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Autoregressive Prewhitening on the Nonparametric Regression Model of Water Discharge in the Jangkok Watershed, Lombok Island

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Abstract

This study aimed to build a river water discharge model in the Jangkok Watershed in Lombok Island based on precipitation and temperature simulation data with the kernel nonparametric statistical downscaling model. The river water discharge modeling was done using the multivariable kernel nonparametric regression approach. The autoregressive prewhitening transformation was carried out in the modeling process to overcome the correlation to the error. The resulting model is relatively good with a small root mean square error value and is better than the resulting model without the transformation. The autoregressive prewhitening transformation eliminates the effect of correlation error to improve the resulting model.

Keywords: autoregressive prewhitening; error correlation; kernel nonparametric regression; precipitation and temperature simulation; statistical downscaling.

1. INTRODUCTION

Lombok Island is one of the areas prone to flood and drought disasters. Therefore, various adaptation and mitigation efforts are made to these disasters. One of the efforts made is to formulate a good water resource management plan in the Jangkok Watershed, one of the vital watersheds on Lombok Island. An accurate prediction of river water discharge is required in planning the management of water resources. River water discharged model with high accuracy is needed to obtain the correct prediction. The modeling of river water discharge is based on the factors that influence it, including precipitation and temperature, which can be obtained by simulating data. Modeling of river water discharge can be done with a nonparametric regression approach, in which a pre-whitening transformation, especially autoregressive prewhitening, can be carried out.

To improve the accuracy of river water discharge predictions, many river discharge models are currently being developed that utilize global-scale General Circulation Model (GCM) data (Gagnon, et al., 2005; Samadi, et al., 2012; Sachindra, et al., 2015). GCM output data is less precise when used for local-scale forecasting because GCM data on a global scale has high dimensions. For this reason, it is necessary to carry out a regionalization process using the downscaling technique. One of the downscaling techniques is Statistical Downscaling (SD). It is a static downscaling process with data on large-scale grids in a certain period used as the basis for determining data on smaller scale grids (Wigena, 2006; Hadijati, et al., 2021).

The SD method used is based on the nature of the GCM and climate data which are generally nonstationary, nonlinear, and not normally distributed. GCM data is spatial and temporal data that is a curse of dimensionality. This characteristic can lead to the spatial correlation between data on different grids in one domain. Therefore, in SD modeling, pre-processing of GCM data is needed to reduce its dimensions. One method used is the Classification and Regression Tree (CART) algorithm which does not require data normality assumptions (Zorita & Storch, 1999; Kannan & Ghosh, 2013; Hadijati, et al., 2015). The SD method that follows the nature of the GCM output data and climate data is a method that does not require stationary, linear, and normal distribution assumptions. This method is generally a nonparametric regression method; one of the approaches is the nonparametric kernel regression (Kannan & Ghosh, 2013; Hadijati, et al., 2016; Singh, et al., 2016).

Based on the statistical model of climate variable downscaling, a simulation of climate data (rainfall and temperature) was carried out in the Jangkok Watershed Lombok Island. Simulation results data are used as predictor variables in modeling river water discharge. The modeling is done by using the kernel nonparametric regression approach, which is based on the assumption that the errors are identically and independently distributed. The data used in this research is time-influenced so that it can cause correlated or non-independent errors. The correlated error causes the kernel bandwidth selection to be corrupted. The increase in correlation causes the selected bandwidth to be smaller so that the estimator becomes less and less smooth (Opsomer et.al., 2001). To solve this problem, an autoregressive prewhitening transformation that considers the error correlation structure in the model can be carried out (Xiao, et.al., 2003; Hadijati & Budiantara, 2005).

Based on the previous background, this study aims to build a model of river water discharge in the Jangkok Watershed, Lombok Island, based on precipitation and temperature simulation data. Data simulation is performed based on the kernel nonparametric statistical downscaling model. The simulation results are used to model river water discharge using a multivariable kernel nonparametric regression approach. To overcome the correlated error condition, an autoregressive prewhitening transformation is also carried out.

2. MATERIALS AND METHODS

This study used the GCM output data (precipitation and temperature GCM output), local climate data (precipitation and temperature in the Jangkok Watershed - *Daerah Aliran Sungai* (DAS) Jangkok), and data on river water discharge in the Jangkok Watershed. GCM output data are the predictor variables in SD modeling of precipitation and temperature, where the response variables are precipitation and temperature data in the Jangkok Watershed. In river water discharge modeling, the data on river water discharge in the Jangkok Watershed is the response variable, and the predictor variables are precipitation and temperature prediction results based on the kernel nonparametric SD model. All data used are daily data for two years, from January 2016 to December 2017.

The GCM output data used (precipitation and temperature) are Climate Prediction Centre (CPC) Global Precipitation (Daily Total Precipitation) data obtained from https://www.esrl.noaa.gov/psd/data/gridded/data.cpc.globalprecip.html and CPC Global Daily Temperature (Daily Maximum Temperature) data obtained from https://www.esrl.noaa.gov/psd/data/gridded/data.cpc.globaltemp.html. These data are total daily precipitation and temperature data at certain latitude and longitude locations, with an 8×8 domain above the coordinates of the Lombok Island location.

Moreover, this study has two modelings: precipitation and temperature SD modeling for precipitation and temperature simulation of the Jangkok Watershed and river water discharge modeling based on the simulation results of climate data. The methods used in the research will be explained as follows:

2.1. Statistical Downscaling

The downscaling technique is a process of transforming data from large-scale units to smaller-scale units. There are two approaches to downscaling techniques: Dynamic Downscaling and Statistical Downscaling (SD). SD is a static downscaling process where data on large-scale grids in a certain period are used as the basis for determining data on smaller-scale grids (Wigena, 2006). This approach uses global data to obtain functional relationships between local and global GCM data. The functional form is generally formulated by (Lembang, et al., 2009):

$$\boldsymbol{Y} = f(\boldsymbol{Z}) + \boldsymbol{\varepsilon} \quad (1)$$

Y indicates the response variable, $\boldsymbol{Z}$ represents the predictor variables resulting from the reduction of the GCM variables, and $\boldsymbol{\varepsilon}$ shows the model error. Various SD methods have been developed and classified into three categories: the transfer function, weather typing, and weather generator (Wilby, et al., 2004).

2.2. Classification and Regression Tree Algorithm

Classification and Regression Tree (CART) is a classification method that uses historical data called a learning sample to construct a decision tree. The response variable data are classified into new, more homogeneous groups based on several predictor variables with CART (Timofeev, 2004). The CART algorithm aims to obtain a model for predicting the response based on the new predictor variables (Loh, 2011). There are three stages of the CART algorithm, namely (Timofeev, 2004):

1) Construct a maximum tree, where each time data is divided into two parts with maximum homogeneity of child nodes. Based on the Gini rule (Gini index), the maximum homogeneity of the right and left child nodes will be equivalent to maximizing changes in the following impurity function $\Delta i(t)$.

$$\Delta i(t) = -\sum_{k=1}^{K} p^2\left(k|t_p\right) + P_l \sum_{k=1}^{K} p^2(k|t_l) + P_r \sum_{k=1}^{K} p^2(k|t_r) \tag{2}$$

   $k = 1,2,\cdots,K$ are the class index, $p(k|t)$ represents the class k conditional probability when in node t, and P_l and P_r are the probability of the left (l) and the right (r) vertices.

2) Choose the right tree size. Determining the appropriate tree size can be done using the Cross-Validation (CV) method by minimizing the following cost-complexity functions.

$$\mathrm{R}_\alpha(\mathrm{T}) = \mathrm{R}(\mathrm{T}) + \alpha\left(\tilde{\mathrm{T}}\right) \tag{3}$$

   $R(T)$ shows the misclassification error of tree T and $\alpha\left(\tilde{T}\right)$ indicates the complexity size which depends on $\left(\tilde{T}\right)$ – the total number of terminal nodes in the tree.

3) Classify the new data using the constructed trees.

2.3. Multivariable Kernel Nonparametric Regression

The multivariate kernel nonparametric regression model is generally written as follows:

$$Y_t = m(\boldsymbol{X}) + \varepsilon_t, \quad t = 1, 2, \dots, T \tag{4}$$

with error ε_t and fulfills $E(\varepsilon_t) = 0$. The function $m(.)$ is assumed to have an unknown shape but smooth, $\boldsymbol{X}$ represents some random predictor variables, and Y_t indicates the response variables. The regression curve $m(\boldsymbol{X})$ is estimated using the kernel estimator, namely the Nadaraya-Watson estimator for the case of random predictor variables (Eubank, 1998). The Nadaraya-Watson estimator for the multivariable case is written as follows (Hardle & Muller, 1997):

$$\widehat{m}_{\boldsymbol{H}}(x) = \frac{\sum_{t=1}^{T} K_{\boldsymbol{H}}(X_t - x) Y_t}{\sum_{t=1}^{T} K_{\boldsymbol{H}}(X_t - x)} \tag{5}$$

$K(\cdot)$ is the kernel function and $\mathbf{H}$ is the kernel bandwidth vector of h. In this study, the kernel function used was the quartic kernel function which is formulated as:

$$K(u) = \frac{15}{16}(1 - u^2)^2,\ I(|u| \leq 1) \tag{6}$$

The bandwidth value influences the kernel estimator curve; if the bandwidth is too small, the curve will be rough (under smooth), and if the bandwidth is too large, the curve will be over smooth. So it is necessary to choose the optimum bandwidth to get the best kernel estimator. In this study, bandwidth selection was based on the Generalized Cross-Validation (GCV) criteria, and the process was carried out using a genetic algorithm (Sarda and Vieu, 2000; Gen, et al., 2000).

2.4. Kernel Estimator with Prewhitening

In estimating the function $m(\cdot)$ in Equation (4), a procedure is used that does not ignore the correlation structure of the error form through the original regression model prewhitening, so that the filtered regression has an uncorrelated error form. This is based on a correlated error form and is assumed to follow the autoregressive-moving average (ARMA) model, especially the autoregressive (AR) model. The research steps carried out were (Box, et al., 1994; Hadijati & Budiantara, 2005; Wei, 2006).

a. Determine the conventional estimator of $\boldsymbol{m}(\boldsymbol{X})$ using the kernel nonparametric regression between $\boldsymbol{Y_t}$ and $\boldsymbol{X}$. The estimation is done based on Equation (5), with a kernel function $\boldsymbol{K_0}$ and optimum bandwidth $\boldsymbol{h_0}$.

b. Calculate the estimated error, with the following formula:

$$\widehat{\boldsymbol{u}}_{\boldsymbol{t}} = \boldsymbol{Y_t} - \widehat{\boldsymbol{m}}(\boldsymbol{X}) \tag{7}$$

c. Suppose that $\boldsymbol{\tau} = \boldsymbol{\tau}(\boldsymbol{T})$ are some truncation parameters that are relatively small for a sample size $\boldsymbol{T}$ but large enough to avoid serious bias. By taking $\widehat{\boldsymbol{u}}_{\boldsymbol{t}} = \boldsymbol{z_t}$, the $\boldsymbol{\tau}$-th order autoregression form of $\boldsymbol{z_t}$ is made as follows:

$$\boldsymbol{z_t} = \boldsymbol{a_1 z_{t-1}} + \boldsymbol{a_2 z_{t-2}} + \ldots + \boldsymbol{a_\tau z_{t-\tau}} + \boldsymbol{residual} \tag{8}$$

Define $\widehat{\boldsymbol{a}} = (\widehat{\boldsymbol{a}}_{\boldsymbol{1}}, \dots, \widehat{\boldsymbol{a}}_{\boldsymbol{\tau}})'$from $\boldsymbol{a} = (\boldsymbol{a}_{\boldsymbol{1}}, \dots, \boldsymbol{a}_{\boldsymbol{\tau}})'$, where $\widehat{\boldsymbol{a}} = (\boldsymbol{Z}_{\boldsymbol{\tau}}'\boldsymbol{Z}_{\boldsymbol{\tau}})^{-\boldsymbol{1}} \cdot \boldsymbol{Z}_{\boldsymbol{\tau}}'\boldsymbol{z}$, $\boldsymbol{z} = (\boldsymbol{z}_{\boldsymbol{\tau}}, \dots, \boldsymbol{z}_{\boldsymbol{T}})'$, and $\boldsymbol{Z}_{\boldsymbol{\tau}}$ is a matrix of size $(\boldsymbol{T} - \boldsymbol{\tau}) \times \boldsymbol{\tau}$ of the regressor with a typical element $\boldsymbol{z}_{\boldsymbol{t-j}}$.

d. Determine the approximation $\widehat{\boldsymbol{Y}}_{\boldsymbol{t}}^{*}$ as follows:

$$\widehat{\boldsymbol{Y}}_{\boldsymbol{t}}^{*} = \boldsymbol{Y}_{\boldsymbol{t}} - \sum_{\boldsymbol{j=1}}^{\boldsymbol{\tau}} \widehat{\boldsymbol{a}}_{\boldsymbol{j}} \left(\boldsymbol{Y}_{\boldsymbol{t-j}} - \widehat{\boldsymbol{m}}(\boldsymbol{X}_{\boldsymbol{t-j}}) \right) \quad (9)$$

e. Determine the estimator of $m(\boldsymbol{X})$, using kernel nonparametric regression $\widehat{Y}_t^*$ against $\boldsymbol{X}$, which is the estimator of the prewhitening results.

3. RESULTS AND DISCUSSION

3.1. Dimensional Reduction Results with CART Algorithm

Dimensional reduction of GCM precipitation data was based on precipitation data at two observation posts in the Jangkok Watershed, namely Jurang Malang post and Sesaot-Aiknyet post. Meanwhile, the dimension reduction of GCM temperature data was based on temperature data at the Kopang post, which is estimated to affect the temperature conditions of the Jangkok Watershed. The dimensional reduction results using the CART algorithm can be seen in Table 1.

Table 1 Dimensional Reduction Results of GCM Data using CART Algorithm

Observed Post	Number of Knots	Number of Terminal Nodes	Classification Accuracy	Variables in the Model	Importance Level (%)
Jurang Malang	5	3	89.2	X_{25}	100.0
				X_1	53.3
Sesaot-Aiknyet	5	3	83.8	X_{25}	100.0
				X_5	27.5
Kopang	5	3	64.4	X_7	100.0
				X_8	97.7

Table 1 shows that the classification results obtained have high accuracy (more than 80%) except for the temperature classification at the Kopang post. Also, from the CART analysis result, GCM precipitation data that affect precipitation at Jurang Malang post are variable X_{25} (Global precipitation on grid 9.250°S lat. and 116.75°E lon.) and variable X_1 (6.750°S lat. and 113.75°E lon.), meanwhile, GCM precipitation data that influence precipitation at Sesaot-Aiknyet post are variable X_{25} (9.250°S lat. and 116.75°E lon.) and variable X_5 (7.250°S lat. and 113.75°E lon.). Moreover, the GCM temperature data that affect the temperature at Kopang post are variable X_7 (Global temperature on grid 8.750°S lat. and 116.25°E lon.), and X_8 (8.750°S lat. and 116.75°E lon.). The variables resulting from the GCM data projection were then used as predictors in modeling precipitation and temperature in Jangkok Watershed using the kernel nonparametric statistical downscaling.

3.2. Precipitation and Temperature of the Jangkok Watershed Statistical Downscaling Nonparametric Kernel Model

There is a process for selecting the optimum bandwidth to obtain the best regression model in the SD modeling process with the kernel nonparametric regression approach. The results of choosing the optimum bandwidth can be seen in Table 2.

Table 2 Optimum Bandwidths for the Climate Models of the Jangkok Watershed

Climate Variable	Optimum Bandwidths		Minimum GCV
	h_1	h_2	
Precipitation at Jurang Malang Post	0.30467	0.43135	263.31029
Precipitation at Sesaot-Aiknyet Post	0.33969	0.49092	245.86631
Temperature at Kopang Post	0.78369	0.42355	1.41016

Based on the optimum bandwidths, two precipitation models and one temperature model for the Jangkok Watershed were built. The model obtained is used to predict precipitation and temperature data in the Jangkok Watershed. The prediction accuracy was measured by the Root Mean Square Error (RMSE) value of the model, as shown in Table 3. It shows that the prediction results obtained have a quite high accuracy, seen from the relatively small RMSE values. Thus, the climate variable can be used to predict river water discharge.

Table 3 RMSE Values of Climate Variables in the Jangkok Watershed

Climate Variable	RMSE Value
Precipitation at Jurang Malang Post	9.03120
Precipitation at Sesaot-Aiknyet Post	7.81206
Temperature at Kopang Post	0.80272

3.3. Nonparametric Model of Kernel Flow Rate of Jangkok River Watershed with Autoregressive Prewhitening Transformation

In this study, two modeling of river water discharge based on climate variables in the Jangkok Watershed were carried out, namely the modeling of river water discharge observed at the Jurang Malang dan Sesaot-Aiknyet post. In modeling the water discharge of the Jurang Malang post, the simulation data of Jurang Malang post precipitation and Kopang temperature were used as predictor variables. Meanwhile, in modeling the water discharge of the Sesaot-Aiknyet post, the predictor variables were the simulation data of the Sesaot-Aiknyet post precipitation and Kopang post temperature. First, a conventional kernel estimator was determined based on the optimum bandwidths in Table 4.

Table 4 The Optimum Bandwidths of the Conventional Kernel Estimator

Observed Post	Optimum Bandwidths		Minimum GCV
	h_1	h_2	
Jurang Malang	0.83044	0.80776	0.14262
Sesaot-Aiknyet	0.81521	0.68822	6.52534

Based on the optimum bandwidths obtained, we acquired the conventional kernel estimators. The model's residuals were then assumed to follow a certain autoregressive (AR) model, with the parameter estimates and value of the Akaike Information Criterion (AIC) shown in Table 5.

Table 5 Estimation of the Autoregressive Parameters

Observed Post	Best Model	Estimators			AIC
		a_1	a_2	a_3	
Jurang Malang	$AR(3)$	0.2920	0.4070	0.1000	-511.77
Sesaot-Aiknyet	AR(3)	0.4880	0.1513	0.1502	2415.92

We then determined the prewhitening transformation of the response variable, which was then used in specifying the prewhitening kernel estimator. The optimum bandwidths with the minimum GCV obtained are shown in Table 6.

Table 6 Optimum Bandwidths of Prewhitening Kernel Estimator

Observed Post	Optimum Bandwidths		Minimum GCV
	h_1	h_2	
Jurang Malang	0.78266	0.81780	0.09686
Sesaot-Aiknyet	0.57959	0.64349	3.85819

Furthermore, Table 7 compares the RMSE values for the conventional kernel and the prewhitening kernel model at the Jurang Malang and Sesaot-Aiknyet post.

Table 7 The RMSE Values of the Models at Jangkok Watershed

Observed Post	RMSE (Conventional Model)	RMSE (Prewhitening Model)
Jurang Malang	0.23711	0.153345
Sesaot-Aiknyet	1.78054	1.109572

Table 7 shows that the model of river water discharge in the Jangkok watershed obtained through the prewhitening transformation gives better results than the conventional kernel model without prewhitening. This conclusion is indicated by the relatively smaller RMSE value of the kernel prewhitening model. Moreover, the comparison of these two approaches is shown in Figure 1 and Figure 2, by comparing the actual data with the predicted data using the conventional kernel model and the prewhitening kernel model at Jurang Malang (Figure 1) and Sesaot-Aiknyet Post (Figure 2) of Jangkok Watershed.

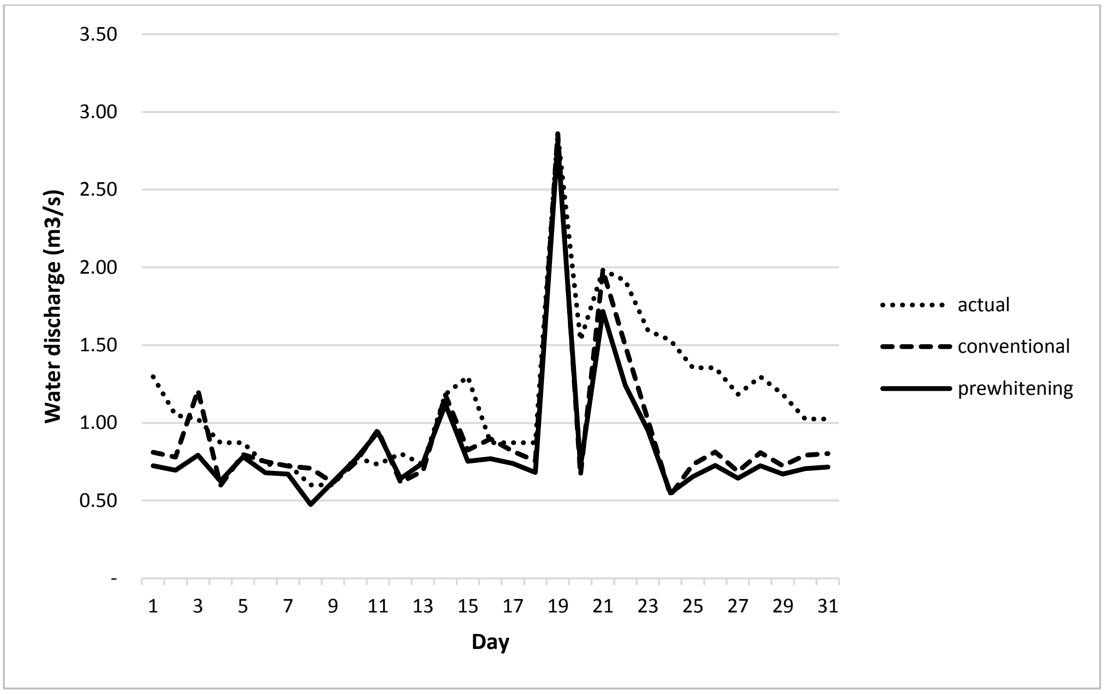

Figure 1 Actual and Prediction River Water Discharge Data Pattern at Jurang Malang Post of Jangkok Watershed

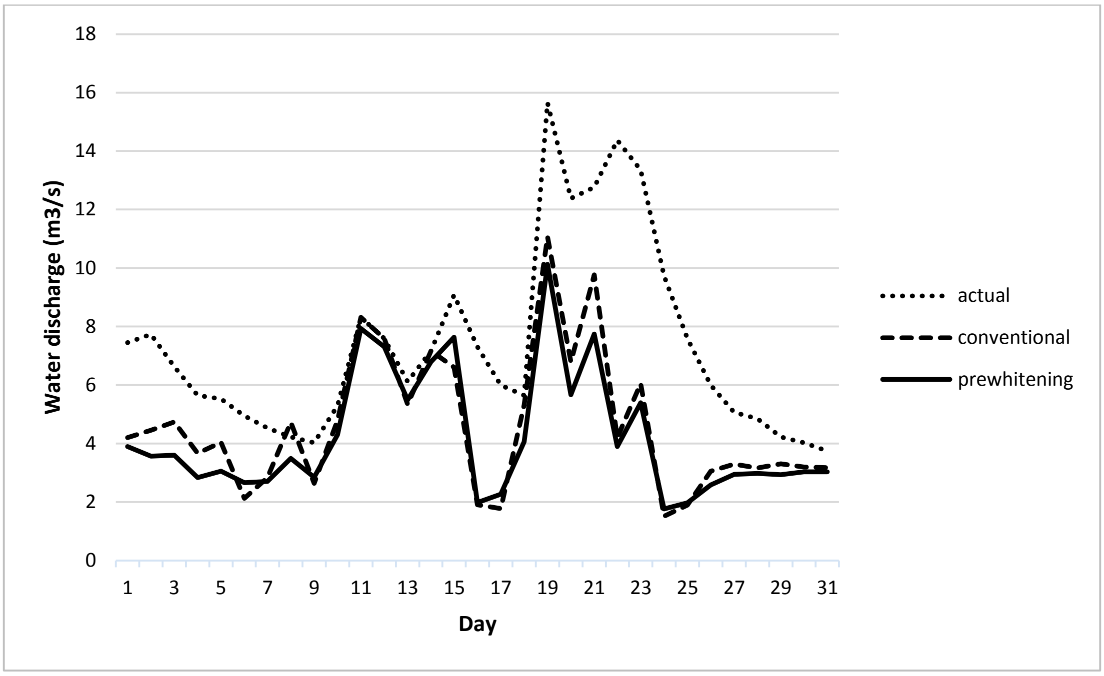

Figure 2 Actual and Prediction River Water Discharge Data Pattern at Sesaot-Aiknyet Post of Jangkok Watershed

Based on Figure 1 and Figure 2, the prediction results show patterns that are not much different between the conventional kernel model and the prewhitening transformation of the river water discharge model at Jangkok Watershed.

4. CONCLUSION

The statistical model of precipitation and temperature downscaling provides simulation results of precipitation and temperature in the Jangkok Watershed that were good enough to model river water discharge. The river water discharge model produced based on the results of the climate prediction in the Jangkok Watershed also offers good prediction results. In addition, river water discharge modeling carried out with autoregressive prewhitening transformation gives better results than the conventional model.

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