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Center-Smooth One Complement Domination of Some Graphs

A. Anto Kinsley[a], J. Joan Princiya[b]

[a]*Department of mathematics,St.Xavier's (Autonomous) College, Palayamkottai. Affiliated to Manonmaniam Sundaranar University, Tirunelveli,, India.*

[b]*Assistant Professor, Department of mathematics, St.Joseph's college of arts and science for women, Hosur, Affliated to periyar university,Salem, India.*

Abstract

Let S be a dominating set of a center smooth graph G and the set of vertices RS^c be the restrict complement of S. The set RS^c is called a *center smooth one complement* (1^c) *dominating set* of a center smooth graph G if for every vertex in S^c has at least one neighbor in S. The number of vertices in RS^c of G is called *center smooth 1^cdomination number* and it is denoted by $\gamma_1^c cs(G)$. In this paper, we introduce the new concept of center-smooth 1^c domination number and establish some results on this new parameter. It is proved that for any graph G,$\gamma_1^c cs(G) \leq p - m$ where m is the number of vertices adjacent to pendent vertices and $\gamma_1^c cs(G) \leq diam(G) + k + 1$ where k is distance of any two vertices in S is ≥ 2. Finally some bounds on $\gamma_1^c cs$ for some classes of graphs are found.

Keywords: Radius, diameter, Center smooth graph, *Restrict S^c*-set, Center-smooth 1^c domination number.

1. INTRODUCTION

We consider only finite simple undirected connected graphs. For the graph G, $V(G)$ denotes its vertex set and $E(G)$ denotes its edge set. As usual, $p=|V|$ and $q= |E|$ denote the number of vertices and edges of a graph G, respectively. For a connected graph $G(V,E)$and a pair u, v of vertices of $G,$ the *distance* $d(u, v)$ between u and v is the length of a shortest u-v path in G. The *degree* of a vertex u, denoted by *deg(u)* is the number of vertices adjacent to u. A vertex$u$ of a graph G is called a *universal vertex*if u is adjacent to all other vertices of G. A graph G is *universal graph* if every vertex in G is universal vertex. For example, the complete graph K_p is universal graph.The set of all vertices adjacent to u in a graph G, denoted by $N(u)$, is the *neighborhood* of the vertex u. The *eccentricity* $e(u)$ of a vertex u is the distance to a vertex farthest from u. Thus, $e(u) = max\{ d(u, v)/v\in V(G)\}$. A vertex v is an *eccentric vertex* of u if $e(u)= d(u, v)$. The radius $r(G)$ is the minimum eccentricity of the vertices, whereas the diameter $diam(G)$ is the maximum eccentricity.The center of G, $C(G)=\{ v\in V(G)/e(v)= r(G)\}$.

Definition 1.1

2-packing is a subset of vertices, in which no two vertices are adjacent or have a common neighbor. The *packing number* $\rho(G)$ is the cardinality of a maximum packing.

Definition 1.2

A *clique* of a graph G is a complete subgraph of G, and the clique of largest possible size is referred to as a maximum clique (which has size known as the clique number $\omega(G)$).

Definition 1.3

A *vertex cover* of G is a set of vertices that covers all the edges. The vertex covering number α_o(G) is minimum cardinality of a vertex cover.

Definition 1.4

The *vertex cut* or separating set of G is a set of vertices whose removal results in a disconnected. The *connectivity* or *vertex connectivity* of a graph G, denoted by $\mathcal{K}(G)$(where G is not complete) is the size of a smallest vertex cut.

Definition 1.5

The *S-eccentricity* $e_S(v)$ of a vertex v in G is $\max_{x\in S}(d(v,x))$. The *S-center* of G is $C_S(G) = \{v\in V \,|\, e_S(v) \leq e_S(x) \forall\, x\in V\}$.

Example 1

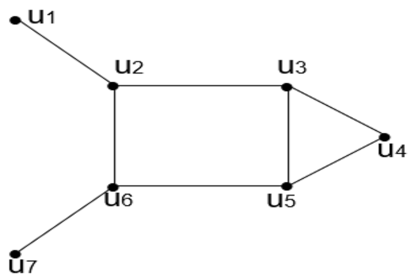

Figure 1. Center smooth graph

In figure 1, $S =\{u_1, u_3, u_6\}$ and $V\text{-}S =\{ u_2, u_4, u_5, u_7\}$.The S-eccentricity $e_S(u_1)=3$, $e_S(u_2)=1$,$e_S(u_3)=3$,$e_S(u_4)=3$,$e_S(u_5)=3$,$e_S(u_6)=2$,$e_S(u_7)=3$. Then the S-center $C_S(G) = \{u_2\}$.

Definition 1.6

The S_1-*eccentricity*, $e_{S_1}(v)$ of a vertex v in S is $\max_{x\in V-S}(d(v,x))$. The S_1 *center* of G is $C_{S_1}(G) = \{v\varepsilon V \mid e_{S_1}(v) \leq e_{S_1}(x) \text{for all} x\varepsilon V\}$.

Example 2 : In figure 1, $S=\{u_1, u_3, u_6\}$ and V-$S=\{u_2, u_4, u_5, u_7\}$.The S_1-eccentricity $e_{S_1}(u_1)$=3, $e_{S_1}(u_2)$=1,$e_{S_1}(u_3)$=3,$e_{S_1}(u_4)$=3,$e_{S_1}(u_5)$=3,$e_{S_1}(u_6)$=2,$e_{S_1}(u_7)$=3. Then the S_1-center, $C_{S_1}(G) = \{u_2\}$.

Definition 1.7

Let G be a graph and S be a proper set of G. G is called a *center-smooth graph* if $C_S(G) = C_{S_1}(G)$ and the set S is said to be a *center-smooth set*.

Example 3: In figure 1, $C_S(G) = \{u_2\} = C_{S_1}(G)$.

Definition 1.8

An $S\subseteq V$ is a *dominating set* in G if every vertex not in S is adjacent to at least one vertex of S. If S is a dominating set then V- S (*inversedominating set*) need not be a dominating set.

Definition 1.9

A set S is *independent* if no two vertices in it are adjacent. An *independent dominating set* of G is a set that is both dominating and independent in G. *Independence domination number* $(\gamma_i(G))\beta_o$(G) of G is the maximum (minimum) cardinality taken over all independent dominating sets of G.

Definition 1.10

A set S is called 1-*dominating set* if for every vertex in V-S, there exists exactly one neighbor in S. The minimum cardinality of a 1-dominating set is denoted by $\gamma_1(G)$

Definition 1.11

Let S be a dominating set of center smooth graph G. Then the *Restrict*-S^c(RS^c) set of a graph G is defined by $RS^c=\begin{cases} v \in RS^c; & |N(v) \cap S| = 1 \\ v \notin RS^c; & |N(v) \cap S| > 1 \end{cases}$ and the number of RS^c - set of G is denoted by $nR(G)$. If RS^c - set is independent set then the number of RS^c - set of G is denoted by $niR(G)$.

2. Center Smooth 1^c –Domination

Definition 2.1

Let S be a dominating set of G and $RS^c\subseteq V(G)$. Then the set $RS^c$is called a *center smooth* 1^c*dominating set* of a center smooth graph G if for every vertex in S^c has atleast one neighbor in S. The number of vertices in RS^c of a center smooth graph G is called *center smooth 1^{c}domination number* and it is denoted by$\gamma_1^c cs(G)$.

Theorem 2.1

If S is 1-dominating set of G, then every inverse dominating set is a center-smooth 1^c dominating set of G.

Proof: Let S is 1- dominating set of G. Then there exists a vertex $u \in S$ such that v is adjacent to u. Therefore $N(v) \cap S = \{u\}$ for every $v \in V - S$. By the definition of RS^c, $u \in RS^c$. Which implies $|RS^c| = |N(v) \cap S| = 1$ and hence the result. ■

Observation 2.1.

(i) $\gamma(G) \leq \gamma_1^c cs(G)$
(ii) $\gamma(G) + \gamma_1^c cs(G) \leq p$

Proof: (i). Let$S$ be a dominating set of G with $|S| = \gamma(G)$. Further, if $u \in S$, then there exists a vertex $v \in V-S$ such that v is adjacent to u. Therefore $|N(v) \cap S| = 1$. Then clearly, $|S| = 1$ and $|V - S| = 1$.Then by the theorem 2.1, $\gamma_1^c cs(G) = 1$ and therefore $\gamma(G) = \gamma_1^c cs(G) \rightarrow (1)$. Then there exists a vertex $w \neq v$ such that w is adjacent to u, therefore $|N(w) \cap S| = 1$, $w \in V-S$. Then, $|V - S| = 2$ and $\gamma(G) < \gamma_1^c cs(G) \rightarrow (2)$. From (1) and (2), we get $\gamma(G) \leq \gamma_1^c cs(G)$.

(ii). Let $S \subseteq V(G)$ be the set of vertices which dominates all the vertices in G. Clearly, $N[S] = V(G)$ and S forms a γ-set of G. Further, let $RS^c \subseteq V(G)$ be the center-smooth 1^c dominating set of G and then $|S \cup RS^c| \subseteq V(G)$.Which implies $|S| + |RS^c| \leq |V(G)|$.Hence, $\gamma(G) + \gamma_1^c cs(G) \leq p$. ■

Theorem 2.2

For any graph G, $\gamma_1^c cs(G) = p-1$ if and only if one of the following holds:

(a) There exist a vertex $u \in V(G)$ such that every vertex of G is within distance one of u.
(b) For any u, $v \in V(G)$ is at distance one.

Proof: Suppose $\gamma_1^c cs(G) = p-1$. We prove that any one of the above condition holds. On the contrary, G does not hold any of the above conditions. Let $RS^c \subseteq V(G)$ be the center-smooth 1^c dominating set of G. Then there exist, two non-adjacent vertices u, $v \in G$ which are adjacent to x and y such that x is adjacent to y in G. It implies that $RS^c - \{u, v\}$ be a center smooth 1^c dominating set of G. Which is a contradiction. Hence, G holds one of the above conditions.Conversely, suppose that G holds one of the above conditions. Let $u \in V(G)$ be a universal vertex in G. Clearly, all the vertices in G dominated by a vertex u. Therefore, $d(u, v) = 1, \forall v \in V(G)$. Hence, condition (a) holds. ■

Theorem 2.3

If a connected graph G having a vertex v is not in RS^c and not in S, then v is adjacent to more than one vertex in S.

Proof: From the theorem (2.1), $|N(v) \cap S| \neq 1, \forall v \in V\text{-}S$ (since $v \notin RS^c$ and $v \notin V\text{-} RS^c$). Clearly, a vertex v is not adjacent to exactly one vertex in S. That is, a vertex v is adjacent to more than one vertex in S. From the theorem (2.1), hence a vertex v is adjacent to more than one vertex in S. ■

Theorem 2.4.

For any graph $G, \gamma_1^c cs(G) \leq p - m$ where m is the number of vertices adjacent to pendent vertices.

Proof: Let $RS^c \subseteq V(G)$ be the center-smooth 1^c dominating set of G . Further, let S be the set of all vertices which are adjacent to pendent vertices with $|S|=m$. Let $u \in RS^c$. If $u \in S$, then $|RS^c| = p\text{-}m$ and the inequality holds. If $u \notin RS^c$, then there exists a vertex $v \in RS^c$ is adjacent to u. Further, all vertices which are connected to v not through u also belonging to RS^c. This implies that S has at most$m$ vertices and the inequality holds. ■

Theorem 2.5.

If $2 \leq \delta(G) \leq p\text{-}1$, then$\gamma_1^c cs(G) \leq q\text{-}1$ where δ(G) is the minimum degree of G.

Proof: Let G be any graph. By the assumption on $\delta(G)$, $\delta(G)$=1 or p-1, then$\gamma_1^c cs(G) \nleq q\text{-}1$.Let $|S| =1$, then $|V\text{-} S| =p\text{-}1$.That is, $V\text{-}S = \{v_1, v_2, \ldots, v_{p-1}\}$. Further, let $V\text{-}S$ is a$\gamma_1^c cs$-set of G. Therefore $p\text{-}1 = \gamma_1^c cs(G) \nleq q\text{-}1$ and so for $\delta(G)$=1 or p-1,$\gamma_1^c cs(G) \nleq q\text{-}1$. Otherwise,$\gamma_1^c cs(G) \leq q\text{-}1$. ■

Theorem 2.6

For any graph $G = K_p, \gamma_1^c cs(K_p) = p\text{-}1$ for $p \geq 2$.

Proof: When $G = K_p$, radius=diameter=1. Hence any vertex $u \in V(G)$ dominate other vertices and RS^chas p-1 vertices. Therefore,$\gamma_1^c cs(K_p)=p\text{-}1$. ■

Theorem 2.7

For any cycle C_{2p+1}, p =1, 3, 5,……,$\gamma_1^c cs(C_{2p+1}) \leq p\text{-}1$.

Proof: Every cycle C_p with p vertices and $q=p$ edges in which each vertex is of degree 2. That is each vertex dominates two vertices and in any odd cycle C_{2p+1}, p=1,3,5,…,$deg(p_i)=deg(p_j)=2$ for all $p_i, p_j \in V(C_{2p+1})$. Further, let S be a dominating set of C_{2p+1}.Then we have two cases:

Case (i): If $RS^c = V$-S, then $\gamma_1^c cs(C_{2p+1})=p$-1 or$\gamma_1^c cs(C_{2p+1}) < p$-1. Hence it follows that $\gamma_1^c cs(C_{2p+1}) \leq p$-1.

Case (ii): If $RS^c \subseteq V$- S, then $\gamma_1^c cs(C_{2p+1}) \leq p$-1. ■

Theorem 2.8.

For any complete bipartite graph $G = K_{m,n}$ with p vertices,$\gamma_1^c cs(G)=p$-2.

Proof: Let $G= K_{m,n}$ be a complete bipartite graph. For $V(G)= V_1 \cup V_2$, $|V_1|=q$ and $|V_2|= p$ such that each element of V_1 is adjacent to every vertex of V_2 and vice versa. Let $RS^c = \{u, v\}$, $u \in V_1$, $v \in V_2$. Then, clearly u dominates all the vertices of V_2. Similarly, v dominates all the vertices of V_1. Hence RS^c be a center-smooth 1[c] dominating set and G has $|RS^c|+2$ vertices. That is, $p= |RS^c|+2$. This implies $|RS^c|=p$-2. Hence it follows that $\gamma_1^c cs(G) =p$-2. ■

Theorem 2.9

For any graph G,$\gamma_1^c cs(G) = p$ -1 iff $G = K_{1,p-1}$.

Proof: Suppose that $\gamma_1^c cs(G) = p$ -1. Let $S = \{u\}$ be the dominating set of G such that $|S|=1$. Further, let G has p -1 end vertices. That is, p -1 vertices of degree one. That is, u dominates all the end vertices of G. This implies, every edge of G is incident to a vertex u. Therefore, a vertex u of degree p -1. Hence it follows that, G is $K_{1,p-1}$.Conversely, suppose $G = K_{1,p-1}$. Let $RS^c = \{u, x, y, v\}$ where u, x, y and v are non-adjacent vertices and w is a central vertex of G. Therefore G has $|RS^c|$ +1 vertices. Hence $\gamma_1^c cs(G) = p$ -1. ■

Theorem 2.10

For any graph $G = B_{m,n}$, $\gamma_1^c cs(G) = p$ -2.

Proof: Let $G=B_{m,n}$ be a bistar. Let $S = \{u, v\}$ be a dominating set and also u and v are the central vertices of G. In a graph G has $m+n+2$ vertices and $m+n+1$ edges and totally $m+n$ pendant vertices and two center vertices. The degree of the central vertices are $m+1$ and $n+1$. Clearly, u dominates m vertices and v dominates n vertices. Therefore G has $|RS^c|+2$ vertices. Hence,$\gamma_1^c cs(G) = p$ -2. ■

Theorem 2.11

For any connected spanning sub graph H of G,$\gamma_1^c cs(H) \leq \gamma_1^c cs(G)$.

Proof: Let V -S be a inverse dominating set of G. Since, $V(G)= V(H)$, Then V -S is also an inverse dominating set of H. Hence an inverse dominating set of H is also an inverse dominating set of G. From the theorem 2.1, hence every center smooth 1[c] dominating set of H is also a center smooth 1[c]dominating set of G. So,$\gamma_1^c cs(H) \leq \gamma_1^c cs(G)$. ■

Theorem 2.12

For any graph G,$\gamma_1^c cs(G) \leq diam(G)+K+1$ where K is distance of any two vertices in S is ≥2.

Proof: Let $S = \{u_1, u_2,\dots,u_k\} \subseteq V(G)$ be the dominating set of G with $d(u_i, u_j) \geq 2$ for all $u_i, u_j \in S$ such that $|S|=K$. Further, there exists a vertex set $V'' \subseteq V'$, where V' is the set of vertices which are adjacent to the vertices of S, constituting the longest path in G such that $|V'|= diam(G)$. Also, let $V\text{-}S= \{v_1, v_2,\dots,v_s\}$ be the inverse dominating set of G. By the theorem 2.1, $V\text{-}S$ is a center smooth 1^c dominating set of G with $|RS^c| =\gamma_1^c cs(G)$. Therefore, it follows that $|RS^c| \leq |S| \cup |V'| +1$ and hence $\gamma_1^c cs(G) \leq diam(G)+K+1$. ■

Theorem 2.13

For any connected graph G with a unicentral vertex u, one of the following holds:

(i) $\gamma_1^c cs(G) = deg(u)$.
(ii) $\gamma_1^c cs(G) \leq p- 2$.

Proof: (i). Let u be a unicentral vertex of G. That is, u is also a cut vertex of G. Then $N(u)$ is a center-smooth 1^c dominating set. Therefore it follows that $\gamma_1^c cs(G) =deg(u)$.

(ii). Let $S=\{x, y\}$ be such that $u \neq x, y$. Now, we shall take two cases.

Case (i): Suppose $u \in RS^c$, then G has $|RS^c|+2$ vertices. Therefore,$\gamma_1^c cs(G) = p\text{-}2$.

Case(ii): Suppose $u \notin RS^c$, then u is adjacent to both the vertices in S. From the theorem (2.3), then $\gamma_1^c cs(G) < p$ -2. In both the cases, $\gamma_1^c cs(G) \leq p$ - 2. ■

Theorem 2.14

Let G be a triangle free graph. If G is of radius 2 with a unique central vertex u, then $\gamma_1^c cs(G) \leq p +1\text{-} deg(u)$.

Proof: If G is of radius 2 with a unique central vertex u, then u dominates $N(u)$. Let $S = \{N(u)\}$ is a dominating set of G. Further, let RS^c contains all vertices in $N(u)$ is an eccentric vertex of G. Since G has no triangles, then the vertices of $N(u)$ are disconnected. Therefore RS^c is a center smooth 1^cdominating set of cardinality $p\text{-}deg(u)+1$ and hence $\gamma_1^c cs(G) \leq p+1\text{-}deg(u)$. ■

3. Bounds on Center Smooth 1^c –Domination number

Theorem 3.1

For any graph G without isolated vertices, $\gamma_1^c cs(G) = p\text{-}\beta_o(G)$.

Proof: Let S be a dominating and also an independent set of G with $\beta_o(G)$ vertices. Then $|S| = \beta_o(G)$. Since G has no isolated vertices, it implies that V-S is an inverse dominating set of G. Clearly, by the theorem 2.1, V-S is a center smooth 1^c dominating set of G.Hence, V -S= RS^c.⇒ $|V$-$S|=|RS^c|$. ⇒ p- $\beta_o(G) = \gamma_1^c cs(G)$.

Corollary 3.1.1.

For any graph G without isolated vertices,$\gamma_1^c cs(G)=\alpha_o(G)$.

Proof: We know that, for any graph G,$\alpha_o(G)+\beta_o(G)= p$. From the main theorem,$\gamma_1^c cs(G) = p - \beta_o(G)$. Therefore, $\gamma_1^c cs(G) = \alpha_o(G)$. ■

Theorem 3.2.

For any connected graph G,$\gamma_1^c cs(G) \geq p - \xi(T)$ where $\xi(T)$ is the maximum number of end vertices in any spanning tree T of G.

Proof: Let $RS^c \subseteq V(G)$ be the center-smooth 1^cdominating set of G . Then for any two vertices u, $v \in V - RS^c$, there exists two vertices u_1, $v_1 \in RS^c$ such that u_1 is adjacent to u and v. Similarly, v_1 is adjacent to u and v. This implies that there exists a spanning tree T of V-RS^c in which each vertex of V-RS^c is adjacent to a vertex of RS^c. This proves that $\xi(T) \geq |V - RS^c|$. Thus, $\xi(T) \geq p - \gamma_1^c cs(G)$. Which implies that $\gamma_1^c cs(G) \geq p - \xi(T)$.

■

Theorem 3.3

For any graph G, $\left\lceil \frac{p}{1+\rho(G)} \right\rceil \leq \gamma_1^c cs(G) \leq p - \rho(G)$ where $\rho(G)$ is the cardinality of a maximum packing.

Proof:Let S be a γ-set of G. First we consider the lower bound. Let $RS^c = \{u_1, u_2, \ldots, u_p\} \subseteq V(G)$ be the $\gamma_1^c cs$-set of G. Further if $|V - RS^c| \geq 2$, then V-RS^c contains at least two vertices such that each vertex can dominate atmost itself and ρ(G) other vertices. Hence, $\gamma_1^c cs(G) \geq \left\lceil \frac{p}{1+\rho(G)} \right\rceil$. For the upper bound, let u, v be a vertices of S. Then the vertices of S dominates $N(u)$ and $N(v)$. By the definition of $\rho(G)$, $N[u] \cap N[v] = \phi$. Therefore, V-$|N[u] \cap N[v]|$ is the $\gamma_1^c cs$-set of G. Hence, $\gamma_1^c cs(G) \leq p - \rho(G)$. ■

Theorem 3.4

For any graph G,$\gamma_1^c cs(G) \leq 2p - \omega(G) - 1$ where $\omega(G)$ is the clique number of G.

Proof: Let RS^c be a $\gamma_1^c cs$-set of G. Further, let S be a set of vertices in G such that $|S|= \omega(G)$.Then $p \geq \frac{|RS^c|+|S|+1}{2}$. Which implies $\gamma_1^c cs(G) \leq 2p - \omega(G) - 1$. ■

Theorem 3.5

For any connected graph G, $\delta(G)$-1 ≤ $\gamma_1^c cs(G)$≤ 2p-$\omega(G)$-1 where $\omega(G)$ is the clique number of G.

Proof: Let RS^c be a $\gamma_1^c cs$-set of G and $u \in V(G)$ such that $deg(u)= \delta(G)$.If $u \in S$, there exist v=$N(u)$ such that $v \in RS^c$, $|N(u)| = \delta$. If G has $diam$=2 then $\gamma(G) \le \delta(G)$. Therefore $\gamma(G) \le \delta(G) \le \gamma_1^c cs(G)$ and it follows that $\delta(G)$-1 ≤ $\gamma_1^c cs(G)$. From the theorem (3.4), we have $\gamma_1^c cs(G) \le 2p-\omega(G)-1$. Hence the result. ■

Theorem 3.6

For any graph G,$\gamma_1^c cs(G) \ge \left\lfloor \frac{2p-q-1}{2} \right\rfloor$. The bound is sharp.

Proof:Let RS^c be a $\gamma_1^c cs$- set of G. Then $q \ge |V- RS^c| + |V- RS^c|$ -1. Therefore, $q \ge 2p$-$2\gamma_1^c cs(G)$-1 and $\left\lfloor \frac{2p-q-1}{2} \right\rfloor \le \gamma_1^c cs(G)$. The bound is sharp for G is $K_{1,\, p\text{-}1}$. ■

Theorem 3.7

For any graph G, 1≤$\gamma_1^c cs(G) \le p$-1. The bounds are sharp.

Proof: Any vertex $u \in V(G)$ dominate other vertices and is also an eccentric point of other vertices. Hence, 1≤$\gamma_1^c cs(G) \le p$-1. The bounds are sharp for $G = K_p$, $p \ge 2$ or $G = T$. ■

4. CONCLUSION

In this paper, S_1-eccentricity of a vertex and center smooth set are defined. Also, the center smooth graph and Restrict S^c set have been introduced. Here we have studied the center smooth 1^c dominating set and center smooth 1^c domination number of some families of graph were enumerated.Also studied some bounds for center smooth 1^c domination number of a graph.

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