Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 16, Number 6 (2020), pp. 759-770

http://www.ripublication.com/gjpam.htm

An Inventory Management System for Deteriorating and Ameliorating items with a Linear Trended in Demand and Partial Backlogging

Dr. Biswaranjan Mandal

Associate Professor of Mathematics, Acharya Jagadish Chandra Bose College, Kolkata, West Bengal, India.

Abstract

Most of the inventory models dealt with deteriorating items. On the contrary, few papers considered inventory system under amelioration environment. This paper discussed the development of an inventory model in presence of both ameliorating and deteriorating items. Here the demand rate is considered as a linearly time-varying over a fixed time horizon and shortages which is partially backlogged. Finally the model is illustrated with the help of a numerical example, some particular cases are derived and a comparative study of the optimal solutions towards different nature of goods is analysed graphically.

Keywords: Inventory, deteriorating, ameliorating, linear trended demand, shortages, and partial backlogging.

Subject classification: AMS Classification No. 90B05

INTRODUCTION:

Several researchers have addressed the importance of the deterioration phenomenon in their field of applications, as a result, many inventory models with deteriorating items have been developed. But due to lack of considering the influence of demand, the ameliorating items for the amount of inventory is increasing gradually.

Amelioration is a natural phenomenon observing in much life stock models. A few researchers have focused on ameliorating system. The vast majority of the traditional stock models depend on the rule that the estimation of stock stays consistent after some time. This is an uncommon sort of stock model where scientists have concentrated on for both ameliorating and deteriorating things, where items ameliorate when stay at breeding yard and deteriorates when in the distribution systems. Hwang [1] developed an inventory model for ameliorating items only. Again Hwang [2] added to a stock model for both ameliorating and deteriorating things independently. Mallick et al. [3] has considered a creation inventory model for both ameliorating and deteriorating items. Professionals did not give much attention for fast growing animals like broiler, ducks, pigs etc. in the poultry farm, highbred fishes in berry (pond) which are known as ameliorating items. When these items are in storage, the stock increases (in weight) due to growth of the items and also decrease due to death, various diseases or some other factors. At the point when these things are away, the stock increments (in weight) because of development of the things. Furthermore the stock diminishes because of death, different illnesses or due to some different components. Many researchers like Moon et al[4], Law et al [5], L-Q ji[6], Valliathal et al[7], Chen [8], Nodoust [9] are few noteworthy.

The assumption of a constant demand rate may not be always appropriate for many inventory items. For example milk, vegetables etc., the age of inventory has a negative impact on demand due to loss of consumer confidence on the quality of such products. So many researchers like Donaldson [10], Silver [11], Goswami et al [12] etc. are names to only a few to develop linear trended time varying demand models. In this paper we consider the inventory replenishment problem with linearly time-varying demand over a fixed time horizon.

In fact an inventory policy that allows for shortage is always less expensive to operate than a policy without shortage. Many researchers like Goyal et al [13], Benkheronf [14] assumed that shortages are completely backlogged. In practice, some customers would like to wait for backlogging during that shortage period, but other would not. Consequently, the opportunity cost due to lost sales should be considered in the modelling. Wee[15], Giri et al [16], Biswaranjan Mandal [17] etc assumed that the backlogging rate was a fixed fraction of demand rate during this period. However, in some inventory system, for some fashionable commodities , the length of waiting time for next replenishment become main factor for determining whether the backlogging will be accepted or not. The longer the waiting time is, the smaller the backlogging rate would be. Therefore backlogging rate is variable and is dependent on the waiting time for the next replenishment.

For these sort of situations, efforts have been made to develop a realistic inventory model in presence of both ameliorating and deteriorating items. The demand rate is considered as a linearly time-varying demand over a fixed time horizon and shortages which is partially backlogged. Finally the model is illustrated with the help of a numerical example, some particular cases are derived and a comparative study of the optimal solutions towards different nature of inventory items is analysed graphically.

Notations and Assumptions:

The present inventory model is developed under the following notations and assumptions:

Notations:

(i) I(t) : On hand inventory at time t.

(ii) R(t) : Demand rate.

(iii) Q : On-hand inventory.

(iv) θ : The constant deterioration rate where $0 \leq \theta < 1$

(v) A : The constant ameliorating rate.

(vi) T : The fixed length of each production cycle.

(vii) A_0 : The ordering cost per order during the cycle period.

(viii) p_c : The purchasing cost per unit item.

(ix) h_c : The holding cost per unit item.

(x) d_c : The deterioration cost per unit item.

(xi) a_c : The cost of amelioration per unit item.

(xii) c_s : The shortage cost per unit item.

(xiii) o_c : The opportunity cost per unit item.

(xiv) TC : Average total cost per unit time.

Assumptions:

(i). Lead time is zero.

(ii). Replenishment rate is infinite but size is finite.

(iii). The time horizon is finite.

(iv). There is no repair of deteriorated items occurring during the cycle.

(v). Amelioration and deterioration occur when the item is effectively in stock.

(vi). The demand rate is time dependent linear increasing function

R(t) = a + bt, a, b $\geq$ 0 where a is the initial demand rate, b is the initial rate of change of demand.

(vii). Shortages are allowed and partially backlogged.

Here the proportion of customers who would like to accept backlogging at time t is decreasing with the waiting time (T-t) waiting for the next replenishment. To take care of this situation, we have defined the backlogging rate to $\frac{1}{1+\alpha(T-t)}$, when inventory is negative. The backlogging parameter α is a positive constant.

Model Formulation and Solutions:

The aim of this model is to optimize the total cost incurred and to determine the optimal ordering level. During the period [0, t_1], the stock will be diminished due to the effect of amelioration, deterioration and demand and ultimately falls to zero at t = t_1. The shortages occur during time period [t_1, T] which are partially backlogged. The differential equations which the on-hand inventory I(t) governed by the following :

$$\frac{dI(t)}{dt} + (\theta - A)I(t) = -(a+bt), 0 \le t \le t_1 \quad (1)$$

And $$\frac{dI(t)}{dt} = -\frac{a+bt}{1+\alpha(T-t)}, t_1 \le t \le T \quad (2)$$

The initial condition is $I(0) = Q$ and $I(t_1) = 0$ (3)

The solutions of the equations (1) and (2) using (3) are given by the following

$$\text{I(t)} = Qe^{(A-\theta)t} + \frac{a}{A-\theta}(1-e^{(A-\theta)t}) + \frac{b}{(A-\theta)^2}(1-e^{(A-\theta)t}) + \frac{bt}{A-\theta}, \quad 0 \le t \le t_1 \quad (4)$$

And $$\text{I(t)} = \{\frac{a}{\alpha} + \frac{b}{\alpha}(T + \frac{1}{\alpha})\}\ln\frac{1+\alpha(T-t)}{1+\alpha(T-t_1)} + \frac{b}{\alpha}(t-t_1), \ t_1 \le t \le T \quad (5)$$

Since $I(t_1) = 0$, we get the following expression of on-hand inventory from the equation (4)

$$\text{Q} = \frac{a}{A-\theta}(1-e^{(\theta-A)t_1}) + \frac{b}{(A-\theta)^2}(1-e^{(\theta-A)t_1}) - \frac{bt_1}{A-\theta}e^{(\theta-A)t_1} \quad (6)$$

The total inventory holding during the time interval [0, t_1] is given by

$$I_T = \int_0^{t_1} I(t)dt$$

$$= \int_0^{t_1}\{Qe^{(A-\theta)t} + \frac{a}{A-\theta}(1-e^{(A-\theta)t}) + \frac{b}{(A-\theta)^2}(1-e^{(A-\theta)t}) + \frac{bt}{A-\theta}\}dt$$

Putting the value of Q from (6) and integrating we get

$$I_T = -\frac{a}{(A-\theta)^2}(1-e^{(\theta-A)t_1}) - \frac{b}{(A-\theta)^3}(1-e^{(\theta-A)t_1}) + \frac{bt_1^{\,2}}{2(A-\theta)} + \frac{at_1}{A-\theta} + \frac{bt_1}{(A-\theta)^2}e^{(\theta-A)t_1} \quad (7)$$

The total number of deteriorated units during the inventory cycle is given by

$$D_T = \theta\int_0^{t_1} I(t)dt$$

$$= \theta\{-\frac{a}{(A-\theta)^2}(1-e^{(\theta-A)t_1}) - \frac{b}{(A-\theta)^3}(1-e^{(\theta-A)t_1}) + \frac{bt_1^2}{2(A-\theta)} + \frac{at_1}{A-\theta} + \frac{bt_1}{(A-\theta)^2}e^{(\theta-A)t_1}\} \quad (8)$$

The total number of ameliorating units during the inventory cycle is given by

$$A_T = A\int_0^{t_1} I(t)dt$$

$$= A\{-\frac{a}{(A-\theta)^2}(1-e^{(\theta-A)t_1}) - \frac{b}{(A-\theta)^3}(1-e^{(\theta-A)t_1}) + \frac{bt_1^2}{2(A-\theta)} + \frac{at_1}{A-\theta} + \frac{bt_1}{(A-\theta)^2}e^{(\theta-A)t_1}\} \quad (9)$$

The total number of shortages during the period [t_1, T] is given by

$$S_T = \int_{t_1}^{T} -I(t)dt$$

$$= \{\frac{a}{\alpha} + \frac{b}{\alpha^2} + \frac{bT}{2\alpha} + \frac{bt_1}{2\alpha})\}(T-t_1) - \frac{1}{\alpha}(\frac{a}{\alpha} + \frac{b}{\alpha^2} + \frac{bT}{\alpha})\ln\{1+\alpha(T-t_1)\} \quad (10)$$

The amount of lost sales during the period [t_1,T] is given by

$$L_T = \int_{t_1}^{T} R(t)\{1-\frac{1}{1+\alpha(T-t)}\}dt$$

$$= \int_{t_1}^{T} (a+bt)\{1-\frac{1}{1+\alpha(T-t)}\}dt$$

$$= (a+\frac{b}{\alpha})(T-t_1) + \frac{b}{2}(T^2-t_1^2) - (\frac{a}{\alpha} + \frac{b}{\alpha^2} + \frac{bT}{\alpha})\ln\{1+\alpha(T-t_1)\} \quad (11)$$

Cost Components:

The total cost over the period [0, T] consists of the following cost components:

(1). Ordering cost **(OC)** over the period [0,T] = A_0 (fixed)

(2). Purchasing cost **(PC)** over the period [0,T] = p_c I(0) = p_c Q

$$= p_c\{\frac{a}{A-\theta}(1-e^{(\theta-A)t_1}) + \frac{b}{(A-\theta)^2}(1-e^{(\theta-A)t_1}) - \frac{bt_1}{A-\theta}e^{(\theta-A)t_1}\}$$

(3). Holding cost for carrying inventory **(HC)** over the period [0,T] $= h_c I_T$

$$= h_c\{-\frac{a}{(A-\theta)^2}(1-e^{(\theta-A)t_1}) - \frac{b}{(A-\theta)^3}(1-e^{(\theta-A)t_1}) + \frac{bt_1^{\,2}}{2(A-\theta)} + \frac{at_1}{A-\theta} + \frac{bt_1}{(A-\theta)^2}e^{(\theta-A)t_1}\}$$

(4). Cost due to deterioration **(CD)** over the period [0,T] = $d_c D_T$

$$= d_c\theta\{-\frac{a}{(A-\theta)^2}(1-e^{(\theta-A)t_1}) - \frac{b}{(A-\theta)^3}(1-e^{(\theta-A)t_1}) + \frac{bt_1^{\,2}}{2(A-\theta)} + \frac{at_1}{A-\theta} + \frac{bt_1}{(A-\theta)^2}e^{(\theta-A)t_1}\}$$

(5).The amelioration cost **(AMC)** over the period [0,T] $= a_c A_T$

$$= a_c A\{-\frac{a}{(A-\theta)^2}(1-e^{(\theta-A)t_1}) - \frac{b}{(A-\theta)^3}(1-e^{(\theta-A)t_1}) + \frac{bt_1^{\,2}}{2(A-\theta)} + \frac{at_1}{A-\theta} + \frac{bt_1}{(A-\theta)^2}e^{(\theta-A)t_1}\}$$

(6). Cost due to shortage **(CS)** over the period [0,T] $= c_s S_T$

$$= c_s[\{\frac{a}{\alpha} + \frac{b}{\alpha^2} + \frac{bT}{2\alpha} + \frac{bt_1}{2\alpha})\}(T-t_1) - \frac{1}{\alpha}(\frac{a}{\alpha} + \frac{b}{\alpha^2} + \frac{bT}{\alpha})\ln\{1+\alpha(T-t_1)\}]$$

(7). Opportunity Cost due to lost sales **(OPC)** over the period [0,T] $= o_c\ L_T$

$$= o_c[(a+\frac{b}{\alpha})(T-t_1) + \frac{b}{2}(T^2 - t_1^{\,2}) - (\frac{a}{\alpha} + \frac{b}{\alpha^2} + \frac{bT}{\alpha})\ln\{1+\alpha(T-t_1)\}]$$

The average total cost per unit time of the system during the cycle [0,T] will be

TC(t_1) = $\frac{1}{T}$ **[OC +PC+HC+CD+AMC+CS+OPC]**

$$= \frac{1}{T}[\ A_0 + p_c\{\frac{a}{A-\theta}(1-e^{(\theta-A)t_1}) + \frac{b}{(A-\theta)^2}(1-e^{(\theta-A)t_1}) - \frac{bt_1}{A-\theta}e^{(\theta-A)t_1}\} +$$

$$(h_c + \theta d_c + Aa_c)\{-\frac{a}{(A-\theta)^2}(1-e^{(\theta-A)t_1}) - \frac{b}{(A-\theta)^3}(1-e^{(\theta-A)t_1}) + \frac{bt_1^{\,2}}{2(A-\theta)}$$

$$+ \frac{at_1}{A-\theta} + \frac{bt_1}{(A-\theta)^2}e^{(\theta-A)t_1}\}$$

$$+ c_s[\{\frac{a}{\alpha} + \frac{b}{\alpha^2} + \frac{bT}{2\alpha} + \frac{bt_1}{2\alpha})\}(T-t_1) - \frac{1}{\alpha}(\frac{a}{\alpha} + \frac{b}{\alpha^2} + \frac{bT}{\alpha})\ln\{1+\alpha(T-t_1)\}]$$

$$+ o_c[(a+\frac{b}{\alpha})(T-t_1) + \frac{b}{2}(T^2 - t_1^{\,2}) - (\frac{a}{\alpha} + \frac{b}{\alpha^2} + \frac{bT}{\alpha})\ln\{1+\alpha(T-t_1)\}]] \qquad (12)$$

For minimum, the necessary condition is $\frac{dTC(t_1)}{dt_1}=0$

This gives $p_c(a+bt_1)e^{(\theta-A)t_1}+\frac{h_c+\theta d_c+Aa_c}{A-\theta}(a+bt_1)(1-e^{(\theta-A)t_1})$

$$-(o_c+\frac{c_s}{\alpha})(a+\frac{b}{\alpha}+bt_1)+(o_c+\frac{c_s}{\alpha})(a+\frac{b}{\alpha}+bT)\frac{1}{1+\alpha(T-t_1)}=0 \qquad (13)$$

For minimum the sufficient condition $\frac{d^2TC(t_1)}{dt_1^{\,2}}$ >0 would be satisfied.

Let $t_1=t_1^*$ be the optimum value of t_1.

The optimal values Q^* of Q and TC^* of TC are obtained by putting the value $t_1=t_1^*$ from the expressions (6) and (12).

Some Particular Cases:

(a). Absence of deterioration :

If the deterioration of items is switched off i.e. $\theta=0$, then the expressions (6) and (12) of on-hand inventory(Q) and average total cost per unit time (TC(t_1)) during the period [0,T] become

$$Q=\frac{1}{A}(a+\frac{b}{A})(1-e^{-At_1})-\frac{bt_1}{A}e^{-At_1} \qquad (14)$$

and TC (t_1) = $\frac{1}{T}$**[** A_0 + $p_c\ \{\frac{1}{A}(a+\frac{b}{A})(1-e^{-At_1})-\frac{bt_1}{A}e^{-At_1}\}$

$$+\ (h_c+Aa_c)\{-\frac{1}{A^2}(a+\frac{b}{A})(1-e^{-At_1})+\frac{bt_1^2}{2A}+\frac{at_1}{A}+\frac{bt_1}{A^2}e^{-At_1}\}$$

$$+\ c_s[\{\frac{a}{\alpha}+\frac{b}{\alpha^2}+\frac{bT}{2\alpha}+\frac{bt_1}{2\alpha})\}(T-t_1)-\frac{1}{\alpha}(\frac{a}{\alpha}+\frac{b}{\alpha^2}+\frac{bT}{\alpha})\ln\{1+\alpha(T-t_1)\}]$$

$$+\ o_c[(a+\frac{b}{\alpha})(T-t_1)+\frac{b}{2}(T^2-t_1^2)-(\frac{a}{\alpha}+\frac{b}{\alpha^2}+\frac{bT}{\alpha})\ln\{1+\alpha(T-t_1)\}]\,\mathbf{]} \qquad (15)$$

The equation (13) becomes

$$p_c(a+bt_1)e^{-At_1}+\frac{h_c+Aa_c}{A}(a+bt_1)(1-e^{-At_1})+$$

$$-(o_c+\frac{c_s}{\alpha})(a+\frac{b}{\alpha}+bt_1)+(o_c+\frac{c_s}{\alpha})(a+\frac{b}{\alpha}+bT)\frac{1}{1+\alpha(T-t_1)}=0 \qquad (16)$$

This gives the optimum value of t_1.

(b). Absence of amelioration:

If the amelioration of items is switched off i.e. $A = 0$, then the expressions (6) and (12) of on-hand inventory(Q) and average total cost per unit time (TC(t_1)) during the period [0,T] become

$$Q = \frac{1}{\theta}(-a+\frac{b}{\theta})(1-e^{\theta t_1}) + \frac{bt_1}{\theta}e^{-At_1} \tag{17}$$

$$\text{and TC}(t_1) = \frac{1}{T}\Big[\ A_0 + p_c\ \{\frac{1}{\theta}(-a+\frac{b}{\theta})(1-e^{\theta t_1}) + \frac{bt_1}{\theta}e^{\theta t_1}\}$$

$$+\ (h_c+\theta d_c)\{\frac{1}{\theta^2}(-a+\frac{b}{\theta})(1-e^{\theta t_1}) - \frac{bt_1^{\,2}}{2\theta} - \frac{at_1}{\theta} + \frac{bt_1}{\theta^2}e^{\theta t_1}\}$$

$$+\ c_s[\{\frac{a}{\alpha}+\frac{b}{\alpha^2}+\frac{bT}{2\alpha}+\frac{bt_1}{2\alpha}\}(T-t_1) - \frac{1}{\alpha}(\frac{a}{\alpha}+\frac{b}{\alpha^2}+\frac{bT}{\alpha})\ln\{1+\alpha(T-t_1)\}]$$

$$+\ o_c[(a+\frac{b}{\alpha})(T-t_1) + \frac{b}{2}(T^2-t_1^{\,2}) - (\frac{a}{\alpha}+\frac{b}{\alpha^2}+\frac{bT}{\alpha})\ln\{1+\alpha(T-t_1)\}]\Big] \tag{18}$$

The equation (13) becomes

$$p_c(a+bt_1)e^{\theta t_1} - \frac{h_c+\theta d_c}{\theta}(a+bt_1)(1-e^{\theta t_1})$$

$$-(o_c+\frac{c_s}{\alpha})(a+\frac{b}{\alpha}+bt_1) + (o_c+\frac{c_s}{\alpha})(a+\frac{b}{\alpha}+bT)\frac{1}{1+\alpha(T-t_1)} = 0 \tag{19}$$

This gives the optimum value of t_1.

(c). If the demand rate is constant then b = 0

The expressions (6) and (12) of on-hand inventory (Q) and average total cost per unit time (TC(t_1)) during the period [0,T] become

$$Q = \frac{a}{A-\theta}(1-e^{(\theta-A)t_1}) \tag{20}$$

$$\text{TC}(t_1) = \frac{1}{T}\Big[\ A_0\ +\ p_c\ \{\frac{a}{A-\theta}(1-e^{(\theta-A)t_1})\} +$$

$$(h_c+\theta d_c+Aa_c)\{-\frac{a}{(A-\theta)^2}(1-e^{(\theta-A)t_1}) + \frac{at_1}{A-\theta}\}\ +\ c_s[\frac{a}{\alpha}(T-t_1) - \frac{a}{\alpha^2}\ln\{1+\alpha(T-t_1)\}]$$

$$+\ o_c[a(T-t_1) - \frac{a}{\alpha}\ln\{1+\alpha(T-t_1)\}]\Big] \tag{21}$$

The equation (13) becomes

$$p_c a e^{(\theta-A)t_1} + \frac{h_c+\theta d_c+Aa_c}{A-\theta}a(1-e^{(\theta-A)t_1}) - a\alpha(o_c+\frac{c_s}{\alpha})\frac{T-t_1}{1+\alpha(T-t_1)} = 0 \tag{22}$$

This gives the optimum value of t_1.

Numerical Example:

To illustrate the developed inventory model, let the values of parameters be as follows:

A_0 = \$500 per order; a = 20; b = 3; θ = 0.8; A= 0.01; α = 10; p_c = \$15 per unit, h_c = \$4 per unit; d_c = \$9 per unit; a_c = \$6 per unit; c_s = \$10 per unit; o_c = \$12 per unit; T = 1 year

Solving the equation (13) with the help of computer using the above values of parameters, we find the following optimum outputs

t_1^* = 0.04 year; Q^* = 0.82 units and TC^* = Rs. 713.78

It is checked that this solution satisfies the sufficient condition for optimality.

Keeping with the above parameters, the following solutions are also made.

Particular cases	Q^* **(units)**	TC^* **(\$)**
Absence of deterioration	0.80	713.40
Absence of amelioration	0.81	713.71
The constant demand rate	0.33	698.77

Comparative study of the optimal solutions towards different nature of inventory items:

The comparative study is also furnished graphically to illustrate the particular cases of the inventory model by varying nature of inventory goods.

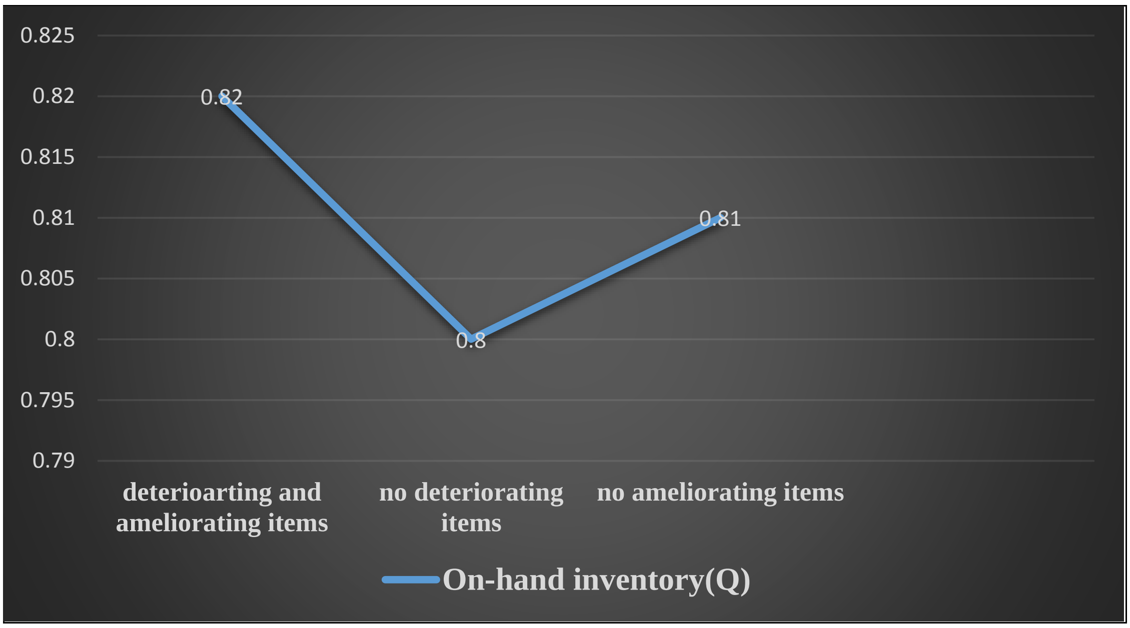

Fig. 1: Inventory items vs Optimal On-hand Inventory

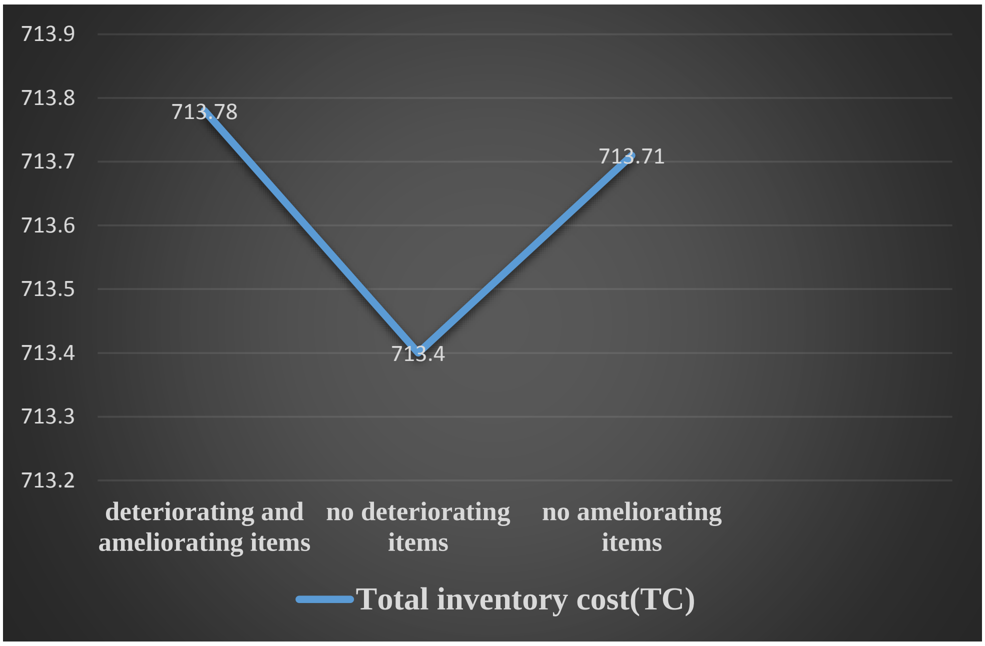

Fig. 2: Inventory items vs Optimal Inventory Total Cost

CONCLUDING REMARKS

In this model, an inventory management policy has been framed model in presence of both ameliorating and deteriorating items under time-varying linear trended in demand. Shortages are allowed which are partially backlogged. The model is developed analytically as well as computationally with graphical representation.

Efforts are given on comparative study graphically between optimal inventory total cost and optimal on-hand inventory considering deteriorating and ameliorating, no deteriorating and no ameliorating inventory items. Analyzing Fig. 1 and Fig. 2, it is observed that optimality of inventory total cost and on-hand inventory level are moderately changing for all kinds of inventory goods. Moreover it is also observed that the optimal inventory total cost is minimum for the inventory model where the deteriorating items is switched off.

REFERENCES:

(1). Hwang, H.S. **:** A study of an inventory model for items with Weibull ameliorating. Computers and Industrial Engineering, 33, 1997, pp.701-704.

(2). Hwang, H.S**.:** A stochastic set-covering location model for both ameliorating and deteriorating items, Computers and Industrial Engineering, 46, 2004, pp.313-319..

(3). Mallick M., Mishra S., Mishra U. K. and Paikray S.K. : Optimal inventory control for ameliorating, deteriorating items under time varying demand condition, Journal of Social Science Research, 3(1), 2018, pp.166-173.

(4).Moon I., Giri B.C. and B. Ko : Economic order quantity models for ameliorating/deteriorating items under inflation and time discounting, European Journal of Operational Research, vol. 162,no. 3, 2005, pp. 773–785.

(5). Law S.-T. and Wee H.M. : An integrated production-inventory model for ameliorating and deteriorating items taking account of time discounting, Mathematical and Computer Modelling, vol. 43, no. 5-6, 2006, pp. 673–685.

(6) L.-Q. Ji : The influences of inflation and time-value of money on an EOQ model for both ameliorating and deteriorating items with partial backlogging, in Proceedings of the 2008 International Conference on Wireless Communications, Networking and Mobile Computing, WiCOM 2008, Dalian, China.

(7). Valliathal M. and Uthayakumar R.: The production-inventory problem for ameliorating/deteriorating items with non-linear shortage cost under inflation and time discounting, Applied Mathematical Sciences, vol. 4, no. 5-8, 2010, pp. 289–304.

(8). Chen J. : An inventory model for ameliorating and deteriorating fresh agricultural items with ripeness and price dependent demand, in Proceedings of the 2011 International Conference on Information Technology, Computer Engineering and Management Sciences, ICM 2011, Jiangsu, China, pp. 228–231.

(9). Nodoust S., Mirzazadeh A. and Weber G. : An evidential reasoning approach for production modeling with deteriorating and ameliorating items, Operational Research, 2017.

(10).Donaldson W.A. : Inventory replenishment policy for a linear trend in demand- an analytical solution. Operational Research Quarterly, 28, 1977, pp. 663-670.

(11). Silver, E.A. : A simple inventory replenishment decision rule for a linear trend in demand. Journal of Operational Research Society. 30, 1979, pp.71-75.

(12). Goswami, A. and Chaudhri, K.S. : An EOQ model for deteriorating items with a linear trend in demand. Journal of Operational Research Society. 42(12), 1991, pp.1105-1110.

(13). Goyal S.K., Morin D. and Nebebe F. : The finite horizon trended inventory replenishment problem with shortages, J Opl. Res. Soc., 43, 1992, pp. 1173-1178.

(14). Benkherouf L. : On an inventory model with deteriorating items and decreasing time- varying demand and shortages, Eur. J. Opl. Res., 86, 1995, pp.293-299.

(15). Wee H.M. : A deterministic lot-size inventory model for deteriorating items with shortages and declining market, Comp. Oper. Res., 22, 1995, pp.345-356.

(16). Giri B.C., Jalan A.K. and Chaudhuri K.S. : An economic production lot size model with increasing demand , shortages and partial backlogging, Int. Trans. Oper. Res., 12(2), 2005, pp.235-245.

(17). Biswaranjan Mandal: An inventory model for random deteriorating items with a linear trended in demand and partial backlogging", Res. J. Business Mgmt. and Accounting, 2(3), 2012, 2010, pp.48-52.