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Review on Centrality Measurements in Urban Networks

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Abstract

In urban networks, planning and transportation management of cities generate a large quantity of information. So, it is critical to classify the vertices of urban networks and vital for the urban street. Centrality can assist in understanding the structural properties of dense traffic networks that get into human life and city activity. Lots of cities classify urban streets into sidewalks, curbs, and setbacks to provide street guidelines for likely new rehabilitation. The transportation networks are considered street networks, such as the connection between different urban areas. The nature of each component of the urban street network (USN) defined the street functionality classification. Few factors such as land use mix, design goal, accessible service, and administrator policies may affect the activity of urban travelers. This paper reviews the existing centrality measurements and their applications in urban networks.

Keywords: Urban networks, page rank centrality, betweenness centrality, eigenvector centrality, degree centrality, closeness centrality.

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1. INTRODUCTION

To analyzes, visualize and understand the characteristics of complex systems like cities, complex networks have appeared like a model. Many complex networks, such as urban networks, can be modeled by graph theory, where a city generates information. The city is a source of virtual and physical data, which build a vital part of it. So, the study of centrality measures is essential as it considers both characteristics, the connectivity of the nodes and the information attached with data. For this, comparing existing centrality measures in urban networks considering the impact of factors, topology, and data is vital. This significant fact is the main motivation that drove me to study this paper. Determining the important vertex (or edge) in a network is the fundamental question. We associate the concept of an important vertex with the mathematical idea of centrality measure. Network analysts have introduced and developed different centrality measures that give us the important vertices in the network consequent to its criteria. These measures have played a vital role in analyzing social, urban, citation, and computer networks. Depending on the type of the network, they act as a representative of the structural importance of a vertex for the total functioning of the network. Centrality measure is mainly studied ideas in network analysis. Many centrality measures have been introduced, including degree centrality, betweenness centrality, closeness centrality, eigenvector centrality, information centrality, Katz centrality, network similarity, and so on.

In 2003 Holme showed that traffic congestion is the biggest problem in the road network in our daily life. Each traveler wants the lowest cost and shortest path when traveling. In 2017 Jayaweera et al. showed that the degree, closeness, betweenness, and eigenvector centrality are very useful in recognizing the traffic congestion in the road networks. Transportation networks such as urban streets, roads, railways, airlines, electric paths, water paths, etc., are valid for transportation, telecommunication, and business. In 2017 Wang and Xiufen analyzed urban transport networks by degree, closeness, and betweennes centrality. In 2005 Guimera et al. analyzed the global structure of the worldwide air transportation network by betweenness centrality.

The organization of the paper is as follows. In Section 1, I describe the introductory part of centrality measurements. Section 2 gives the development of centrality measurements. In Section 3, I describe some notations used in my paper. Section 4 presents some centrality measurements related to urban networks. The last section gives the conclusion of the paper.

2. DEVELOPMENT OF CENTRALITY MEASUREMENTS

Lots of centrality measurements were introduced and developed to solve many real-life problems. In 1948, Bavelas first introduced centrality measurements on the connected graph and applied them to the communication network. In 1954 Shaw introduced betweenness centrality. Shimbel defined stress centrality as the shortest path in 1953. In 1953 Katz also defined Katz centrality to measure the influence of a node in the network. In 1965 Beauchamp improved the Bavelas's centrality measurements. In 1966 Sabidussi improved Beauchamp's centrality index and gave the definition. Nieminen proposed the centrality index of an undirected graph and modified the definition of the centrality of Sabidussi in 1974. Freeman first gave the formulae of betweenness centrality, relative betweenness centrality, and graph centrality in 1977. These measures were experimented with within small groups.. Stephenson and Zelen introduced a new centrality based on information about connected networks in 1989. In 1991 Freeman et al. found centrality in the idea of network flows. White and Borgatti generalized betweenness centrality for directed graphs in 1994. In 1999 Everett and Borgatti classified the standard three centrality measurements for groups and individuals. Freeman introduced centrality measurements on binary networks and weighted graphs. Brandes proposed a faster algorithm related to betweenness centrality in 2001. In 2003 Costenbader and Valente described the stability of centrality measurements. In 2005 Estrada and Rodriguez-Velazquez discussed subgraph centrality. Rodriguez et al. generalized the subgraph centrality as functional centrality in 2006. In 2007 Bonacich introduced eigenvector centrality. In 2010 Opsahl et al. improved centrality measurements for the weighted network. Joyce et al. introduced leverage centrality to analyze the human brain network in 2010. In 2010 Kitsak et al. introduced another centrality called k-shell decomposition. Klein proposed edge centrality in 2010. Zeng and Zhang introduced mixed degree decomposition by the degree of residual in 2013. Liu et al. proposed the ranking list in 2014. Bae and Kim introduced neighborhood coreness centrality in 2014. In 2015 Liu et al. introduced neighborhood centrality to know influential nodes in complex networks. In 2017 Wang et al. introduced weighted neighborhood centrality. Csato introduced generalized degree centrality with the help of the Laplacian matrix in 2017. Duron introduced heatmap centrality, and Singh et al. introduced node-weighted centrality in 2020.

3. SOME NOTATIONS

V	:	set Set of vertices of the network.
E	:	set of edges of the network.
n	:	cardinality of V.
m	:	cardinality of E.
$d(u)$	:	degree of the node u.
$d(u,v)$	:	distance between u and v.
σ_{st}	:	number of shortest paths between s and t.
$\sigma_{st}(u)$	:	number of shortest paths between s and t passes through u.
$D_C(u)$	:	degree centrality of the node u.

$C_C(u)$	:	closeness centrality of the node u.
$B_C(u)$	:	betweenness centrality of the node u.
$P_C(u)$	:	page rank centrality of the node u.
$S_C(u)$	:	straightness centrality of the node u.
$E_C(u)$	:	eigenvector centrality of the node u.

4. DIFFERENT TYPES OF CENTRALITY MEASUREMENTS RELATED TO THE URBAN NETWORK

Centrality measurement is used as a mathematical tool in urban network analysis to identify influential nodes in the network. Many centrality measurements have been introduced and applied to various topics within networks. The following centrality measurements are used to analyze the urban networks.

4.1. Degree centrality in the urban network

In 1954 Shaw introduced degree centrality to identify the important vertex in a network. In 1974 Nieminen first defined the degree centrality. However, he did not give the formula. In 1978 Freeman first developed the mathematical formula of degree centrality. In 1987 Bonacich generalized the degree centrality by power and centrality. In 2004 Barrat et al. and Newman generalized the degree of a vertex. In 2009 Opsahl and Panzarasa extended the degree of a vertex by the sum of weights replace of number ties. In 2010 Opsahl et al. generalized the degree centrality by the degree and strength of vertex. In 2016 Bardar et al. established the relation between research performance and degree centrality in a domestic co-authorship network. In 2017 Csato introduced generalized degree centrality through the Laplacian matrix. In 2021 Zhang et al. established the relation between degree centrality, L-index, and scientific performance. The degree centrality of a vertex is the number of vertices adjacent to this vertex. The mathematical formula of degree centrality is $D_C(v) = d(v)$, where $D_C(v)$ and $d(v)$ is the degree centrality and degree of the vertex v, respectively. For normalization, degree centrality $D'_C(v) = \frac{d(v)}{n-1}$ where n is the number of nodes of the network. The time complexity of degree centrality for the unweighted network is $O(m)$, where m is the number of edges in the networks. The limitation of degree-based centrality is that it does not give the global information of the vertex of a network. The advantage of degree centrality is that it is less complex than other centrality measurements. In the urban street network, streets are considered vertices, and the two vertices are adjacent, or an edge exists if the two streets are connected. The degree of the street indicates the number of streets that are directly accessed from that street, and it can measure the accessibility of the streets in the urban network. By this measurement, the direction and length (weight) of streets should also be taken in a directed and weighted urban network.

4.2. Closeness centrality in the urban network

In 1948 Bavelas first gave the idea of closeness centrality, and in 1966 Sabidussi first defined closeness centrality by the inverse of the sum of distances between every pair of vertices in the network. In 1979 Freeman gave the mathematical formula of closeness centrality. In 2001 Newman generalized the closeness centrality on weighted graphs by Dijkstra's shortest paths algorithm. Mathematically, the closeness centrality $C_C(x)$ is defined by $C_C(x) = \frac{1}{\sum_{y \in V} d(x,y)}$ where V is the set of vertices in the network, $d(x, y)$ is the distance between the vertices x and y. For normalization, closeness centrality $C'_C(x) = \frac{n-1}{\sum_{y \in V} d(x,y)}$ where n is the number of nodes of the network. In 2001 Brandes presented an $O(mn)$ time algorithm to measure the closeness centrality of all vertices in a network. This measure is more acceptable than degree centrality, as it counts indirect connections also. The motive behind this measurement was to recognize the nodes which could attain others more quickly. Closeness centrality represents the connectivity between a street and all other neighbourhood streets in the network and measures their accessibility.

4.3. Betweenness centrality in the urban network

In 1954 Shaw first introduced the concept of betweenness centrality. In 1977 Freeman first introduced the mathematical formula of betweenness centrality, and in the same article, he also introduced the formula of relative betweenness centrality and graph betweenness centrality. Next year Freeman defined the graph centrality for social networks. In 2001 Brandes proposed a faster algorithm to measure betweenness centrality which takes, $O(mn)$ time. The mathematical formula of betweenness centrality $B_C(u)$ of a vertex u is given by $B_C(u) = \sum\limits_{s \neq t \neq u} \frac{\sigma_{st}(u)}{\sigma_{st}}$ where $\sigma_{st}(u)$ is the number of shortest paths between s and t through u, $\sigma_{st}(u) = 1$, if the shortest path between vertices s and t passes through vertex u and $\sigma_{st}(u) = 1$, if the shortest path between vertices s and t does not pass through vertex u and σ_{st} is the number of shortest paths between s and t in the network. A high betweenness centrality node is more attached in directing and transferring flow in the network, so it has a significant role in communications. To find the geopolitical importance of a particular town, we use the betweenness centrality. It is used to recognize the streets with a bridging role between various topological shortest paths and detect high-traffic or Arterial streets in the urban network.

In 2002, Girvan and Newman first introduced the edge betweenness centrality. Edge betweenness centrality is the number of shortest paths that pass via an edge in a network. Every edge of the network has an edge betweenness centrality value. A high edge betweenness centrality value of an edge implies a bridge between the network parts. Removal of the bridge may affect the communication between lots of pairs of vertices.

4.4. Eigenvector centrality in the urban network

In 1972 Bonacich introduced a new centrality measurement which was eigenvector centrality by the eigenvector of the largest eigenvalue of the adjacency matrix of networks. In 2001 Bonacich and Lloyd proposed another method that gives meaningful comparable results. In 2007 Bonacich presented some axiom of eigenvector centrality. Eigenvector centrality is the multiple of the sum of adjacent centralities. Let A be the adjacency matrix of the network where the element a_{ij} of A equal to 1, if the node i is connected to node the j and equal to 0, otherwise. The eigenvector centrality $E_C(i)$ of the node i is $E_C(i) = x_i = \frac{1}{\lambda}\sum_k a_{ki}x_k$ where $\lambda \neq 0$. A node x is important and central if its neighbor is important and highly central. Eigenvector centrality of each vertex of a network can be measured in $O(n^2)$ time. This measure is used in urban networks (Curado et al., 2020). The urban topic significantly affects the importance of spaces in the city. For example, the commercial streets and places with made heritage and other characteristic city spaces form their identity.

4.5. Page Rank centrality in the urban network

Page rank centrality was first invented by Google founders Larry Page and Sergei Brin. The technology of Page Rank centrality is the Google search engine which can decide the topicality and importance of separate web pages. It is computed by a web graph (directed graph) where separate web pages and hotlinks are considered nodes and links, respectively. It is a variant of eigenvector centrality and is designed for ranking web content. Page rank centrality of a node is the number of links it receives. A highly ranked node is one that highly ranked nodes point to; a recursive definition is the basic idea of Page Rank. Page Rank centrality is used to rank separate web pages in a hyperlinked database and solve binary and weighted graphs. It is defined as follows, Page rank centrality $P_C(i)$ of a vertex i is $P_C(i) = pr(i) = \frac{(1-d)}{n} + d\sum_{j\in ON(i)} \frac{pr(j)}{n_j}$ where n is the cardinality of nodes, $ON(i)$ is the outline neighbors, $pr(i)$ and $pr(j)$ are rank scores of nodes i and j, n_j is the number of outline nodes of node j and d (=0.85 for ranking web pages) is a damping factor. Page rank centrality is used to help a person for identification of randomly chosen routes in the urban street networks. The streets with high page rank are much possible to be those passing routes.

4.6. Straightness centrality in the urban network

Straightness centrality S_C is the ability to communicate between two nodes i and j is equal to the inverse of the length $d(i,j)$ of shortest path between them. The straightness centrality of a node i is as follows, $S_C(i) = \frac{1}{n-1}\sum_{j\neq i}\frac{(d(i,j))^{Eu}}{d(i,j)}$ where $(d(i,j))^{Eu}$ is the euclidean distance between i and j along a straight line. This measure receives to which latitude the connecting route between nodes i and j moves from the virtual straight road.

5. CONCLUSION

Centrality measurements have an important role in network analysis. For this, lots of centrality measurements inventing and developing. Most of the researchers focused on their works and showing their centrality measurements are different and better than others. So we need information, proper selection, and applications. In this work, my objective is to provide information regarding centrality measurements that help us analyze the urban network.

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