO assignment cost.

SPECIFIC CASES

When an RCM is of size 2 × 2, then there can be three possibilities (two 0-entries, three 0-entries, four 0-entries) of 0-entry cells in it.

1. If the RCM has only two 0-entry cells, which are at diagonally opposite positions, then select both the cells for two individual assignments.
2. If the RCM has three 0-entry cells, then select only the two cells which are at diagonally opposite positions for two individual assignments.
3. If the RCM has all four 0-entry cells, then select any two cells which are at diagonally opposite positions for two individual assignments. Such a situation creates alternative solutions

Phase #2 (Optimality testing and optimizing of the obtained solution via Phase #1 using the existing NILA technique)

We use the following notations in Phase #2:

C_{ij} – Assignment cost at the cell (i, j)

C^* – Largest assignment cost

Z – Overall assignment cost

Z^* – Minimum overall assignment cost

NCC – Net Cost Change

D – Overall Net Cost Change value

Step 1: Test the optimality of the obtained solution

(i) Construct the current solution table

Consider the assignment cost matrix marked / encircled with the obtained assignments (solution) via Phase #1as the 'current solution table'.

(ii) Find the first / next set of shifts to reduce the overall assignment cost Z

In the current solution table, identify the assigned cell with the largest assignment cost (C^*) for the first shift. Let it be in the i^{th} row (and j^{th} column), say (i, j). At first, we try to shift this assignment (i, j)at the j^{th} column to k^{th} column in the same i^{th} row corresponding to the smaller cost next to C^*. Let it be (i, k). Due to the 'unique assignment property' in a row and column, this shift will induce the current assignment in the k^{th} column, say (l, k) to shift to another suitable new column, say (l, n). Thus, shift the assignment from the currently assigned cell (l. k) to a suitable cell (l, n). Shift the currently assigned cells in this way until to get a new assignment in the j^{th} column from which we have started our first shift. Now, the process of possible shifts starting and ending at the j^{th} column is over. The complete set of shifts is shown in Table 1.

Table 1: Set of shifts starting and ending at the j^{th} column

Currently assigned cell	Newly assigned cell	NCC value due to the shift
(i, j)	(i, k)	$C_{ik} – C_{ij}$
(l, k)	(l, m)	$C_{lm} – C_{lk}$
(n, m)	(n, p)	$C_{mp} – C_{nm}$
(q, p)	(q, r)	$C_{qr} – C_{qp}$
…	…	…
(s, t)	(s, j)	$C_{sj} – C_{sj}$

Note that the sequence of shifts starts and ends at the j^{th} column. In general, while shifting an assignment from a currently assigned cell (i, j) with cost C_{ij} to a new cell (i, k) with cost C_{ik}, the following three situations may arise:

1. $C_{ik} < C_{ij}$
2. $C_{ik} = C_{ij}$
3. $C_{ik} > C_{ij}$

To select the cell (i, k) for a new assignment, one of the following three priority rules are used: The situations and their priorities to select the new cell are shown in Table 2.

Table 2: Priority rules for the selection of a new cell (i, k)

Situation arose	Priority #
$C_{ik} < C_{ij}$	1
$C_{ik} = C_{ij}$	2
$C_{ik} > C_{ij}$	3

The Priority #1 is to select the cell (i, k) with $C_{ik} < C_{ij}$. Here the relation '<' represents 'just smaller than'. The Priority #3 is to select the cell (i, k) with $C_{ik} > C_{ij}$. Here the relation '>' represents 'just greater than'.

NOTE:

In the current solution table, for the first shift if tie occurs among certain assigned cells with the same C^*, then consider each such assigned cell as a separate case for the first shift and finally choose the best solution among them.

(iii) Compute the net cost changes for the set of shifts made

The 'net cost change value' (NCC value) is defined as the difference between the assignment costs at the newly assigned cell and the currently assigned cell. For each shift, compute the corresponding NCC value. An NCC value can be negative, zero or positive. Compute the 'overall NCC value' (D) for the set of shifts made, which is the sum of all the NCC values computed for the individual shifts. D also can be can be negative, zero or positive.

(iv) Test the optimality

For the set of shifts, if D is negative, then definitely there will be a decrease in the overall assignment cost (Z) achieved from the new acquired solution. Go to Step 3 for further decrease of Z, if possible.

For the set of shifts, if D is positive, then definitely there will not be a further decrease in Z caused from the new acquired solution. Go to Step 1(ii) for making next set of shifts by selecting another unassigned cell (i, k) with smaller cost next to next to C^* for first shift.

For the set of shifts, if D is zero, then this indicates that there will be neither a decrease nor an increase in Z effected from the new acquired solution. That is, the current solution is an optimal one. Write the optimal solution and compute the associated minimum overall cost of assignment (Z^*).

NOTE:

In a set of shifts, if the first shift from a currently assigned cell (i, j) with C^* to an unassigned cell (i, k) with 'least cost' in the i^{th} row and its subsequent induced shifts does not decrease in Z, then in the 'next set of shifts' select the currently assigned cell with smaller cost next to C^* for the first shift.

Step 2: Write the modified assignment plan as the new solution

Write the corresponding modified assignment plan, which is a new acquired solution, and compute the associated new overall assignment cost.

Step 3: R**epeat the process**

Consider the assignment cost matrix marked / encircled with the new acquired solution as the 'current solution table' and repeat Steps 1 and 2 until there is no further decrease in the overall assignment cost. That is, the current solution is an optimal one. Write the optimal solution and compute the associated minimum overall cost of assignment (Z^*).

Alternative optimal solutions

In an optimal solution of an AP, if D is zero for a set of shifts, then this indicates that there will be neither a decrease nor an increase in Z^*. This set of shifts results in an alternative optimal solution to a given AP. If the given AP has alternative optimal solutions, n in number, all such optimal solutions can also be derived from the generated optimal solution by using Phase #2.

3. NUMERICAL ILLUSTRATION

Proper illustrative example helps the readers to know the process of generating optimal solution to a given AP by the proposed SVS method. Keeping in mind, two benchmark APs from the literature has been illustrated.

Example-1: Consider the following *cost minimization type balanced AP* with six jobs and six machines, as given in Table 1.

Table 1: The given cost minimization balanced AP

Job	**Machine**					
	1	**2**	**3**	**4**	**5**	**6**
1	20	23	18	10	16	20
2	50	20	17	16	15	11
3	60	30	40	55	8	7
4	6	7	10	20	25	9
5	18	19	28	17	60	70
6	9	10	20	30	40	55

4.1 SOLUTION BY THE PROPOSED SVS METHOD

Phase #1 (Obtaining a solution)

By applying the Phase #1 of the proposed SVS method on the given balanced AP, we can obtain the following two distinct solutions as shown in Table 2 and Table 3 with the overall assignment cost of Z =$70:

Table 2: Solution-1 obtained by Phase #1 of the SVS method

Assignment / Assigned cell	Assignment cost in $
(1, 4)	10
(2, 5)	15
(3, 6)	07
(4, 3)	10
(5, 1)	18 = C^*
(6, 2)	10
Overall assignment cost (Z)	**70**

Table 3: Solution-2 obtained by Phase #1 of the SVS method

Assignment / Assigned cell	Assignment cost in $
(1, 4)	10
(2, 5)	15
(3, 6)	07
(4, 3)	10
(5, 2)	19
(6, 1)	09
Overall assignment cost (Z)	**70**

Phase #2 (Testing the optimality and optimizing the obtained solution)

Consider the assignment cost matrix marked (*) with the obtained Solution-1, as shown in Table 4, as the 'current solution table' and apply the Phase #2 for further reduction in the overall cost of assignment (Z).

Table 4: Current solution table (Solution-1)

Job	**Machine**					
	1	**2**	**3**	**4**	**5**	**6**
1	20	23	18	10 *	16	20
2	50	20	17	16	15 *	11
3	60	30	40	55	8	7 *
4	6	7	10 *	20	25	9
5	18 *	19	28	17	60	70
6	9	10 *	20	30	40	55

As of Table 4, the assigned cell with the largest assignment cost (C^* = \$18) is (5, 1). If we make the first shift from (5, 1) to (5, 4) with the least cost \$17 in the 5th row, then the induced assignments are not producing an improved solution. Therefore, we consider the next assigned cell (2, 5) with assignment cost \$15 (next to C^*) for the first shift in a set of shifts. The corresponding set of shifts and the resulting improvement in the solution are shown in Table 5.

Table 5: Shifting of first assignment from (2, 5) to (2, 6) and its induced shifts

Currently assigned cell	Newly assigned cell	NCC value
(2, 5)	(2, 6)	-4
(3, 6)	(3, 5)	+1
	Overall NCC value (D)	-3

As of Table 3, since D is negative (-3), this set of shifts will definitely decrease the overall assignment cost by \$3. The modified and improved assignments with the new overall assignment cost of \$67 are shown in Table 6.

Table 6: Modified and improved solution plan due to Phase #2

Assignment / Assigned cell	Assignment cost in \$
(1, 4)	10
(2, 6)	11
(3, 5)	08
(4, 3)	10
(5, 1)	18 = C^*
(6, 2)	10
Overall assignment cost (Z^*)	**67**

Next, we consider the assignment matrix marked / encircled with the obtained assignments, as shown in Table 6, as the 'current solution table' and apply the Phase #2 for further reduction in the overall cost of assignment $67. The shift from (5, 1) to (5, 4) and its induced assignments and also any other shifts from an assigned cell to unassigned cell will not reduce the overall assignment cost further. Therefore, we stop the further reduction process.

DECISION

Thereby, the assignment plan (solution) shown in Table 6 is an optimal assignment plan with the minimum overall assignment cost Z^*= $67 to the given balanced AP.

Alternative Optimal Solution

Consider the assignment cost matrix marked (*) with the obtained 'Optimal Solution', as shown in Table 6, as the 'current solution table' and apply the Phase #2 for further reduction in the overall cost of assignment (Z).

Table 7: Current optimal solution table

Job	**Machine**					
	1	**2**	**3**	**4**	**5**	**6**
1	20	23	18	10 *	16	20
2	50	20	17	16	15	11 *
3	60	30	40	55	8 *	7
4	6	7	10 *	20	25	9
5	18 *	19	28	17	60	70
6	9	10 *	20	30	40	55

As of Table 7, the assigned cell with the largest assignment cost (C^* = $18) is (5, 1). If we make the first shift from (5, 1) to (5, 2), then the induced assignments will neither reduce nor increase the minimum overall assignment Z^* = $67. This is shown in Table 8.

Table 8: Shifting of first assignment from (5, 1) to (5, 2) and its induced shifts

Currently assigned cell	Newly assigned cell	NCC value
(5, 1)	(5, 2)	+1
(6, 2)	(6, 1)	-1
	Overall NCC value (D)	0

As of Table 8, since D is zero, this set of shifts will definitely neither decrease nor increase the minimum overall assignment cost $67.Therefore, the resulting solution is also an optimal solution to the given AP. Consequently, the given balanced AP has an alternative optimal solution, which is shown in Table 9.

Table 9: Alternative optimal solution due to Phase #2

Assignment / Assigned cell	Assignment cost in $
(1, 4)	10
(2, 6)	11
(3, 5)	08
(4, 3)	10
(5, 2)	19
(6, 1)	09
Overall assignment cost (Z^*)	**67**

DECISION

The given balanced AP has to two distinct optimal solutions with the minimum overall assignment cost Z^*= $67.

NOTE

If we consider the assignment cost matrix marked (*) with the obtained Solution-2, as shown in Table 3, as the 'current solution table' and apply the Phase #2 for further reduction in the overall cost of assignment (Z), then we obtain the Optimal Solution as shown in Table 9. In this Optimal Solution, if we again apply the Phase #2, then we obtain an alternative Optimal Solution as shown in Table 6. Hence, the two solutions obtained via Phase #1 can be improved towards the same two distinct optimal solutions via Phase #2 for the given balanced AP.

Example-2: Consider the following *cost minimization type unbalanced AP* with six jobs and four machines, as given in Table 10.

Table 10: The given cost minimization unbalanced AP

Jobs	**Machines**			
	1	**2**	**3**	**4**
1	6	5	1	6
2	2	5	3	7
3	3	7	2	8
4	7	7	5	9
5	12	8	8	6
6	6	9	5	10

4.2 SOLUTION BY THE PROPOSED SVS METHOD

Phase #1(Obtaining a solution)

Conversion to balanced AP

As (No. of columns < No. of rows), add two dummy columns with zero assignment costs. This will result in the balanced AP as shown in Table 11.

Table 11: The resulting balanced AP

Job	**Machine**					
	1	**2**	**3**	**4**	**5**	**6**
1	6	5	1	6	0	0
2	2	5	3	7	0	0
3	3	7	2	8	0	0
4	7	7	5	9	0	0
5	12	8	8	6	0	0
6	6	9	5	10	0	0

By applying the Phase #1 of the proposed SVS method on the balanced AP shown in Table 11, we can obtain the following two distinct solutions as shown in Table 12 and Table 13 with the overall assignment cost of Z =$16:

Table 12: Solution-1 obtained by Phase #1 of the SVS method

Assignment / Assigned cell	Assignment cost in $
(1, 3)	1
(2, 1)	2
(3, 2)	$7 = C^*$
(4, 6)	--
(5, 4)	6
(6, 5)	--
Overall assignment cost (Z)	**16**

Table 13: Solution-2 obtained by Phase #1 of the SVS method

Assignment / Assigned cell	Assignment cost in $
(1, 3)	1
(2, 1)	2
(3, 6)	--
(4, 2)	7
(5, 4)	6
(6, 5)	--
Overall assignment cost (Z)	**16**

Phase #2 (Testing the optimality and optimizing the obtained solution)

Consider the assignment cost matrix marked (*) with the obtained Solution-1, as shown in Table 12, as the 'current solution table' and apply the Phase #2 for further reduction in the overall cost of assignment (Z). The current assignment table is shown in Table 14.

Table 14: Current solution table (Solution-1)

Job	**Machine**					
	1	**2**	**3**	**4**	**5**	**6**
1	6	5	1 *	6	0	0
2	2 *	5	3	7	0	0
3	3	7 *	2	8	0	0
4	7	7	5	9	0	0 *
5	12	8	8	6 *	0	0
6	6	9	5	10	0 *	0

As of Table 14, the assigned cell with the largest assignment cost ($C^* = \$7$) is (3, 2). If we make the first shift from (3, 2) to (3, 3) with the least assignment cost $2 in the 3rd row, then the induced assignments are producing an improved solution. The corresponding set of shifts and the resulting improvement in the solution are shown in Table 15.

Table 15: Shifting of first assignment from (3, 2) to (3, 3) and its induced shifts

Currently assigned cell	Newly assigned cell	NCC value
(3, 2)	(3, 3)	-5
(1, 3)	(1, 2)	+4
	Overall NCC value (D)	-1

As of Table 15, since D is negative (-1), this set of shifts will definitely decrease the overall assignment cost by $1.The modified and improved assignments with the overall assignment cost of $15areshown in Table 16.

Table 16: Modified and improved solution plan due to Phase #2

Assignment / Assigned cell	Assignment cost in $
(1, 2)	5
(2, 1)	2
(3, 3)	2
(4, 6)	--
(5, 4)	$6 = C^*$
(6, 5)	--
Overall assignment cost (Z^*)	**15**

Then, we consider the assignment cost matrix marked / encircled with the obtained assignments, as shown in Table 16, as the ‘current solution table’ and apply the Phase #2 for further reduction in the overall cost of assignment $15. The shift from (5, 4) to any other cell in the 5th row and its induced assignments and also any other shifts from an assigned cell to unassigned cell will not reduce the overall assignment cost further. Therefore, we stop the further reduction process.

DECISION

Thereby, the assignment plan (solution) shown in Table 16 is an optimal assignment plan with the minimum overall assignment cost Z^*= $15 to the given unbalanced AP.

NOTE

If we consider the assignment cost matrix marked (*) with the obtained Solution-2, as shown in Table 13, as the ‘current solution table’ and apply the Phase #2 for further reduction in the overall cost of assignment (Z), then we obtain the same identical Optimal Solution as shown in Table 16. Hence, the two solutions obtained via Phase

#1 can be improved towards the same identical optimal solution via Phase #2 for the given unbalanced AP. Thereby, the given unbalanced AP has a unique optimal solution with Z^*= $15.

4. NUMERICAL EXAMPLES

To justify the efficiency of the proposed SVS method, we have solved a set of 42 numbers of benchmark APs of balanced and unbalanced category in different sizes, from various literature and textbooks. The first 30 problems are taken from the CASSI method [4] due to Murugesan R. and the additional 22 problems are listed in Table 17. Due to the space limitations, the first 30 problems are not provided here. The readers are requested to refer the article [4] for the first 30 problems.

Table 17: List of additional benchmark APs for testing

Problem #	Problem #
Problem 31 [C_{ij}] 4×4 = [4 5 2 5; 3 1 1 4; 13 1 7 4; 12 6 5 9]	**Problem** 37[C_{ij}] 5×6= [80 140 80 100 56 98; 48 64 94 126 170 100; 56 80 120 100 70 64; 99 100 100 104 80 90; 64 90 90 60 60 70]
Problem 32 [C_{ij}] 4×4 = [62 78 50 101; 71 8561 73; 87 92 111 71; 48 64 87 77]	**Problem38** [C_{ij}] 5×8= [300 250 180 320 270 190 220; 290 310 190 180 210 200 300; 280 290 300 190 190 220 230; 290 300 190 240 250 190 180; 210 200 180 170 160 140 160]
Problem 33 (Maximization AP) [C_{ij}] 5×5 = [40 46 48 36 48; 48 32 36 29 44; 49 35 41 38 45; 30 46 49 44 44; 37 41 48 43 47]	**Problem 39** [C_{ij}] 6×5 = [6 2 5 2 6; 2 5 8 7 7; 7 8 6 9 8; 6 2 3 4 5; 9 3 8 9 7; 9 7 4 6 8]
Problem 34 [C_{ij}] 3×4 = [18 24 28 32; 8 13 17 19; 10 15 19 22]	**Problem 40** [C_{ij}] 6×10= [10 2 14 9 6 7 21 32 18 11; 7 12 9 3 5 6 9 16 54 12; 4 8 6 12 21 9 21 14 45 13; 21 9 12 9 32 10 19 25 16 10; 10 12 30 15 12 17 30 12 12 9; 15 7 34 17 7 16 14 17 9 5]
Problem 35 [C_{ij}] 5×4 = [9 14 19 15; 7 17 20 19; 9 18 21 18; 10 12 18 19; 10 15 21 16]	**Problem 41** [C_{ij}] 7×6= [126 207 254 245 214 243; 229 238 242 228 213 285 ; 118 253 306 218 245 216; 172 247 218 248 217 243; 309 207 105 136 194 139; 99 168 220 140 215 116; 95 174 168 145 249 98]
Problem 36 [C_{ij}] 5×6= [10 8 13 20 16 6; 8 16 23 13 14 10; 9 8 1 6 3 7; 4 12 8 11 11 10; 6 10 9 5 11 8]	**Problem 42** [C_{ij}] 7×10= [21 11 16 9 15 10 12 32 26 16; 14 15 20 10 16 3 6 9 21 14; 9 17 11 31 21 16 7 9 10 11; 16 23 8 15 10 3 6 3 20 23; 12 40 14 36 9 21 14 19 4 13; 8 18 9 42 8 11 19 9 32 20; 21 9 12 9 32 10 19 25 116 10]

5. EVALUATIONAND ANALYSIS

To measure the effectiveness of the proposed Professor SVS method, 42 benchmark instances, listed in Section 4, have been tested and the results are shown in Table 18.

Table 18:Optimal solutions produced by the SVS method

Prob. No.#	**Phase #1**		**Phase #2 (Opt. soln.)**	**No. of Opt. Soln.**	**Prob. No.#**	**Phase #1**	**Phase #2 (Opt. soln.**	**No. of Opt. Soln.**
1.	48		48	1	**22.**	**213 ***	214	1
2.	14		14	2	**23.**	54	54	2
3.	59		59	3	**24.**	15	15	2
4.	65		65	2	**25.**	54	54	1
5.	09		09	1	**26.**	08	08	1
6.	14		14	1	**27.**	**16 ***	15	1
7.	29		29	1	**28.**	**77 ***	73	2
8.	21		21	2	**29.**	870	870	1
9.	24		24	1	**30.**	24	24	2
10.	27		27	1	**31.**	14	14	3
11.	13		13	3	**32.**	254	254	1
12.	900		900	1	**33.**	231	231	4
13.	81		81	1	**34.**	50	50	2
14.	399		399	2	**35.**	54	54	1
15.	**70 ***		67	2	**36.**	30	28	1
16.	392		392	1	**37.**	328	326	1
17.	114		114	1	**38.**	870	870	1
18.	99		99	2	**39.**	16	16	1
19.	248		248	1	**40.**	**36 ***	35	1
20.	191		191	2	**41.**	**952 ***	951	1
21.	50		50	1	**42.**	43	43	2

ANALYSIS

As of Table 18, we decide that out of 42 benchmark APs (25 balanced APs and 17 unbalanced APs) tested, the SVS method has produced optimal solutions directly to 36 problems (23 balanced APs and 13 unbalanced APs) via Phase #1 itself and for the problems numbered with 15, 22, 27, 28, 40 and 41 only we have to go to Phase #2 to improve the solutions towards optimal.

DECISION

Therefore, we conclude that for a given AP, whether it is balanced or unbalanced, the proposed SVS method produces an optimal solution.

6. CONCLUSION

In this research article, we have proposed a new method named 'SVS' for producing the optimal solutions to the assignment problems (APs). The SVS algorithm solves a given AP at most in two phases. In the first phase, a solution which is either optimal or very close to the optimal is found out for a given AP. In the second phase, testing the optimality of the obtained solution and improving it to optimal, if it's not optimal, is carried out. The SVS method has been tested on set of 42 benchmark APs of balanced and unbalanced categories for which it has produced optimal solutions directly to 36 problems by applying the first phase itself and only for 6 problems we have to apply the second phase for improving the near optimal solutions towards optimal. Thus, the SVS method generates tested optimal solutions to the APs. By this means, the SVS method is the best and easiest one to solve a wide range of APs. Moreover, the proposed SVS method will generate all possible numbers of optimal solutions to a given AP, if they exist.

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