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NILA – An Innovative Technique for Optimality Testing and Optimizing a Solution in Assignment Problems

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Abstract

In this research article, we propose a very simple and innovative iterative technique named NILA (equivalent name for the MOON in the classical Tamil language) for optimality testing and also optimization of a solution obtained by a method in assignment problems. The NILA technique has been tested on a number of near optimal and optimal solutions obtained for many balanced and unbalanced assignment problems by several other available methods. Testing results validate that the proposed NILA technique is the ideal one for testing the optimality of an obtained solution and also optimizing that solution, if that's not optimal. Thereby, we obtain the 'tested optimal' solutions to assignment problems. Another added advantage of the proposed technique is that it also generates the possible numbers of alternative optimal solutions to a given assignment problem, it they exist to the problem. Howsoever, we have to accept the reality that for the assignment problems the solutions generated by the 'Hungarian' method and the 'Mantra' technique are 'optimal by default' (not necessary to test its optimality). It is the design (divine!) specialty of the two methods.

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1. INTRODUCTION

The readers of this paper know the essentials of the assignment problems (APs) and the common Hungarian method of solving them. The Hungarian method was developed by Kuhn H.W. [6] in 1955 and it has been the efficient method for solving the balanced and unbalanced APs since its introduction. Howsoever, in the recent years several methods, based on 'zeros assignment approach' as well as on 'ones assignment approach', have been developed to solve the balanced and unbalanced TPs distinctly. Here, we present a brief review of literature from the very recent years on the developed zeros and ones assignment methods.

In 2020, Murugesan R. and Esakkiammal T. [9] introduced a new zeros assignment approach entitled TERM for solving a broad choice of APs with least endeavor of mathematical calculations. Simulation results authenticate that the TERM method is an efficient method which produces optimal solution directly 80% cases.

In 2021, Murugesan R. and Esakkiammal T. [8] introduced a very simple zeros assignment technique, called MANTRA, which generates an optimal solution directly to a given unbalanced AP without converting it into a balanced AP. This technique takes smaller time to solve an unbalanced AP in comparison to the existing Hungarian method.

In 2022 Murugesan R. [11] introduced a new method named CASSI which tests the optimality of an assignment solution obtained by an assignment method and improves it towards optimal, if it is not optimal. This was necessitated because certain methods, such as TERM [9], have not produced the optimal solutions directly to certain problems. In the literature of assignment problems, the CASSI method is of its first kind, which can be used for optimality testing and optimizing a solution obtained through any method based on the 'zeros assignment approach'.

In the same year 2022, Murugesan R. [12] introduced a novel and innovative method entitled E-SOFT for determining the optimal assignment plans to the unbalanced assignment problems (UAPs) and viewed its performance with the existing 'Hungarian' method and the 'Mantra' technique

In 2012, Hadi Basirzadeh [4] introduced very first time a new approach to APs namely, Ones Assignment Method (OAM) for solving a wide range of such problems. This method can be applied only when all the entries of the cost matrix are nonzero.

In 2013, Ghadle K.P. and Muley Y.M. [3] presented a new method namely, Revised Ones Assignment Method (ROAM) for solving wide range of APs, which is different from the OAM.

In 2014, Gamal M.D.H. [2] had indicated out some drawbacks in the OAM due to Hadi Basirzadeh [4], and provided a remedial strategy to overcome the case when some entries of the cost matrix are zeros. The author also provided examples of the APs where the OAM fails to find their optimal solution.

In the same year 2014, Khalid M. et al. [5] introduced the New Improved Ones Assignment Method (NIOAM), which leads to brief computation time comparatively and will attain an exact optimal solution. Besides, this improved version will overcome the drawbacks as specified by Gamal M.D.H. [2].

In 2020, Murugesan R. and Esakkiammal T. [7] exposed that the ROAM as well as the NIOAM for solving APs do not present optimal solution at all times. The authors also provided examples of the APs where the ROAM and the NIOAM fail to generate their optimal solution.

In 2021, Esakkiammal T. and Murugesan R. [1] introduced a new 'ones assignment method' namely MASS for obtaining optimal solutions to a broad choice of APs. This method obtains the optimal solution to a given AP in two phases. In the first phase, a solution is generated using the ones assignment technique. Optimality testing and optimizing of the obtained solution is passed out in the second phase.

In 2021, Esakkiammal T. and Murugesan R. [10] compared certain 'zeros assignment approaches' with the 'ones assignment approaches' and showed by some benchmark assignment problems that 'ones assignment approaches' have failed to produce optimal assignment plans directly, whereas the 'zeros assignment approach' has produced optimal assignment plans directly.

The paper is a beautiful masterpiece, which is organized as follows: In Section 1, brief introduction about the recent methods developed for solving the APs based on the zeros and ones assignment approaches is given. In Section 2, the algorithm of the proposed NILA technique is presented. In Section 3, three benchmark APs from the literature has been illustrated. Section 4 lists a set of 10 benchmark APs from the literature, which are all tested by the zeros and ones assignment methods as well as improved towards optimal by the proposed NILA technique. Finally, in Section 5 conclusion is drawn.

2. ALGORITHM FOR THE PROPOSED NILA TECHNIQUE

NILA is a very simple 'shifting of assignments' technique which can be used for testing the optimality and also optimize a solution (assignment plan) for assignment problems. The innovative way of improving a near optimal solution to an optimal solution by NILA technique is based on shifting initially a currently assigned cell with largest assignment cost to another unassigned cell and its subsequent induced assignments. This technique has been dedicated to our granddaughter NILA at USA during her birth year. For the proposed technique the name NILA has been crowned from her name. Actually, this technique was developed during April to May 2022 when we were at Winston Salem, North Carolina, USA. The natural satellite MOON reflects the light coming from the SUN and gives the more pleasant light to the earth. Likewise, the proposed technique 'NILA' (equivalent name for MOON in the classical Tamil language) reflects the optimal solution to us from a solution coming from any method in assignment problems. The following are the sequence of steps involved in it:

We use the following notations in the development of the algorithm of the NILA technique:

C_{ij} – Assignment cost at the cell (i, j)

C^* – Largest assignment cost

Z – Overall assignment cost

Z^* – Minimum overall assignment cost

NCC – Net Cost Change

D – Overall Net Cost Change value

Step 1: Obtain a solution to the given AP

For the given AP, first find a solution with its overall assignment cost Z using either a ‘zeros assignment approach’ or a ‘ones assignment approach’.

Step 2: Test the optimality of the obtained solution

(i) Construct the current solution table

Consider the assignment cost matrix marked / encircled with the obtained assignments (solution) as the current solution table.

(ii) Find the first / next set of shifts to reduce the overall assignment cost Z

In the current solution table, identify the assigned cell with the largest assignment cost (C^*) for the first shift. Let it be in the i^{th} row (and j^{th} column), say (i, j). At first, we try to shift this assignment (i, j) at the j^{th} column to k^{th} column in the same i^{th} row corresponding to the smaller cost ‘next to’ C^*. Let it be (i, k). Due to the ‘unique assignment property’ in a row and column, this shift will induce the current assignment in the k^{th} column, say (l, k) to shift to another suitable new column say (l, n). Thus, shift the assignment from the currently assigned cell (l. k) to a suitable cell (l, n). Shift the currently assigned cells in this way until to get a new assignment in the j^{th} column from which we have started our first shift. Now, the process of possible shifts starting and ending at the j^{th} column is over. The complete set of shifts is shown in Table 1.

Table 1. Set of shifts starting and ending at the j^{th} column

Currently assigned cell	Newly assigned cell	NCC value due to the shift
(i, j)	(i, k)	$C_{ik} – C_{ij}$
(l, k)	(l, m)	$C_{lm} – C_{lk}$
(n, m)	(n, p)	$C_{np} – C_{nm}$
(q, p)	(q, r)	$C_{qr} – C_{qp}$
…	…	…
(s, t)	(s, j)	$C_{sj} – C_{st}$

Note that the sequence of shifts starts and ends at the j^{th} column. In general, while shifting an assignment from a currently assigned cell (i, j) with cost C_{ij} to a new cell (i, k) with cost C_{ik}, the following three situations may arise:

1. $C_{ik} < C_{ij}$
2. $C_{ik} = C_{ij}$
3. $C_{ik} > C_{ij}$

To select the cell (i, k) for a new assignment, one of the following three priority rules is used. The situations and their priorities to select the new cell are shown in Table 2.

Table 2. Priority rules for the selection of a new cell (i, k)

Situation arose	Priority #
$C_{ik} < C_{ij}$	1
$C_{ik} = C_{ij}$	2
$C_{ik} > C_{ij}$	3

The Priority #1 is to select the cell (i, k) with $C_{ik} < C_{ij}$. Here the relation '<' represents 'just smaller than'. The Priority #3 is to select the cell (i, k) with $C_{ik} > C_{ij}$. Here the relation '>' represents 'just greater than'.

NOTE:

In the current solution table, for the first shift if tie occurs among certain assigned cells with the same C^*, then consider each such assigned cell as a separate case for the first shift and finally choose the best solution among them.

(iii) Compute the net cost changes for the set of shifts made

The 'net cost change value' (NCC value) is defined as the difference between the assignment costs at the newly assigned cell and the currently assigned cell. For each shift, compute the corresponding NCC value. An NCC value can be negative, zero or positive. Compute the 'overall NCC value' (D) for the set of shifts made, which is the sum of all the NCC values computed for the individual shifts. D also can be can be negative, zero or positive.

(iv) Test the optimality

For the set of shifts, if D is negative, then definitely there will be a decrease in the overall assignment cost (Z) achieved from the new acquired solution. Go to Step 4 for further decrease of Z, if possible.

For the set of shifts, if D is positive, then definitely there will not be a further decrease in Z caused from the new acquired solution. Go to Step 2(ii) for making next set of shifts by selecting another unassigned cell (i, k) with smaller cost 'next to next to' $C^*$for first shift.

For the set of shifts, if D is zero, then this indicates that there will be neither a decrease nor an increase in Z effected from the new acquired solution. That is, the current solution is an optimal one. Write the optimal solution and compute the associated minimum overall cost of assignment (Z^*).

NOTE:

In a set of shifts, if the first shift from a currently assigned cell (i, j) with C^* to an unassigned cell (i, k) with 'least cost' in the i^{th} row and its subsequent induced shifts does not decrease in Z, then in the 'next set of shifts' select the currently assigned cell with smaller cost 'next to' C^* for the first shift.

Step 3: Write the modified assignment plan as the new solution

Write the corresponding modified assignment plan, which is a new acquired solution, and compute the associated new overall assignment cost.

Step 4: R**epeat the process**

Consider the assignment cost matrix marked / encircled with the new acquired solution as the 'current solution table' and repeat Steps 2 to 3 until there is no further decrease in the overall assignment cost. That is, the current solution is an optimal one. Write the optimal solution and compute the associated minimum overall cost of assignment (Z^*).

Alternative optimal solutions

In an optimal solution of an AP, if D is zero for a set of shifts, then this indicates that there will be neither a decrease nor an increase in Z^*. Thereby, this set of shifts results in an alternative optimal solution to a given AP. If the given AP has alternative optimal solutions, n in number, all such optimal solutions can also be derived from the generated optimal solution by using the NILA technique.

Tested optimal solution

An optimal solution is regarded as 'tested optimal' if it comes out of an algorithm after testing its optimality.

Default optimal solution

An optimal solution is regarded as 'default optimal' if it comes out of an algorithm as optimal directly. Default optimal is optimal by default.

IMPORTANT NOTE

1. For the assignment problems, the optimal solutions generated by the NILA technique are 'tested optimal' and that of generated by the 'Hungarian' method and the 'Mantra' technique are 'default optimal' by nature. The latter two are the 'direct' methods which produce 'default optimal' solutions. It is the design (divine!) specialty of the two methods. The other two methods developed in assignment problems namely CASSI [11] and MASS [1] are also generating optimal solutions which are 'tested optimal'.

2. In transportation problems, the optimal solutions generated through either the MODI method or Stepping Stone method [14] are 'tested optimal'. So far, no direct method has been developed to produce the 'default optimal' solutions directly to

transportation problems. This is the fact from the article titled Neeya? Naana? due to Murugesan R. [13]

3. NUMERICAL ILLUSTRATIONS

Right illustrative explanation helps the readers to understand the NILA technique in an easier way. Keeping in mind, three benchmark APs from the literature has been illustrated.

Example 1: Consider the following *cost minimization type unbalanced AP* with six jobs and four machines, as given in Table 3.

Table 3. The given minimization UAP

Job	Machine					
	1	**2**	**3**	**4**	**5**	**6**
1	80	140	80	100	56	98
2	48	64	94	126	170	100
3	56	80	120	100	70	64
4	99	100	100	104	80	90
5	64	90	90	60	60	70

GENERATION OF OPTIMAL SOLUTION THROUGH 'NILA' TECHNIQUE

Step 1: Obtain a solution to the given AP

To obtain an assignment plan (solution) we use the E-SOFT method due to Murugesan R. [12]. By applying the steps of the E-SOFT method, one can obtain the near optimal assignment plan with the overall assignment cost of $328, as shown in Table 4. Note the dummy row (job 6) with zero assignment costs added in order to have balanced AP and then to solve it.

Table 4. Current solution due to the E-SOFT method

Assignment / Assigned cell	Assignment cost in $
(1, 5)	56
(2, 1)	48
(3, 6)	64
(4, 2)	100
(5, 4)	60
(6, 3) dummy cell	00
Overall assignment cost (Z) =	**328**

NOTE: In an assigned cell (i, j), the symbol i represents a job and j represents a machine.

FIRST ITERATION

Step 2: Test the optimality of the obtained solution

(i) Construct the current solution table

Consider the assignment cost matrix marked / encircled with the obtained assignments as the current solution table. This is shown in Table 5. The cells marked with the * symbol represent the assignments. Note that, each row (job) and each column (machine) has got a single a single assignment. This is known as the 'unique assignment property'.

Table 5. The current solution table

Job	**Machine**					
	1	**2**	**3**	**4**	**5**	**6**
1	80	140	80	100	56*	98
2	48*	64	94	126	170	100
3	56	80	120	100	70	64*
4	99	100*	100	104	80	90
5	64	90	90	60*	60	70
6	0	0	0*	0	0	0

(ii) Find the first set of shifts for reduction in the overall assignment cost Z

From Table 4 / Table 5, we see that among the six assignments, the assignment 4 → 2 or equivalently (4, 2) is having C^* = \$100. Therefore, we take the assignment (4, 2) in the 4th row for our first shift. In the 4th row the cost next to C^* is \$99, which corresponds to the cell (4, 1). Thereby, we make our first shift from (4, 2) to (4, 1). This shift and its corresponding induced shifts are shown in Table 6. Note that, we have started our first shift from the 2nd column and ended with our last shift also at the 2nd column.

Table 6. Shifting of first assignment from (4, 2) to (4, 1) and its induced shifts

Currently assigned cell	Newly assigned cell	Priority rule applied	NCC value
(i, j) = (4, 2)	(i, k) = (4, 1)	1	99 – 100 = -01
(k, l) = (2, 1)	(k, j) = (2, 2)	3	64 – 48 = +16
Overall NCC value			D = +15

(iii) Compute the net cost changes for the set of shifts made

From the first and second columns of Table 6, we have made only two possible shifts. For these shifts we compute the NCC values and the resulting 'overall NCC value (D)'. This is shown in the fourth column of Table 6.

(iv) Test the optimality

Since D is positive (+15), the set of shifts made will not reduce in the overall assignment cost Z = $328.

(ii) Find the second set of shifts for reduction in the overall assignment cost$328

In the 4th row, the smaller cost next to next to C^* is $90, which corresponds to the cell (4, 6). Thereby, we make our first shift from (4, 2) to (4, 6). This shift and its corresponding induced shifts are shown in Table 7. Note that, we have started and ended the set of shifts at the 2nd column.

Table 7. Shifting of assignment from (4, 2) to (4, 6) and its induced shifts

Currently assigned cell	Newly assigned cell	Priority rule applied	Net cost change
(i, j) = (4, 2)	(i, k) = (4, 6)	1	90 – 100 = -10
(l, k) = (3, 6)	(l, m) = (3, 1)	1	56 – 64 = -08
(n, m) = (2, 1)	(n, j) = (2, 2)	3	64 – 48 = +16
Overall NCC value			D = -02

(iii) Compute the net cost changes for the set of shifts made

From the first and second columns of Table 7, we have made a set of three possible shifts. For each of the shifts we compute the NCC value and the resulting D value. This is shown in the fourth column of Table 7.

(iv) Test the optimality

Since D is negative (-02), this set of shifts will definitely decrease the overall assignment cost by $02. Thereby, new Z = $326.

Step 3: Write the modified assignment plan as the new solution

The modified and improved assignments with the new overall assignment cost of Z = $326 are shown in Table 8.

Table 8. Modified and improved assignment plan due to NILA technique

Assignment / Assigned cell	Assignment cost in $
(1, 5)	56
(2, 2)	64
(3, 1)	56

(4, 6)	90
(5, 4)	60
(6, 3)	00
New overall assignment cost(Z)	**326**

SECOND ITERATION

Step 5: R**epeat the process**

Consider the assignment cost matrix marked / encircled with the obtained assignments, as shown in Table 8, as the 'current solution table' and apply the NILA technique for further reduction in the overall cost of assignment Z = $326. The shift from (4, 6) to (4, 5) and its induced assignments will not reduce the overall assignment cost further. Also, in the 4^{th} row the cell (4, 5) is with the least cost of $80. Therefore, we stop the further reduction of process.

DECISION

Thereby, the assignment plan shown in Table 8 is the tested optimal assignment plan with the minimum overall assignment cost Z^*= $326 to the given unbalanced AP. Note that, we have used the term 'tested' rather than 'verified'. Tested optimal solution means, one need not compare the obtained optimal solution via NILA technique with the solution by any other method.

Example 2: Consider the following *cost minimization type balanced AP* with five jobs and five machines, as given in Table 9.

Table 9. The given minimization BAP

Job	**Machine**				
	1	**2**	**3**	**4**	**5**
1	4	6	7	5	11
2	7	3	6	9	5
3	8	5	4	6	9
4	9	12	7	11	10
5	7	5	9	8	11

GENERATION OF OPTIMAL SOLUTION THROUGH 'NILA' TECHNIQUE

Step 1: Obtain a solution to the given AP

To obtain an assignment plan (solution) we use the ones assignment method (OAM) due to Hadi Basirdaeh [4]. By applying the steps of the OAM, one can obtain the near optimal assignment plan with the overall assignment cost of Z = $29, as shown in Table 10.

Table 10. Current solution due to the OAM

Assignment / Assigned cell	Assignment cost in $
(1, 1)	4
(2, 2)	3
(3, 3)	4
(4, 5)	10
(5, 4)	8
Overall assignment cost (Z)	**29**

FIRST ITERATION

Step 2: Test the optimality of the obtained solution

(i) Construct the current solution table

Consider the assignment cost matrix marked / encircled with the obtained assignments as the current solution table. This is shown in Table 11. The cells marked with the * symbol represent the assignments.

Table 11. The current solution table

Job	**Machine**				
	1	**2**	**3**	**4**	**5**
1	4*	6	7	5	11
2	7	3*	6	9	5
3	8	5	4*	6	9
4	9	12	7	11	10*
5	7	5	9	8*	11

(ii) Find the first set of shifts for reduction in the overall assignment cost Z = $29

From Table 10 / Table 11, we see that among the five assignments, the assignment (4, 5) is having C^*= $10. Therefore, we consider the currently assigned cell (4, 5) for our first shift. In the 4th row the smaller cost next to C^* is $9, which corresponds to the cell (4, 1). Thereby, we make our first shift from (4, 5) to (4, 1). This shift and its corresponding induced shifts are shown in Table 12. Note that, we have started and ended the set of shifts at the 5th column.

Table 12. Shifting of assignment from (4, 5) to (4, 1) and its induced shifts

Currently assigned cell	Newly assigned cell	Priority rule applied	Net cost change
(i, j) = (4, 5)	(i, k) = (4, 1)	1	9 – 10 = -1
(l, k) = (1, 1)	(l, m) = (1, 4)	3	5 – 4 = +1
(n, m) = (5, 4)	(n, p) = (5, 2)	1	5 – 8 = -3
(q, p) = (2, 2)	(q, j) = (2, 5)	3	5 – 3 = +2
Overall NCC value			D = -1

(iii) Compute the net cost changes for the set of shifts made

From the first and second columns of Table 12, we have made only four possible shifts. For these shifts we compute the NCC values and the resulting D value. This is shown in the fourth column of Table 12.

(iv) Test the optimality

Since D is negative (-1), the set of shifts made will definitely reduce in the overall assignment cost by $1. That is, new Z = $28

Step 3: Write the modified assignment plan as the new solution

The modified and improved assignments with the new overall assignment cost of Z =$28areshown in Table 13.

Table 13. Modified and improved assignment plan due to NILA technique

Assignment / Assigned cell	Assignment cost in $
(1, 4)	5
(2, 5)	5
(3, 3)	4
(4, 1)	9
(5, 2)	5
Overall assignment cost (Z)	**28**

SECOND ITERATION

Step 2: Test the optimality of the obtained new solution

(i) Construct the current solution table

Consider the assignment cost matrix marked / encircled with the obtained assignments as the current solution table. This is shown in Table 14. The cells marked with the * symbol represent the new assignments.

Table 14. The current solution table

Job	Machine				
	1	**2**	**3**	**4**	**5**
1	4	6	7	5*	11
2	7	3	6	9	5*
3	8	5	4*	6	9
4	9*	12	7	11	10
5	7	5*	9	8	11

(ii) Find the first set of shifts for reduction in the overall assignment cost Z = \$28

From Table 13 / Table 14, we see that among the five assignments, the assignment (4, 1) is having C^*= \$9. Therefore, we consider the assignment (4, 1) for our first shift. In the 4th row the smaller cost next to C^* is \$7, which corresponds to the cell (4, 3). Thereby, we make our first shift from (4, 1) to (4, 3). This shift and its corresponding induced shifts are shown in Table 15.

Table 15. Shifting of assignment from (4, 1) to (4, 3) and its induced shifts

Currently assigned cell	Newly assigned cell	Priority rule applied	Net cost change
(i, j) = (4, 1)	(i, k) = (4, 3)	1	7 – 9 = -2
(l, k) = (3, 3)	(l, m) = (3, 2)	3	5 – 4 = +1
(n, m) = (5, 2)	(n, p) = (5, 1)	3	7 – 5 = +2
Overall NCC value			D = +1

(iii) Compute the net cost changes for the set of shifts made

The computed net cost changes for the shifts made are shown in Table 15.

(iv) Test the optimality

Note that, D = +1. Since D is positive, these set of shifts made will not reduce in Z = \$28.

(ii) Find the second set of shifts for reduction in the overall assignment cost Z = \$28

In the 4th row, the least cost is \$7 and we already have got an assignment in that cell (4, 3). Therefore, we change the second shift in the Table 15 from (3, 3) to (3, 4). Thereby, we make our second shift from (3, 3) to (3, 4) instead of from (3, 3) to (3, 2). This shift and its corresponding induced shifts are shown in Table 16.

Table 16. Shifting of assignment from (4, 1) to (4, 6) and its induced shifts

Currently assigned cell	Newly assigned cell	Priority rule applied	Net cost change
(i, j) = (4, 1)	(i, k) = (4, 3)	1	7 – 9 = -2
(l, k) = (3, 3)	(l, m) = (3, 4)	3	6 – 4 = +2
(n, m) = (1, 4)	(n, j) = (1, 1)	1	4 – 5 = -1
Overall NCC value			D = -1

(iii) Compute the net cost changes for the set of shifts made

The computed net cost changes for the shifts made are shown in Table 16.

(v) Test the optimality

Since D is negative (-1), this set of shifts made will definitely decrease the overall assignment by $1. Thereby, new Z = $27.

Step 3: Write the modified assignment plan as the new solution

The modified and improved assignments with the new overall assignment cost of Z = $27areshown in Table 17.

Table 17. Modified and improved assignment plan due to NILA technique

Assignment / Assigned cell	Assignment cost in $
(1, 1)	4
(2, 5)	5
(3, 4)	6
(4, 3)	7
(5, 2)	5
New overall assignment cost (Z)	**27**

THIRD ITERATION

Step 2: Test the optimality of the obtained new solution

(i) Construct the current solution table

Consider the assignment cost matrix marked / encircled with the obtained assignments as the current solution table. This is shown in Table 18. The cells marked with the * symbol represent the new assignments.

Table 18. The current solution table

Job	Machine				
	1	**2**	**3**	**4**	**5**
1	4*	6	7	5	11
2	7	3	6	9	5*
3	8	5	4	6*	9
4	9	12	7*	11	10
5	7	5*	9	8	11

(ii) Find the first set of shifts for reduction in the overall assignment cost Z =$27

From Table 17 / Table 18, we see that among the five assignments, the assignment (4, 3) is having C^*= $7 and is the least assignment cost in the 4th row. Therefore, we cannot make a change in the assignment (4, 3). In the current solution Table 18, the smaller assigned cost next to C^* is $6, which is at the assigned cell (3, 4). We have already experienced the shift from (3, 3) to (3, 2) which has no further improvement in Z. (Refer Table 15). Next, if we make a shift from (3, 4) to (3, 1) or (3, 4) to (3, 5), then they also makes no further improvement in Z.

DECISION

Therefore, the current solution given in Table17 or Table 18 is the tested optimal solution with the minimum overall assignment cost Z^* = $27 to the given balanced AP.

Example 3: Consider the following *cost minimization type balanced AP* with four jobs and four machines, as given in Table 19.

Table 19. The given minimization BAP

Jobs	Machines			
	1	**2**	**3**	**4**
1	4	5	2	5
2	3	1	1	4
3	13	1	7	4
4	12	6	5	9

GENERATION OF OPTIMAL SOLUTION THROUGH 'NILA' TECHNIQUE

Step 1: Obtain a solution to the given AP

To obtain a solution we use the Revised Ones Assignment Method (ROAM) due to Ghadle K.P. and Muley Y.M. [3] or Newly Improved Ones Assignment Method

(NIOAM) due to M. Khalid et al. [5]. By applying the steps of the ROAM / NIOAM, one can obtain the near optimal solution with the overall assignment cost of Z = $15, as shown in Table 20.

Table 20. Current solution due to the ROAM/NIOAM

Assignment / Assigned cell	Assignment cost in $
(1, 1)	4
(2, 3)	1
(3, 2)	1
(4, 4)	9
Overall assignment cost (Z)	**15**

By applying the NILA technique with shifting the currently assigned cell (4, 4) to a new cell(4, 3) we get an improvement in the overall assignment cost by $1, that results in the tested optimal solution with Z^* = $14 as shown in Table 21.

Table 21. Optimal Solution #1 due to the NILA technique

Assignment / Assigned cell	Assignment cost in $
(1, 1)	4
(2, 2)	1
(3, 4)	4
(4, 3)	5
Minimum overall assignment cost (Z^*)	**14**

Alternative Optimal Solution-1

By applying the NILA technique on the optimal solution given in Table 21 by shifting the currently assigned cell (1, 1) to a new cell (1, 4) we get neither an increase nor a decrease in Z^* = $14, that results in an alternative optimal solution as shown in Table 22.

Table 22. Optimal Solution #2 due to the NILA technique

Assignment / Assigned cell	Assignment cost in $
(1, 4)	5
(2, 1)	3
(3, 2)	1
(4, 3)	5
Minimum overall assignment cost (Z^*)	**14**

Alternative Optimal Solution-2

By applying the NILA technique on the optimal solution given in Table 22 by shifting the currently assigned cell (1, 4) to a new cell (1, 1) we get neither an increase nor an decrease in Z^* = $14, that results in an alternative optimal solution as shown in Table 23.

Table 23. Optimal Solution #3 due to the NILA technique

Assignment / Assigned cell	Assignment cost in $
(1, 1)	4
(2, 4)	4
(3, 2)	1
(4, 3)	5
Minimum overall assignment cost (Z^*)	**14**

DECISION

For the given 4 × 4 AP, the NILA technique has generated all the possible alternative optimal solutions. In fact, the given AP has three distinct optimal solutions.

5. NUMERICAL EXAMPLES AND RESULTS

In order to justify the efficiency of the NILA technique, we have tested ten numbers of benchmark APs in different sizes, from various literatures and textbooks. Table 24 shows the challenging APs for the zeros assignment approach E-SOFT [12] and their optimal solutions generated by the NILA technique. Table 25 displays the challenging APs for the zeros assignment approach TERM [9] and their optimal solutions produced by the NILA technique. Table 26 displays the challenging APs for the ones assignment approaches due to Hadi Basirzadeh [4] / Ghadle K. P. and Muley Y. M. [3] / Khalid M., and et al. [5] and their optimal solutions derived by the NILA technique.

Table 24. Challenging APs to the E-SOFT method and their minimum overall assignment cost (Z^*) by NILA technique

Problem #	**Overall assignment cost (Z) by E-SOFT**	**Min. overall assignment cost (Z^*) by NILA**
Problem 1 [C_{ij}] 5×6= [10 8 13 20 16 6; 8 16 23 13 14 10; 9 8 1 6 3 7; 4 12 8 11 11 10; 6 10 9 5 11 8]	30*	28†
Problem 2 [C_{ij}] 5×6= [80 140 80 100 56 98; 48 64 94 126 170 100; 56 80 120 100 70 64; 99 100 100 104 80 90; 64 90 90 60 60 70]	328*	326†

Problem 3 (Maximization AP) $[C_{ij}]$ 5×5= [40 46 48 36 48; 48 32 36 29 44; 49 35 41 38 45; 30 46 49 44 44; 37 41 48 43 47]	231 ††††	231 ††††
Problem 4 $[C_{ij}]$ 6×6 = [20 23 18 10 16 20; 50 20 17 16 15 11; 60 30 40 55 8 7; 6 7 10 20 25 9; 18 19 28 17 60 70; 9 10 20 30 40 55]	67 ††	67 ††

Table 25. Challenging APs to the TERM method and their minimum overall assignment cost (Z^*) by NILA technique

Problem #	**Overall assignment cost (Z) by TERM**	**Min. overall assignment cost (Z^*) by NILA**
Problem5 $[C_{ij}]$5×4= [9 14 19 15; 7 17 20 19; 9 18 21 18; 10 12 18 19; 10 15 21 16]	55*	54†
Problem 6 $[C_{ij}]$6×4= [6 5 1 6; 2 5 3 7; 3 7 2 8; 7 7 5 9; 12 8 8 6; 6 9 5 10]	16*	15†

Table 26. Challenging APs for the Ones Assignment Method (OAM) and their minimum overall assignment cost (Z^*) by NILA technique

Problem #	**Overall assignment cost (Z) by OAM**	**Min. overall assignment cost (Z^*) by NILA**
Problem 57 $[C_{ij}]$3×4= [18 24 28 32; 8 13 17 19; 10 15 19 22]	51*	50†
Problem 8 $[C_{ij}]$ 6×5= [6 2 52 6; 2 5 8 7 7; 7 8 69 8; 6 2 3 4 5; 9 3 8 9 7; 9 7 4 6 8]	17*	16†
Problem 9 $[C_{ij}]$4×4= [4 5 2 5; 3 1 1 4; 13 1 7 4; 12 6 5 9]	15*	14 †††
Problem 10 $[C_{ij}]$ 5×5= [4 6 7 5 11; 7 3 6 9 5; 8 5 4 6 9; 9 12 7 11 10; 7 5 9 8 11]	29*	27†

Note: In Tables 24, 25 and 26, the symbol * denotes the near optimal solution and † denote the optimal solution. Numbers of † presents denote the numbers of optimal solutions exist.

DECISION

As of the results shown in Tables 24, 25 and 26, we decide that for a given assignment problem with a solution obtained by any zeros assignment approach or ones assignment approach we can test its optimality and also can improve towards optimal solution by the proposed NILA technique. Also, for the problems numbered with 3, 4 and 9 the proposed NILA technique generates all possible numbers of alternative optimal solutions.

Novelty of the proposed NILA technique

In the literature, for solving Transportation Problems only methods such as MODI method and Stepping Stone method are available for testing the optimality of an obtained solution and optimizing it, if it’s not optimal. Thereby, these two methods generate ‘tested optimal’ solutions to transportation problems. But, so far such methods are not available for testing and optimizing the obtained solutions of assignment problems. MASS method and CASSI method are the first of its kind for testing the optimality of an obtained solution separately using ones assignment approach and zeros assignment approach respectively, and optimizing the solution, if it’s not optimal. Howsoever, the proposed NILA technique is of its first kind for testing the optimality of an obtained solution of assignment problems using either ones assignment approach or zeros assignment approach and optimizing the solution, if it’s not optimal. In the MASS and CASSI methods, the testing and optimizing of a solution are done based on the improvement indices computed via forming of loops. But, in the NILA technique the testing and optimizing of a solution are done based on the improvement indices computed via set of shifts. In the MASS and CASSI methods, the incoming cell for new assignment and the outgoing already assigned cell are done based on forming of loops. But, in the NILA technique the incoming cell for new assignment and the outgoing already assigned cell are done based on framing a set of shifts.

6. CONCLUSION

In this paper, we have proposed an innovative iterative technique titled NILA for optimality testing and optimizing a solution found by any available method in assignment problems. The NILA technique has been tested on a number of near optimal solutions obtained by several other available methods for many assignment problems. Testing outcomes authenticate that the proposed NILA technique is the perfect one for testing the optimality of an obtained solution and also optimizing of that solution, if that’s not optimal. Thereby, the NILA technique generates ‘tested optimal’ solutions to assignment problems. Also, the proposed technique generates the alternative optimal solution(s) of a given assignment problem, it they exist to the problem. It is the added benefit of this proposed technique.

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