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Impact of Poynting-Robertson Drag on the Motion of a Geocentric Satellite: Sun-Earth System

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Abstract

To investigate the impact of Poynting-Robertson (P-R) drag on the motion of a geocentric satellite in the Sun-Earth system with the assumption that all the three primaries lie in the ecliptic plane is the main objective of this paper. Following the perturbation technique in Section-2, we have examined the equations of motion by replacing r and $\dot{\theta}$ to their steady state values r_0 and $\dot{\theta}_0$ respectively to reduce the equation in integral form. In Section-3, the series solution of the three-body problem developed by the Lindstedt-Poincare method (L-P Method) has been compared graphically by different values of a, e and $\dot{\theta}_0$. In the Section-4, the periodicity and stability on the motion of a geocentric satellite in the Sun-Earth system under the effects of P-R drag have been derived using Poincare section. It has been concluded that the frequency of oscillation and the peak values of u increase as the coefficient of P-R drag q is increased. Also, it is seen that for same effect of drag, the satellite with greater value of a, e and $\dot{\theta}_0$ has larger frequency of oscillation and larger peak values of u.

Keywords: General three-body problem (G3BP), Three-body problem (3BP), Ecliptic Plane, P-R Drag, Series solution, Poincare section.

2010 Mathematics Subject Classification: 37N05

1. INTRODUCTION

In celestial mechanics G3BP is determining the motions of initial set of positions, velocities and masses of the three bodies (Earth, Sun and Satellite) for a particular point in time in uniformity with the Newton's law of motion and gravity. It can also be termed as a particular case of general *N*-body problem.

The Doppler shift, the radiation pressure, and the Drag (Poynting) have been studied by Poynting(1903) and Robertson(1937) which comprised of a radiation force enforced on a particle by a radiating body. Bhatnagar and Gupta (1976) have discussed the effect of solar radiation pressure on resonance in the restricted problem . Klacka (1993,1994) has evaluated the effect of Solar radiation on the (interplanetary) dust particle and derive the general case of a spherical particle. Murray (1994) has investigated the stability and location of the five Lagrangian equilibrium points in the planar CR3BP when the third body is acted upon by a various drag. He also analysed the use of Jacobi constant of the circular restricted problem as a simple mean of investigating the stability of the equilibrium points in certain circumstances. The 3BP work has been carried on by many learned personalities like Ragos (1995), Ragos and Vrahatis (1955), Liou and Zouk (1995), Salaam (2008) and Kushvah (2009). The enormous contribution of these in the related field of study has served as a guiding light for many.

In this paper, we propose to discuss the G3BP in the Sun-Earth system. It is considered that the Earth as center and Sun and Satellite are moving in elliptic orbits around it, with the assumption that all the three primaries are in an ecliptic plane. First, we derived the equations of motion of the Satellite (polar form) and then by using the perturbation technique to reduce the equations of motion in integral form. Next to this, we proposed to discuss the series solution by using Lindstedt-Poincare method. Finally, we have discussed the impact of P-R drag in the motion of the Satellite on the Sun-Earth system, by using the Poincare section.

2. STATEMENT OF THE PROBLEM AND EQUATIONS OF MOTION

Let S represents the Sun, $\bar{S}$ the Satellite and E the Earth with their masses M_S, $M_{\bar{S}}$ and M_E respectively. The mass of the Satellite is negligible compared to the masses of the Earth and the Sun. The system is revolving with the angular velocity $\vec{\omega}$ and the Satellite is moving in ecliptic plane around the Earth with the same angular velocity $\vec{\omega}$.

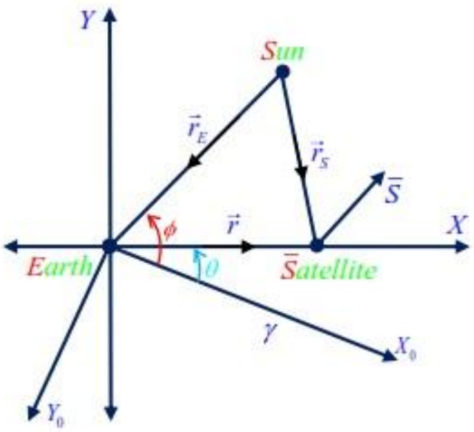

Fig.1 Configuration of the3BP

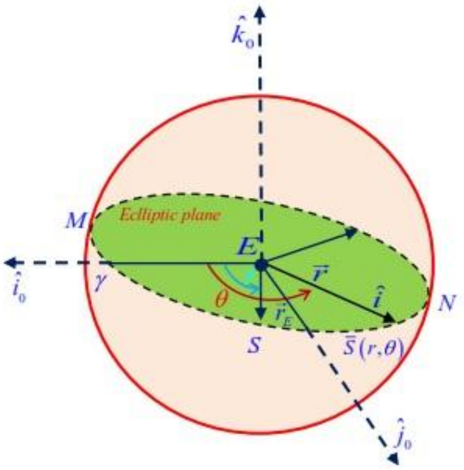

Fig. 2 Configuration of the 3BP with coordinate axis

Let $\vec{r}_E$, $\vec{r}_{\bar{S}}$ and $\vec{r}$ represent the vectors from Sun and Earth, Sun and Satellite and Earth and Satellite respectively; γ be the vernal equinox, θ the angle between direction of the Satellite and the direction of vernal equinox, ϕ the angle between direction of the Sun and the direction of vernal equinox and c the velocity of light. Let origin be at the center of the Earth with x, y and z the co-ordinate system of the Satellite and unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$ along the axes respectively. Let another system with origin at the center of the Earth and x_0, y_0 and z_0 as set of co-ordinate system of the Sun in the same plane, with unit vectors i_0, j_0 and k_0 along the axes respectively (Fig. 2).

2.1 EQUATIONS OF MOTION IN POLAR FORM

$$M_{\bar{S}}\vec{F}_{\bar{S}} = \vec{f}_1 + \vec{f}_2 + \vec{f}_3,$$

where $\vec{f}_1 = F\dfrac{\vec{r}_s}{r_s}$, $\vec{f}_2 = F\dfrac{(\vec{v}\cdot\vec{r}_s)}{c}\dfrac{\vec{r}_s}{r_s}$, $\vec{f}_3 = -F\dfrac{\vec{v}}{c}$, (Ragos,1995)

$F =$ the measure of the radiation pressure, $\vec{v} =$ velocity of $\bar{S}$.

$\vec{F}_{\bar{S}}$ = the P-R drag (per unit mass) due to the Sun acting on the Satellite , as mentioned in Fig.1

The motion (relative) of the Satellite with regard to the Earth is given by

$$\ddot{\vec{r}} = \ddot{\vec{r}}_s - \ddot{\vec{r}}_E = \frac{\vec{F}_{S\bar{S}} + \vec{F}_{E\bar{S}} + \vec{F}_{\bar{S}}\vec{M}_{\bar{S}}}{M_{\bar{S}}} - \frac{\vec{F}_{SE}}{M_E},$$

where

$$\vec{F}_{S\bar{S}} = -\frac{GM_S M_{\bar{S}}}{r_s^3}\vec{r}_S, \;\; \vec{F}_{SE} = -\frac{GM_S M_E}{r_E^3}\vec{r}_E,$$

$$\vec{F}_{E\bar{S}} = -\frac{GM_{\bar{S}} M_E}{r^3}\vec{r}, \;\; G = \text{Gravitational constant.}$$

Thus

$$\ddot{\vec{r}} = -qF_g\frac{\vec{r_s}}{r_s} - \frac{GM_E}{r^3}\vec{r} + \frac{GM_S}{r_E^{\,3}}\vec{r}_E - (1-q)F_g \times \left(\frac{(\vec{v}\cdot\vec{r}_s)}{c}\frac{\vec{r}_s}{r_s} - \frac{\vec{v}}{c}\right).$$

Where $q = 1 - p, \;\; p = \frac{F_{\bar{S}}}{F_g}, \;\; F_g = \frac{GM_S}{r_s^2}.$

$\dot{\phi}^2 = \frac{GM_S}{r_E^3}$ (Motion of the Earth relative to the Sun) , also

$$\vec{r} = r\hat{i}, \;\; \vec{r}_E = r_E\hat{r}_E, \;\; \hat{r}_E = \cos\phi\,\hat{i}_\circ + \sin\phi\,\hat{j}_\circ, \;\; \vec{r}_E = r_E\cos\phi\,\hat{i}_\circ + r_E\sin\phi\,\hat{j}_\circ.$$

Using the above values in the equations of motion (in vector form) can be written as

$$\ddot{\vec{r}} = -qGM_S\frac{\vec{r}_s}{r_s^3} - \frac{GM_E}{r^3}\vec{r} + \dot{\phi}^2 r_E(\cos\phi\,\hat{i}_\circ + \sin\phi\,\hat{j}_\circ) - (1-q)F_g\left\{\frac{(\vec{v}\cdot\vec{r}_s)}{c}\frac{\vec{r}_s}{r_s} - \frac{\vec{v}}{c}\right\}. \quad (1)$$

By rotating frame of reference, we get

$$\ddot{\vec{r}} = \frac{\partial^2 r}{\partial t^2}\hat{i} + 2\frac{\partial r}{\partial t}\left(\vec{\omega}\times\hat{i}\right) + r\left(\frac{\partial\vec{\omega}}{\partial t}\times\hat{i}\right) + r\left\{\left(\vec{\omega}\cdot\hat{i}\right)\vec{\omega} - \left(\vec{\omega}\cdot\vec{\omega}\right)\hat{i}\right\}, \quad (2)$$

where $\vec{\omega} = \dot{\theta}\hat{K}$.

By applying the procedure of Bhatnagar and Mehra (1986), the equations of motion are given as

$$\frac{d^2 r}{dt^2} - r\dot{\theta}^2 + \frac{GM_E}{r^2} = \dot{\phi}^2 r_E\cos(\dot{\theta}-\dot{\phi}) - qGM_S\frac{\vec{r_s}\cdot\hat{i}}{r_s^3} - (1-q)\frac{GM_S}{(r_s)^2}\left\{\frac{(\vec{v}\cdot\vec{r}_s)}{cr_s}(\vec{r}_s\cdot\hat{i}) + \frac{\left(\vec{v}\cdot\hat{i}\right)}{c}\right\}, \quad (3)$$

$$\frac{d\left(r^2\dot{\theta}\right)}{dt} = -\dot{\phi}^2 r_E r\sin(\dot{\theta}-\dot{\phi}) - qGM_S\, r\frac{\vec{r}_s\cdot\hat{j}}{r_s^3} - (1-q)\frac{r\,GM_S}{r_s^2}\left\{\frac{(\vec{v}\cdot\vec{r}_s)}{cr_s}(\vec{r}_s\cdot\hat{j}) + \frac{\left(\vec{v}\cdot\hat{j}\right)}{c}\right\}. \quad (4)$$

These are required equations of motion in Polar form and in the synodic coordinate system.

2.2 EQUATIONS OF MOTION AFTER PERTURBATION

Equations (3) and (4) are not integrable hence the perturbation technique is followed by us and replacing r and θ to their steady state values r_0 and $\dot{\theta}_0$ respectively. Placing these steady state values in the right-hand side of equations (3) and (4), we get

$$\frac{d^2r}{dt^2} - r\dot{\theta}^2 + \frac{GM_E}{r^2} = \dot{\phi}^2 r_E \cos(\dot{\theta}_0 - \dot{\phi})t - qGM_S \frac{\vec{r}_s \cdot \hat{i}}{r_s^3} - (1-q)\frac{GM_S}{r_s^2}\left\{\frac{(\vec{v}\cdot\vec{r}_s)}{cr_s}(\vec{r}_s \cdot \hat{i}) + \frac{(\vec{v}\cdot\hat{i})}{c}\right\}, \quad (5)$$

$$\frac{d\left(r^2\dot{\theta}\right)}{dt} = -\dot{\phi}^2 r_E r_0 \sin(\dot{\theta}_0 - \dot{\phi})t - qGM_S r_0 \frac{\vec{r}_s \cdot \hat{j}}{r_s^3} - (1-q)r_0 \frac{GM_S}{r_s^2}\left\{\frac{(\vec{v}\cdot\vec{r}_s)}{cr_s}(\vec{r}_s \cdot \hat{j}) + \frac{(\vec{v}\cdot\hat{j})}{c}\right\}. \quad (6)$$

Now $\vec{v} = \dfrac{\partial \vec{r}_s}{\partial t} + \vec{\omega}\times\vec{r}_s = r_E\dot{\phi}\sin\left(\dot{\theta}_0 - \dot{\phi}\right)t\,\hat{i} + \left\{r_E\dot{\phi}\cos\left(\dot{\theta}_0 - \dot{\phi}\right)t + \dot{\theta}_0 r_0\right\}\hat{j}.$

With the help of above values and the transformations:

$$\hat{i} = (\cos\theta)\hat{i}_0 + (\sin\theta)\hat{j}_0, \ \hat{j} = (-\sin\theta)\hat{i}_0 + \cos\hat{j}_0, \ \hat{k} = \hat{k}_0.$$

We obtain the following:

$$\begin{aligned}\ddot{r} - r\dot{\theta}^2 = &-\frac{GM_E}{r_0^2} + r_E\dot{\phi}^2\cos\left(\dot{\theta}_0 - \dot{\phi}\right)t - \frac{GM_S q\cos\left(\dot{\theta}_0 - \dot{\phi}\right)t}{r_s^3} - \frac{GM_S q r_0}{r_s^3} \\ &+ \frac{GM_S(1-q)r_0 r_E^2\left(\dot{\theta}_0 - \dot{\phi}\right)\sin 2\left(\dot{\theta}_0 - \dot{\phi}\right)t}{2cr_s^3} \\ &+ \frac{GM_S(1-q)r_0^2 r_E(\dot{\theta}_0 - \dot{\phi})\sin\left(\dot{\theta}_0 - \dot{\phi}\right)t}{cr_s^3} - \frac{GM_S(1-q)r_E\dot{\phi}\sin\left(\dot{\theta}_0 - \dot{\phi}\right)t}{cr_s^2},\end{aligned} \quad (7)$$

Taking $r^2\dot{\theta}$ = constant, $r = 1/u$, we have

$$\begin{aligned}\frac{d^2u}{d\theta^2} + u = &\frac{GM_E}{r_0^4} - \frac{r_E\dot{\phi}^2u^2\cos\left(\dot{\theta}_0 - \dot{\phi}\right)t}{\dot{\theta}_0^2} + \frac{GM_S q u^2\cos\left(\dot{\theta}_0 - \dot{\phi}\right)t}{r_s^3\dot{\theta}_0^2} + \frac{GM_S qu}{r_s^3\dot{\theta}_0^2} \\ &- \frac{GM_S(1-q)ur_E^2(\dot{\theta}_0 - \dot{\phi})\sin 2\left(\dot{\theta}_0 - \dot{\phi}\right)t}{2cr_s^3\dot{\theta}_0^2} - \frac{GM_S(1-q)r_E(\dot{\theta}_0 - \dot{\phi})\sin\left(\dot{\theta}_0 - \dot{\phi}\right)t}{cr_s^3\dot{\theta}_0^2} \\ &- \frac{GM_S(1-q)u^2 r_E\dot{\phi}\sin\left(\dot{\theta}_0 - \dot{\phi}\right)t}{cr_s^2\dot{\theta}_0^2}.\end{aligned} \quad (8)$$

The solution of unperturbed system $\dfrac{d^2u}{d\theta^2} + u = \dfrac{GM_E}{r_0^4\dot{\theta}_0^2}$, is given by

$\dfrac{l}{r} = 1 + e\cos(\theta - \omega)$, where $r^2\dot{\theta} = \text{constant}$, $l = a(1-e^2)$; and

$e, \omega =$ constants of integration, $u = \dfrac{1 + e\cos(\theta - \omega)}{a(1-e^2)}.$

Let us consider $\theta-\omega=\dot{\theta}_0 t=\Lambda_0 t$, say. Since $e<1$, we have $(1+e\cos\Lambda_0 t)^{k_1}\approx 1+k_1 e\cos\Lambda_0 t$.

On simplifying the equation (8) we get

$$\begin{aligned}\frac{d^2u}{dt^2}+\Lambda_0^2 u \;=\;& h_1+h_2\cos\Lambda_0 t+h_3\cos\dot{\phi}t+h_4\sin\dot{\phi}t+h_5\cos\left(\Lambda_0-\dot{\phi}\right)t\\ &+h_6\sin\left(\Lambda_0-\dot{\phi}\right)t+h_7\sin 2\left(\Lambda_0-\dot{\phi}\right)t+h_8\cos\left(2\Lambda_0-\dot{\phi}\right)t+h_9\sin(2\Lambda_0\dot{\phi})t\\ &+h_{10}\cos\left(\dot{\phi}+\Lambda_0\right)t+h_{11}\sin\left(\dot{\phi}+\Lambda_0\right)t+h_{12}\cos\left(3\Lambda_0-\dot{\phi}\right)t+h_{13}\sin\left(3\Lambda_0-\dot{\phi}\right)t\\ &+h_{14}\sin\left(\Lambda_0-2\dot{\phi}\right)t+h_{15}\sin\left(3\Lambda_0-2\dot{\phi}\right)t \qquad (9)\end{aligned}$$

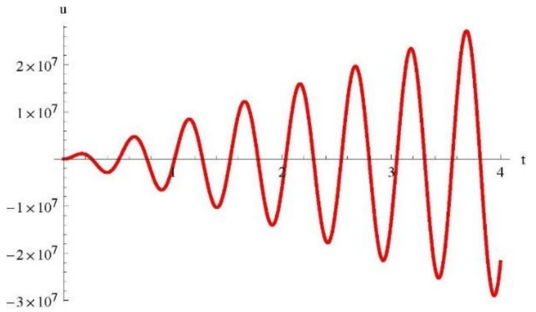

Fig. 3. Graph of Eq. (9) for $a=0.0000469251$, $e=0.0071$, $\dot{\theta}_0=708.102$ and $q=0.25$.

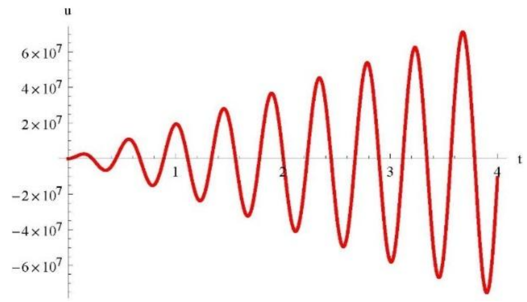

Fig. 4. Graph of Eq. (9) for $a=0.0000469251$, $e=0.0071$, $\dot{\theta}_0=708.102$ and $q=0.75$.

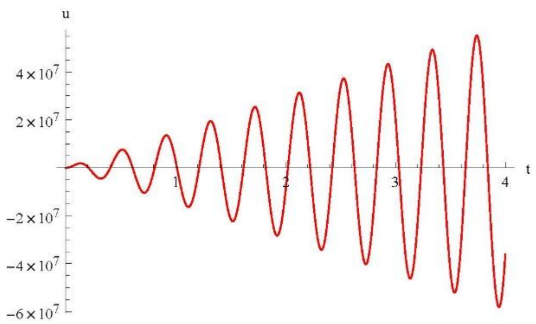

Fig. 5. Graph of Eq. (9) for $a=0.0000526604$, $e=0.0635$, $\dot{\theta}_0=890.602$ and $q=0.25$

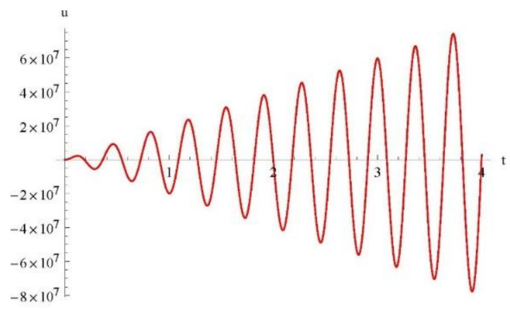

Fig. 6. Graph of Eq. (9) for $a=0.0000526604$, $e=0.0635$, $\dot{\theta}_0=890.602$ and $q=0.75$

Using the complementary function and particular integral, the complete solution of equation (9) is given by

$$
\begin{aligned}
u \;=\; & A\cos\left(\Lambda_0 t-\alpha\right)+\frac{C_1}{\Lambda_0^2}-\frac{C_2 t}{2\Lambda_0}\sin t-\left\{\frac{C_3}{\Lambda_0^2-(\dot{\phi}-\Lambda_0)^2}\right\}\cos(\dot{\phi}-\Lambda_0)t \\
& -\left\{\frac{C_4}{\Lambda_0^2-(\dot{\phi}-\Lambda_0)^2}\right\}\sin(\dot{\phi}-\Lambda_0)t \\
& -\left\{\frac{C_5}{\Lambda_0^2-(2\dot{\phi}-2\Lambda_0)^2}\right\}\cos 2(\dot{\phi}-\Lambda_0)t-\left\{\frac{C_6}{\Lambda_0^2-(2\dot{\phi}-2\Lambda_0)^2}\right\}\sin 2(\dot{\phi}-\Lambda_0)t \\
& -\left\{\frac{C_7}{\Lambda_0^2-(2\dot{\phi}-\Lambda_0)^2}\right\}\cos 2(2\dot{\phi}-\Lambda_0)t-\left\{\frac{C_8}{\Lambda_0^2-(2\dot{\phi}-\Lambda_0)^2}\right\}\sin(2\dot{\phi}-\Lambda_0)t \\
& -\left\{\frac{C_9}{\Lambda_0^2-(2\dot{\phi}-3\Lambda_0)^2}\right\}\cos(2\dot{\phi}-3\Lambda_0)t-\left\{\frac{C_{10}}{\Lambda_0^2-(2\dot{\phi}-3\Lambda_0)^2}\right\}\sin(2\dot{\phi}-3\Lambda_0)t \\
& -\left\{\frac{C_{11}}{(\Lambda_0^2-\dot{\phi}^2)}\right\}\cos\dot{\phi}t-\left\{\frac{C_{12}}{(\Lambda_0^2-\dot{\phi}^2)}\right\}\sin\dot{\phi}t-\left\{\frac{C_{13}}{\Lambda_0^2-(\dot{\phi}-2\Lambda_0)^2}\right\}\cos(\dot{\phi}-2\Lambda_0)t \\
& -\left\{\frac{C_{14}}{\Lambda_0^2-(\dot{\phi}-2\Lambda_0)^2}\right\}\sin(\dot{\phi}-2\Lambda_0)t \qquad (10)
\end{aligned}
$$

3. SERIES SOLUTION BY L-P METHOD

In this section, we have to derive the approximate solution for u as function of time t. For the first approximation, we suppose that the energy of the Satellite which is is bound to near the Earth as origin. For this let us write Equation (9) in the standard form as

$$\frac{d^2u}{dt^2}+\Lambda_0^2 u-h(t)=0, \qquad (11)$$

where

$$
\begin{aligned}
h(t) \;=\; & h_1+h_2\cos\Lambda_0 t+h_3\cos\dot{\phi}t \\
& + h_4\sin\dot{\phi}t+h_5\cos(\Lambda_0-\dot{\phi})t+h_6\sin(\Lambda_0-\dot{\phi})t+h_7\sin 2(\Lambda_0-\dot{\phi})t \\
& + h_8\cos(2\Lambda_0-\dot{\phi})t+h_9\sin(2\Lambda_0\dot{\phi})t+h_{10}\cos(\dot{\phi}+\Lambda_0)t+h_{11}\sin(\dot{\phi}+\Lambda_0)t \qquad (12) \\
& + h_{12}\cos(3\Lambda_0-\dot{\phi})t+h_{13}\sin(3\Lambda_0-\dot{\phi})t+h_{14}\sin(\Lambda_0-2\dot{\phi})t+h_{15}\sin(3\Lambda_0-2\dot{\phi})t.
\end{aligned}
$$

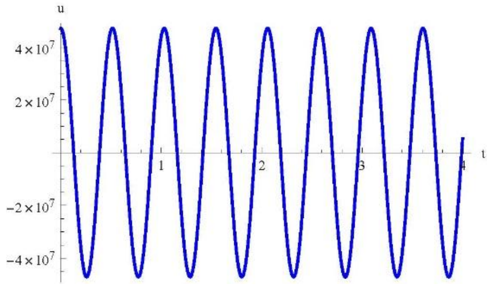

Fig. 7. Graph of Eq. (10) for
$a = 0.0000469251$, $e = 0.0071$,
$\dot{\theta}_0 = 708.102$ and $q = 0.25$

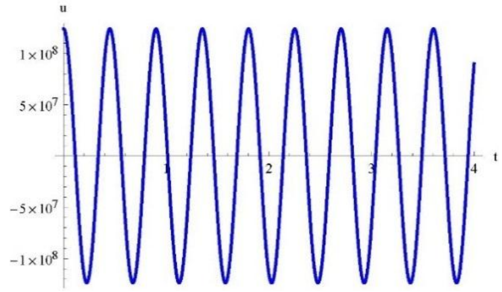

Fig. 8. Graph of Eq. (10) for
$a = 0.0000469251$ $e = 0.0071$,
$\dot{\theta}_0 = 708.102$ and $q = 0.75$

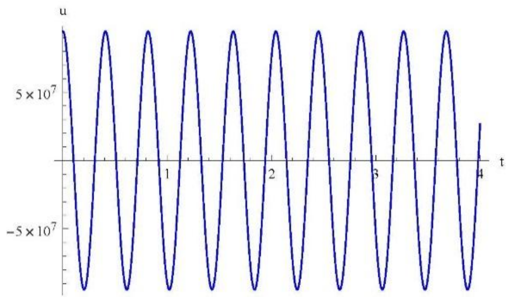

Fig. 9. Graph of Eq. (10) for
$a = 0.0000526604$, $e = 0.0635$,
$\dot{\theta}_0 = 890.602$ and $q = 0.25$

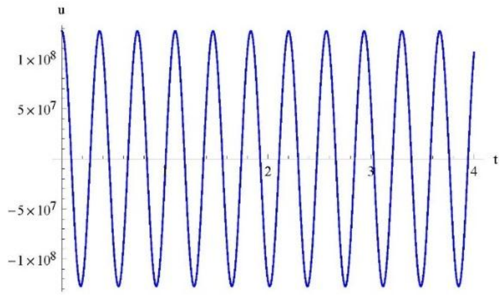

Fig. 10. Graph of Eq. (10) for
$a = 0.0000526604$ $e = 0.0635$,
$\dot{\theta}_0 = 890.602$ and $q = 0.75$

Parameters $h_{i'}s$ refer to Appendix. Let us write

$$u = u_0 + h(t)u_1 + h(t)^2 u_2 + h(t)^3 u_3 + \cdots \tag{13}$$

Now we define an independent variable τ as $\tau = \Lambda t$, where

$$\Lambda = \Lambda_0 + h(t)\Lambda_1 + h(t)^2 \Lambda_2 + + h(t)^3 \Lambda_3 + \cdots \tag{14}$$

Using $\tau = \Lambda t$ in the equation (11) we found

$$\Lambda^2 \ddot{u} + \Lambda_0^2 u - h(t) = 0 \tag{15}$$

Using the equations (13) and (14) in equation (15), we get

$$\Lambda_0^2 \left(\ddot{u}_0 + u_0\right) = 0, \tag{16}$$

$$\Lambda_0^2 \left(\ddot{u}_1 + u_1\right) + 2\Lambda_0 \Lambda_1 \ddot{u}_0 - 1 = 0, \tag{17}$$

$$\Lambda_0^2 \left(\ddot{u}_2 + u_2\right) + 2\Lambda_0 \Lambda_1 \ddot{u}_1 + (\Lambda_1^2 + 2\Lambda_0 \Lambda_2)\ddot{u}_0 = 0, \tag{18}$$

$$\Lambda_0^2 \left(\ddot{u}_3 + u_3\right) + 2\Lambda_0 \Lambda_1 \ddot{u}_2 + (\Lambda_1^2 + 2\Lambda_0 \Lambda_2)\ddot{u}_1 + 2(\Lambda_1 \Lambda_2 + \Lambda_1 \Lambda_3)\ddot{u}_0 = 0. \tag{19}$$

The general solution of the equation (16) is $u_0 = C_1 \cos\tau + C_2 \sin\tau$, where C_1 and C_2 are constants of integration. Taking initial conditions $u_0(0) = C$ and $\dot{u}_0(0) = 0$ the complete solution of the equation (16) can be given as

$$u_0 = C\cos\tau \tag{20}$$

Putting the values of u_0 and $\ddot{u}_0$ in the equation (17) we found

$$\ddot{u}_1 + u_1 = \frac{1}{\Lambda^2} + \frac{2\Lambda_1}{\Lambda_0} C\cos\tau \tag{21}$$

Equating the coefficient of $\cos\tau$ to zero, we get

$$\ddot{u}_1 + u_1 = \frac{1}{\Lambda^2},\ \Lambda_1 = 0 \tag{22}$$

The general solution of the equation (22) is

$$u_1 = C_3 \cos\tau + C_4 \sin\tau, \tag{23}$$

where C_3 and C_4 are constants of integration. Taking initial conditions $u_1(0) = C$ and $\dot{u}_1(0) = 0$ the complete solution of the equation (22) can be given as

$$u_1 = C\cos\tau + \frac{(1-\cos\tau)}{\Lambda_0^2}. \tag{24}$$

Substituting the values of u_0, $\ddot{u}_0$, u_1, $\ddot{u}_1$ and $\Lambda_1 = 0$ in the equation (18) we get

$$\ddot{u}_2 + u_2 = \frac{2\Lambda_2}{\Lambda_0} C\cos\tau. \tag{25}$$

Equating the coefficient of $\cos\tau$ to zero, we get

$$\ddot{u}_2 + u_2 = 0, \Lambda_2 = 0 \tag{26}$$

The general solution of the equation (26) is

$$u_2 = C_5 \cos\tau + C_6 \sin\tau, \tag{27}$$

where C_5 and C_6 are constants of integration. Taking initial conditions $u_2(0) = C$ and $\dot{u}_2(0) = 0$ the complete solution of the equation (26) can be given as

$$u_2 = C\cos\tau. \tag{28}$$

Substituting the values of u_0, $\ddot{u}_0$, u_1, $\ddot{u}_1$, u_2, $\ddot{u}_2$, Λ_1 and Λ_2 in the equation (19), we get

$$\ddot{u}_3 + u_3 = \frac{2\Lambda_3}{\Lambda_0} C\cos\tau. \tag{29}$$

Equating the coefficient of $\cos\tau$ to zero, we get

$$\ddot{u}_3 + u_3 = 0,\ \Lambda_3 = 0. \tag{30}$$

The general solution of the equation (30) is

$$u_3 = C_7 \cos\tau + C_8 \sin\tau, \tag{31}$$

where C_7 and C_8 are constants of integration. Taking initial conditions $u_3(0) = C$ and $\dot{u}_3(0) = 0$, the complete solution of the equation (30) can be given as

$$u_3 = C\cos\tau. \tag{32}$$

Proceeding in the same way and taking the values of $u_i (i = 0,1,2,3,\ldots)$ in the equation (13), the values of u can be given as

$$\begin{aligned}
u \;=\; & C\cos\Lambda\ t + \{h_1 + h_2\cos\Lambda_0 t + h_3\cos\dot{\phi}t + h_4\sin\dot{\phi}t + h_5\cos\left(\Lambda_0 - \dot{\phi}\right)t \\
& + h_6\sin\left(\Lambda_0 - \dot{\phi}\right)t + h_7\sin 2\left(\Lambda_0 - \dot{\phi}\right)t + h_8\cos\left(2\Lambda_0 - \dot{\phi}\right)t + h_9\sin(2\Lambda_0\dot{\phi})t \\
& + h_{10}\cos\left(\dot{\phi} + \Lambda_0\right)t + h_{11}\sin\left(\dot{\phi} + \Lambda_0\right)t + h_{12}\cos\left(3\Lambda_0 - \dot{\phi}\right)t + h_{13}\sin\left(3\Lambda_0 - \dot{\phi}\right)t \\
& + h_{14}\sin\left(\Lambda_0 - 2\dot{\phi}\right)t + h_{15}\sin\left(3\Lambda_0 - 2\dot{\phi}\right)t\}\left\{C\cos\Lambda t + \frac{1}{\Lambda_0^2}(1 - \cos\Lambda t)\right\} \\
& + \{h_1 + h_2\cos\Lambda_0 t + h_3\cos\dot{\phi}t + h_4\sin\dot{\phi}t + h_5\cos\left(\Lambda_0 - \dot{\phi}\right)t \\
& + h_6\sin(\Lambda_0 - \dot{\phi})t + h_7\sin 2\left(\Lambda_0 - \dot{\phi}\right)t + h_8\cos\left(2\Lambda_0 - \dot{\phi}\right)t \\
& + h_9\sin(2\Lambda_0\dot{\phi})t + h_{10}\cos\left(\dot{\phi} + \Lambda_0\right)t + h_{11}\sin\left(\dot{\phi} + \Lambda_0\right)t \\
& + h_{12}\cos\left(3\Lambda_0 - \dot{\phi}\right)t + h_{13}\sin\left(3\Lambda_0 - \dot{\phi}\right)t + h_{14}\sin\left(\Lambda_0 - 2\dot{\phi}\right)t \\
& + h_{15}\sin\left(3\Lambda_0 - 2\dot{\phi}\right)t\}^2 C\cos\Lambda\ t\{h_1 + h_2\cos\Lambda_0 t + h_3\cos\dot{\phi}t + h_4\sin\dot{\phi}t \\
& + h_5\cos\left(\Lambda_0 - \dot{\phi}\right)t + h_6\sin\left(\Lambda_0 - \dot{\phi}\right)t + h_7\sin 2\left(\Lambda_0 - \dot{\phi}\right)t + h_8\cos\left(2\Lambda_0 - \dot{\phi}\right)t \\
& + h_9\sin(2\Lambda_0\dot{\phi})t + h_{10}\cos\left(\dot{\phi} + \Lambda_0\right)t + h_{11}\sin\left(\dot{\phi} + \Lambda_0\right)t + h_{12}\cos\left(3\Lambda_0 - \dot{\phi}\right)t \\
& + h_{13}\sin(3\Lambda_0 - \dot{\phi})t + h_{14}\sin(\Lambda_0 - 2\dot{\phi})t + h_{15}\sin(3\Lambda_0 - 2\dot{\phi})t\}^3 C\cos\Lambda t + \cdots
\end{aligned} \tag{33}$$

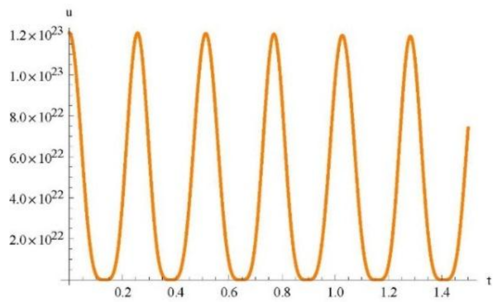

Fig. 11. Graph of Eq. (33) for $a = 0.0000469251$, $e = 0.0071$, $\dot{\theta}_0 = 708.102$, $q = 0.25$, $0 \le t \le 1.5$

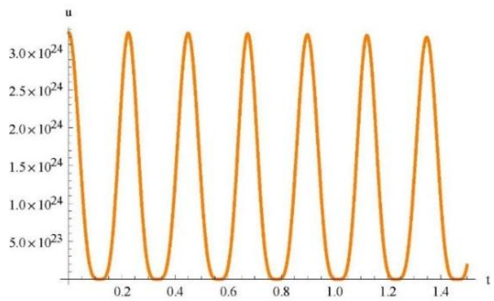

Fig. 12. Graph of Eq. (33) for $a = 0.0000469251$ $e = 0.0071$, $\dot{\theta}_0 = 708.102$, $q = 0.75$, $0 \le t \le 1.5$

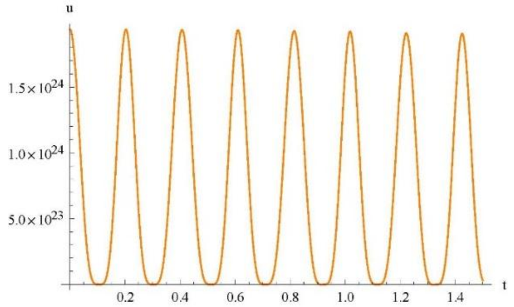

Fig. 13. Graph of Eq. (33) for $a = 0.0000526604$, $e = 0.0635$, $\dot{\theta}_0 = 890.602$, $q = 0.25$, $0 \leq t \leq 1.5$

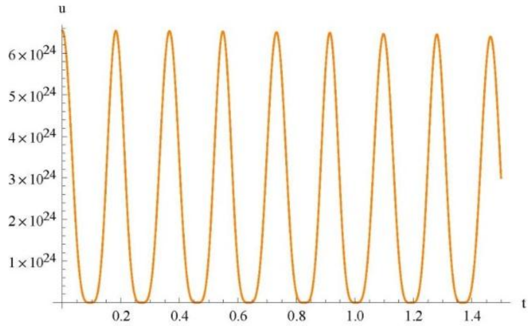

Fig. 14. Graph of Eq. (33) for $a = 0.0000526604$ $e = 0.0071$, $\dot{\theta}_0 = 708.102$, $q = 0.75$, $0 \leq t \leq 1.5$

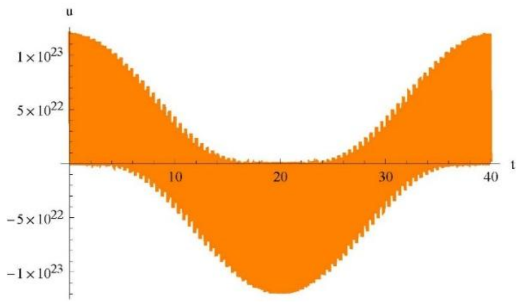

Fig. 15. Graph of Eq. (33) for $a = 0.0000469251$, $e = 0.0071$, $\dot{\theta}_0 = 708.102$, $q = 0.25$, $0 \leq t \leq 40$

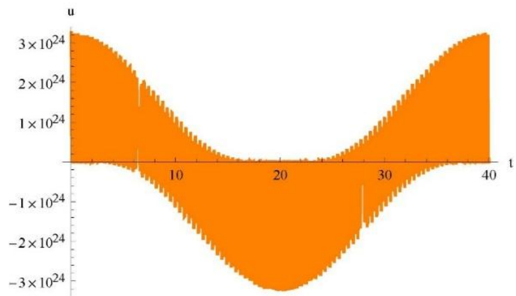

Fig. 16. Graph of Eq. (33) for $a = 0.0000469251$ $e = 0.0071$, $\dot{\theta}_0 = 708.102$, $q = 0.75$, $0 \leq t \leq 40$

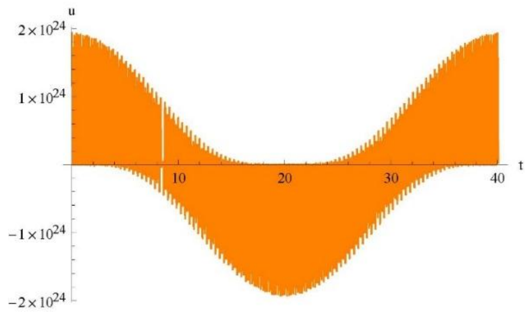

Fig. 17. Graph of Eq. (33) for $a = 0.0000526604$, $e = 0.0635$, $\dot{\theta}_0 = 890.602$, $q = 0.25$, $0 \leq t \leq 40$

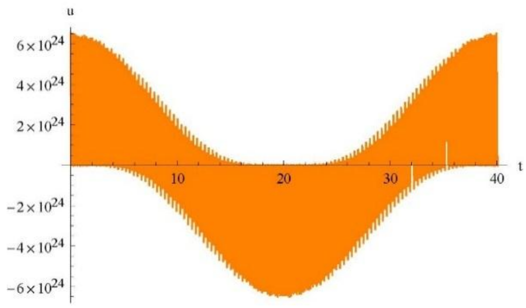

Fig. 18. Graph of Eq. (33) for $a = 0.0000526604$ $e = 0.0071$, $\dot{\theta}_0 = 708.102$, $q = 0.75$, $0 \leq t \leq 40$

4. POINCARE SECTION: Discussions

We have analyzed the impacts of P-R drag by putting different values of a, e and $\dot{\theta}_0$ for two different Satellites on the motion of a geocentric satellites in the Sun-Earth system in light of (a) motion of the 3BP, (b) solution of the three-body problem, (c) series solution by Lindstedt-Poincare method.

For the datum of the Satellite a, e and $\dot{\theta}_0$ mentioned in Fig. (3) and (4), it is observed that as P-R drag increases, the frequency of oscillation of the Satellite and the values of u decreases. Similarly for the datum of another Satellite a, e and $\dot{\theta}_0$ mentioned in Fig. (5) and Fig. (6), it is also observed that as P-R drag increases, the frequency of oscillation of the Satellite as well as u decreases. From Fig. set (3 and 5) and Fig. set (4 and 6), it can also be concluded that the Satellite with greater value of a, e and $\dot{\theta}_0$ has larger frequency of oscillation and larger peak values of u.

For the datum of the Satellite a, e and $\dot{\theta}_0$ mentioned in Fig. (7) and Fig. (8), it is further observed that as P-R drag increases in the solution of the 3BP, the frequency of oscillation of the satellite and the values of u increases. Similarly for the datum of next Satellite a, e and $\dot{\theta}_0$ mentioned Fig. (9) and Fig. (10), it is also observed that as P-R drag increases, the frequency of oscillation of the satellite and the values of u increases. From Fig. set (7 and 9) and Fig. set (8 and 10), it can also be concluded that the Satellite with greater value of a, e and $\dot{\theta}_0$ has larger frequency of oscillation and larger peak values of u.

For the datum of the satellite a, e and $\dot{\theta}_0$ mentioned in the captions of Fig. (11) and Fig. (12), the graphs of the series solution of Lindstedt-Pioncare within time interval $0 \le t \le 1.5$, it is observed that as P-R drag increases, the frequency of oscillation and the values of u increases. Similarly for the datum of next satellite a, e and $\dot{\theta}_0$ mentioned in the captions of Fig. (13) and Fig. (14), it is observed that as P-R drag increases, the frequency of oscillation and the values of u increases. From Fig. set (11and 13) and Fig. set (12 and 14), it can also be concluded that the satellite with greater value of a, e and $\dot{\theta}_0$ has larger frequency of oscillation and larger peak values of u.

For the datum of the satellite a, e and $\dot{\theta}_0$ mentioned in Fig. (15) and Fig. (16), within time interval $0 \le t \le 40$, the frequency of oscillation of the satellite increases and the values of u increases and decreases as the value of P-R drag increased. It is observed that when time interval lies between 0 and 4, values of t decreases with increase in the values of u. When t increased from 4 to 16, values of u decreases positively and increases negatively. Further, when t increased from 16 to 25, values of u decreases negatively. When the values of t increased from 25 to 38, values of u increase positively and decreases negatively. Furthermore, if t increased from 38 to 40, the values of u increases positively and the graphs gets repeated after that.

Similarly, the satellite with greater values of a, e and $\dot{\theta}_0$ mentioned in Fig. (17) and Fig. (18) has larger frequency of oscillation and larger peak values of u within the same interval. In short, it can be seen that the graphs look like similar to the Fig. (15 and 16) within the same sub time intervals. Thus, we conclude that as we increase P-R drag ($q = 0.75$) throughout the manuscript, its effects are clearly visible in various figures.

APPENDIX

$$h_1 = \frac{GM_s q}{r_s^3 a(1-e^2)},\; h_2 = \frac{GM_s qe}{r_s^3 a(1-e^2)},\; h_3 = -\frac{r_E e(\dot{\phi}^2 r_s^3 - GM_s q)}{r_s^3 a^2 (1-e^2)^2},\; h_4 = -\frac{r_E e\dot{\phi} GM_s (1-q)}{r_s^2 ca^2 (1-e^2)^2},$$

$$h_5 = -\frac{r_E (1+e^2)(\dot{\phi}^2 r_s^3 - GM_s q)}{r_s^3 a^2 (1-e^2)^2},\; h_6 = -\frac{\left(\left(1-e^2\right)^2 a^2 \left(\Lambda_0 - \dot{\phi}\right) + r_s \dot{\phi}\left(1+\frac{e^2}{2}\right)\right)}{cr_s^3 a^2 (1-e^2)^2} \times r_E GM_s (1-q),$$

$$h_7 = -\frac{(\Lambda_0 - \dot{\phi})(r_E)^2 GM_s (1-q)}{2r_s^3 ca(1-e^2)},\; h_8 = \frac{\dot{\phi}^2 r_E e}{a^2 (1-e^2)^2} - \frac{GM_s qer_E}{r_s^3 a^2 (1-e^2)^2},$$

$$h_9 = -\frac{GM_s (1-q) e\dot{\phi} r_E}{r_s^2 ca^2 (1-e^2)^2},\; h_{10} = -\frac{r_E}{e^2}\left(\dot{\phi}^2 r_s^3 - GM_s q\right) 4r_s^3 a^2 (1-e^2)^2 = h_{12},$$

$$h_{11} = -\frac{r_E e^2 \dot{\phi} GM_s (1-q)}{4r_s^2 ca^2 (1-e^2)^2} = h_{13},\; h_{14} = -\frac{e\left(\Lambda_0 - \dot{\phi}\right) r_E^2 GM_s (1-q)}{4cr_s^3 a(1-e^2)} = h_{15}.$$

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