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Soft Rough Set With Covering Based

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Abstract

Research in mathematics and computer science has progressed in a widespread way to introduce new theories as efficacious model for plowing the difficulties of growing knowledge and information in varied sphere of real life. Soft set and rough set theory have been amalgamated by researchers as soft rough set to address the problems of imprecision and uncertainty . Soft Rough set nicely deals the complex issue of impreciseness and vagueness of information. This paper is brilliant attempt proposing soft rough set with covering as a new model to grapple the incipient matter of imprecision more easily presenting definition in varied manners with the help of two approximation operators.

Keywords: Rough set ,soft set, soft rough set ,minimum description, covering based soft rough set.

1. INTRODUCTION

Mathematics is the by name of knowledge, study and learning. It has been regarded as "the Queen of Sciences" by some mathematicians and promotes the main driving

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force to carry forward the scientific discovery although modern philosophers claim that mathematics is not a science rather a field of knowledge to address abstract ideas and topics.

In the age of knowledge expansion and information explosion , the mathematicians find it herculean to take on the challenging topic of imperfect knowledge and the vagueness of objects. Philosophers as well as logicians equally face embarrassment in respective areas in real life situations. To solve the problem of imperfect knowledge in social life mathematicians proposed new mathematical theories such as fuzzy set theory, rough set theory, and soft set theory that comprehensively deal impreciseness and vagueness.

The introduction of the fuzzy set theory in 1965 by L.A.Zadeh, a great mathematician, computer scientist and artificial intelligence researcher advances the mathematical model to handle vague concepts and inspirers the mathematicians and computer scientists to explore new theoretical account. As a result , Professor Z Pawlak introduced the rough set theory in 1982 that expands upon the model of set theory for the study of imperfect knowledge with regard to uncertain , imprecise , incomplete and vague information. Its application has become useful both in science and engineering domain such as chemistry, biology and engineering control. Furthermore, Molodstove proposed the concept of the soft set theory in 1999, as a new mathematical tool to handle vagueness conveniently in life. There after, in 2010, Feng et al introduced the advanced mathematical model soft rough set combining the soft set and rough sets to deal more effective the complex problems of uncertainty found in different areas in our day to day life. The combination of theories further advances by researchers in mathematics and computer science resulted in modeling new theories such as the rough fuzzy set, fuzzy rough set, soft rough set, rough soft set, fuzzy soft set, rough fuzzy soft set ect.

In this paper we aim at introducing a new concept of rough soft set with covering to tackle the challenges of impreciseness of objects, imperfect and incomplete knowledge in a more convenient way. The paper presents appropriate definitions with examples deriving the basic properties along with approximation space and both lower and upper approximation operators. The new covering based soft rough set would certainly encourage further research on the soft rough set theory as the problems in real life situations originate from uncertainty in the form of enigma.

2. ROUGH SET

2.1. Definition

Let U a finite set , be a universe of discourse and R be an equivalence relation (knowledge) on U. R will provide a partition on U. U/R be the set of all equivalence classes generated by R. Then the pair (U, R) is called approximation space.

2.2. Definition(Rough Set[4],[5])

The lower and upper approximation of X on U are defined by
$LR(X) = \cup\{Yi \in U/R|Y_i \subseteq X\}$ and $HR(X) = \cup\{Y_i \in U/R|Y_i \cap X \neq \phi\}$,where $X \subseteq U$ respectively. The set X is termed as Rough set with respect to R if $LR(X) \neq HR(X)$, otherwise X is said to be R - definable or exact set.

3. SOFT SET

3.1. Definition:([3])

Let U be the universe of discourse, a finite set. E denotes set of parameter, $P(U)$ indicate power set of U. In defining a mapping $F : A \rightarrow P(U)$, where $A \subseteq E$, then a pair (F, A) is called a soft set over U. For $e \in A$, $F(e)$ is considered as set of e-approximate elements of the soft set (F, A).

3.2. Definition:[6]

Let $S = (F, A)$ be a soft set over U. Then S is said to be a full soft set if $\cup_{a\in A}F(a) = U$.

4. SOFT ROUGH SET :

4.1. Definition:[1]

Let $S = (F, A)$ be a soft set over U.The pair $P = (U, S)$ is called a soft approximation space. Based on the soft approximation space P, we define the following two operations for $X \subseteq U$,
$L(apr_p(X)) = \{u \in U : \exists\, a \in A|u \in F(a) \subseteq X\}$.
$H(apr_p(X)) = \{u \in U : \exists\, a \in A|u \in F(a), F(a) \cap X \neq \phi\}$,
are called the soft P - lower and P - upper approximation of X with respect to P.

4.2. Definition:[1]

The soft P - positive region ,the soft P - negative region and the soft P - boundary region of $X \subseteq U$, can be defined as
$POS_P(X) = L(apr_p)(X),\ \ NEG_P(X) = U - H(apr_p(X))$,
$BND_P(X) = H(apr_p(X)) - L(apr_p(X))$, respectively. X is said to be soft P - rough set if $L(apr_p(X)) \neq H(apr_p(X))$, otherwise X is said to be soft P-definable.
Through the definition ,we have X as a soft definable set, if $BND_P(X) = \phi$ and also $L(arp_p(X)) \subseteq X$ and $L(apr_p(X)) \subseteq H(apr_p(X))$ for all $X \subseteq U$.

4.3. Definition:[2]

Let $S = (F, A)$ be a soft set over U. Then the triplet $P = (U, F, A)$ is called soft approximation space. The soft Lower and upper approximation defined as , for $X \subseteq U$,
$L(apr_F(X)) = \{u \in U : \exists\ e \in A | u \in F(e) \subseteq X\} = \cup_{e \in A}\{F(e) : F(e) \subseteq X\}$,
and

$$H(apr_F(X)) = \begin{cases} M, for X \subseteq M \\ M \cup N for X \nsubseteq M \end{cases}$$

respectively, where
$M = \{u \in U : \exists\ e \in A | u \in F(e), F(e) \cap X \neq \phi\} = \cup\{F(e) : F(e) \cap X \neq \phi\}$, and $N = \cap\{F'(e) : e \in (-A)\}$.
The $L(apr_F(X))$ and $H(apr_F(X))$ are referred to soft rough lower and upper approximation of X with respect to parameterized mapping F_A, where $F_A : A \to P(U)$ be the illustrated mapping. The set $X \subseteq U$ is called F-soft rough set, if $L(apr_F(X)) \neq H(apr_F(X))$, otherwise X is called F-soft definable set .Here we denote the notation F_A to indicate the parameter set A and the mapping F. That is F_A and F have the same mapping from A to $P(U)$. In this article we use the notation F instead of F_A everywhere.
The F-soft positive region, F-Soft negative region and F-soft boundary region of X may be defined as
$POS_F(X) = L(apr_F(X))$,
$NEG_F(X) = \cup_{e \in A}\{F(e) : F(e) \cap X \neq \phi\} = U - M$, for $X \subseteq M$,
$BND_F(X) = H(apr_F(X)) - L(apr_F(X))$, respectively.

4.4. Examples:

4.4.1. Example :

Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$, $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$, considering $A = \{e_1, e_2, e_3, e_4\}$, where $F(e_1) = \{u_1, u_6\}$, $F(e_2) = \{u_3\}$, $F(e_3) = \phi$, $F(e_4) = \{u_1, u_2, u_5\}$, Let $G = (F, A)$ be a soft set over U and then $P = (U, G)$ is the soft approximation space.
For $X = \{u_3, u_4, u_5\} \subseteq U$. We have
$L(apr_p(X)) = \{u_3\}$,
$H(apr_p(X)) = \{u_1, u_2, u_3, u_5\}$,
Here $L(apr_p(X)) \neq H(apr_p(X))$, So X is a soft rough set. But $X \nsubseteq H(apr_p(X))$, and
$L(apr_F(X)) = \{u_3\} \subseteq X$

$M = \{u_1, u_2, u_3, u_5\}$
$A = \{e_1, e_2, e_3, e_4\}$, So - $A = \{-e_1, -e_2, -e_3, -e_4\}$
$N = F'(e_1) \cap F'(e_2) \cap F'(e_3) \cap F'(e_4) = \{u_4\}$
$H(apr_F(X)) = M \cup N = \{u_1, u_2, u_3, u_4, u_5\}$, as $X \nsubseteq M$. Then $X \subseteq H(apr_F(X))$.
However, from this example we infer that the definition given in ([2]) is better than the previous one.

4.5. Proposition:[2]

Let $S = (F, A)$ be a soft set over U and $A \subseteq E$ be a set of parameters and $P = (U, F, A)$ be the corresponding soft approximation space. The F - Rough lower and upper approximation satisfy the following properties for every $X, Y \subseteq U$.

(i) $L(apr_F(\phi)) = \phi, \qquad H(apr_F(\phi)) = \phi$
(ii) $L(apr_F(U)) = U, \qquad H(apr_F(U) = U$
(iii) $L(apr_F(X \cap Y)) = L(apr_F(X)) \cap L(apr_F(Y))$
(iv) $L(apr_F(X \cup Y)) \supseteq L(apr_F(X)) \cup L(apr_F(Y))$
(v) $X \subseteq Y \Rightarrow L(apr_F(X)) \subseteq L(aprF(Y))$ and $H(apr_F(X)) \subseteq H(apr_F(Y))$
(vi) $H(apr_F(X \cup Y) = H(apr_F(X)) \cup H(apr_F(Y))$
(vii) $H(apr_F(X \cap Y)) \subseteq H(apr_F(X)) \cap H(apr_F(Y))$.

5. COVERING BASED SOFT ROUGH SET:

5.1. Definition

let $C = \{C_1, C_2, C_3, ..., C_n\}, C_i \subset U$ for each i , then C is called covering of U if $\cup\, C_i = U$.

5.2. Definition:

Let $K = (F, A)$ be a soft set over U. Then K is called covering soft set over U if $F(a) = \cup\, C_i$ for some i , for each $a \in A$ and $C_i \in C$.

5.3. Definition:

Let C is a covering of U and $K = (F, A)$ be a soft set over U. And (U, C, K) be a soft covering approximation space. Then minimum description of $x \in U$, can be defined as $Md_A(x) = \{K : K \in F(A) \wedge K \in C_x \wedge (\forall S \subseteq C_x(S \subseteq K \Rightarrow K = S))\}$, where $C_x = \{K \subseteq C : x \in K\}$.

5.4. First type of covering based soft Rough set:

5.4.1. Definition:

Let $P = (U, C, K)$ be a Soft covering approximation space for a set $X \subseteq U$, the soft covering Lower and upper approximation are , respectively defined as
$FL_F(X) = \cup\{C_i : C_i \subseteq F(e) \cap X\}$
$FH_F(X) = FL_F(X) \cup \{\cup Md_A(x) : x \in X - FL_F(X)\}$.
If $FL_F(X) \neq FH_F(X)$, then X is said to be the First type of covering soft Rough Set, otherwise X is called definable covering based soft set.

5.5. Second type of covering based soft rough set.

5.5.1. Definition :

Let $P = (U, C, K)$ be a soft covering approximation space. For a set $X \subseteq U$, the soft covering lower and upper approximation are respectively defined as :
$SL_F(X) = \cup\{C_i : Ci \subseteq F(e) \cap X\}$
$SH_F(X) = \cup_{e\in A}\{C_i \subseteq F(e) : F(e) \cap X \neq \phi\}$.
If $SL_F(X) \neq SH_F(X)$, then X is said to be second type of covering soft rough set, otherwise X is called definable covering based soft set.

5.6. New type of covering based soft rough set

5.6.1. Definition :

Let $P = (U, C, K)$ be a soft covering approximation space. For a set $X \subseteq U$, the soft covering lower and upper approximation are , respectively defined as :
$NL_F(X) = \cup\{C_i : C_i \subseteq F(e) \cap X\}$
and

$$NH_F(X) = \begin{cases} M, for X \subseteq M \\ M \cup N for X \nsubseteq M \end{cases}$$

where
$M = \cup_{e\in A}\{C_i \subseteq F(e) : F(e) \cap X \neq \phi\}$, and
$N = \cap\{F'(e) : e \in (-A)\}$. If $NL_F(X) \neq NH_F(X)$, then X is said to be new type covering based soft Rough set, otherwise X is said to be soft covering based definable.

5.7. Examples:

5.7.1. Example:

Let $U = \{h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8, h_9, h_{10}\}$, $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$. Let $A = \{e_1, e_2, e_3, e_5\}, C_K$ is covering of U, where

$C_1 = \{h_1, h_7\}, \quad C_2 = \{h_2, h_8\}, \quad C_3 = \{h_3, h_6, h_9\},$
$C_4 = \{h_4, h_7\}, \quad C_5 = \{h_1, h_7, h_{10}\}, \quad C_6 = \{h_2, h_9\},$
$C_7 = \{h_2, h_4, h_6\}, \quad C_8 = \{h_2, h_3, h_5\}$, and
F is a mapping from A to C_K such that
$F(e_1) = C_1 \cup C_2, \quad F(e_2) = C_3, \quad F(e_3) = C_4, \quad F(e_4) = C_7 \cup C_8, \quad F(e_5) = C_2 \cup C_6, \ F(e_6) = C_5 \cup C_1, F(e_7) = C_3 \cup C_8.$
Let us consider $X = \{h_1, h_2, h_5, h_7, h_{10}\}$,then to find out first , second and new type of soft covering lower and upper approximation.
$FL_F(X) = C_1 = \{h_1, h_7\} = \ SL_F(X) = \ NL_F(X),$
$Md(h_1) = C_1, \ Md(h_2) = C_2 \cup C_6,$
$Md(h_5) = \phi, \quad Md(h_7) = C_4, \ Md(h_{10}) = \phi,$
$X - FL_F(X) = \{h_2, h_5, h_{10}\},$
$\cup Md(x) = C_2 \cup C_6 \cup \phi = \{h_2, h_8, h_9\}$ for all $x \in (X - FL_F(X)).$
$FH_F(X) = C_1 \cup (C_2 \cup C_6) = \{h_1, h_2, h_7, h_8, h_9\}.$
$SH_F(X) = C_1 \cup C_2 \cup C_4 \cup C_6 = \{h_1, h_2, h_4, h_7, h_8, h_9\},$
$M = C_1 \cup C_2 \cup C_4 \cup C_6 = \{h_1, h_2, h_4, h_7, h_8, h_9\},$
$X \nsubseteq M.$
Then we have to find out N.
$N = F(e_1)' \cap F(e_2)' \cap F(e_3)' \cap F(e_5)' = \{h_3, h_4, h_5, h_6, h_9, h_{10}\} \cap \{h_1, h_2, h_4, h_5, h_7, h_8, h_{10}\}$
$\cap \{h_1, h_2, h_3, h_5, h_6, h_8, h_9, h_{10}\} \cap \{h_1, h_3, h_4, h_5, h_6, h_8, h_9, h_{10}\}$
$= \{h_5, h_{10}\}$
$NH_F(X) = M \cup N = \{h_1, h_2, h_4, h_5, h_7, h_8, h_9, h_{10}\}.$
Here $X \nsubseteq FH_F(X), X \nsubseteq SH_F(X), But X \subset NH_F(X)$. So New type covering is better than that of above two.

6. CONCLUSION

The New type Soft Rough with covering based is better than that of First and second type of Soft Rough Set with covering Based. We are considering covering based without full Soft set but getting the new concepts. Combination of theories has not only advanced research but also helped in tackling the issues of uncertainties in real life situations. To conclude along with introducing new extended models, this article also presents the different problem in covering based Soft Rough set inviting further investigations to find the properties related to these three types of Soft Rough Set with covering based.

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REFERENCES

[1] Feng Feng, Xiaoyan Liu, Violeta Leoreanu-Fotea, and Young Bae Jun. Soft sets and soft rough sets. *Information Sciences*, 181(6):1125–1137, 2011.

[2] Debadutta Mohanty. Comparative analysis of hybrid soft set methods. In *Global Trends in Intelligent Computing Research and Development*, pages 360–383. IGI Global, 2014.

[3] Dmitriy Molodtsov. Soft set theory first results. *Computers & Mathematics with Applications*, 37(4-5):19–31, 1999.

[4] Zdzisław Pawlak. Rough sets. *International journal of computer & information sciences*, 11(5):341–356, 1982.

[5] Zdzisław Pawlak. *Rough sets: Theoretical aspects of reasoning about data*, volume 9. Springer Science & Business Media, 2012.

[6] Roy AR P.K.Majhi and Biswal R. Soft set theory. *Computer and Mathematics with Applications*, 45(4-5):555–562, 2003.