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Solution of Some Non Linear Partial Differential Equation by New Integral Transform Combined with ADM

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Abstract

Integral Transform method is very useful Technique for finding solutions of Differential Equations, Partial Differential and Integral Equations. Sadik Transform is new Integral Transform. Laplace Transform, Sumudu Transform, Elzaki Transform, Tarig Transform, Kamal Transform, Laplace-Carson Transform, Aboodh Transform are special cases of Sadik Integral Transform. In this Paper we derive formula for two dimensional Non-Homogeneous non-linear Partial Differential Equation by using Sadik transform combine with Adomian Decomposition Method. Different applications of this method also presented in this paper.

Keywords: The Sadik Transform, Adomian Polynomial and two dimensional Non Homogeneous non linear Partial Differential Equation.

1. INTRODUCTION

There are a lot of methods to solve partial differential equations, to solve linear partial differential equations the most powerful method is an integral transformation method. But for solution of non linear differential and partial differential equations we have

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method of integral transform combined with Adomian polynomial method. Some Non linear partial differential has been solved by different integral transform like Laplace Transform, Sumudu transform, Natural transform, Elzaki transform, Aboodh transform, Kashuri and Fundo transform, ZZ transform Combined with Adomian Method [7].
Laplace transform is the most effective tool to solve some kinds of ordinary and partial differential equations. Actually an electric engineer Oliver Heaviside made Laplace transform popular by developing its operational calculus. After Laplace transform, in 1993 again an electrical engineer Watugula in [1] proposed a new integral transform named the Sumudu transform and used it for solving problems in control engineering, it is similar to the Laplace transform having the preservation property of unit and change of scale. After that, T. Elzaki [6] introduced a new integral transform named Elzaki transform and applied it for solving partial differential equations, Shaikh Sadikali has been applied Elzaki transform for solving integral equations of convolution type see in [5]. Likewise many integral transforms have been proposed which are similar to the Laplace transform, and each new transform claimed its own superiority over the Laplace transform. Shaikh Sadikali presented solution of some linear partial differential by Sadik Transform [8]. In this paper we considered a new integral transform named the Sadik transform [3], [4]. It is similar to the Laplace transform but the Laplace transform, the Sumudu transform, Elzaki transform and all integral transforms with kernel of an exponential type are particular cases of the Sadik transform. Due to the very general and unified nature of the Sadik transform, we can transport a problem of partial differential equations into the known transformation technique which is available in the literature through the Sadik transform.

2. SADIK TRANSFORM

If,
1 $f(t)$ is piecewise continuous on th intervel $0 \leq t \leq A$,for any $A \geq 0$
2 $f(t)| \leq K$, when $t \geq M$, for any real constant A. and some positive constant K and M.
Then Sadik Transform of $f(t)$ is defined by

$$F(\upsilon^{\alpha}, \beta) = S[f(t)] = \frac{1}{\upsilon^{\beta}} \int_0^{\infty} e^{-\upsilon^{\alpha} t} f(t)\, dt, for Re(\upsilon^{\alpha}) > w^{\alpha}$$

Where,
υ is complex variable,
α is any non zero real numbers, and
β is any real number.

Method of applying Sadik transform to partial derivative

If $G(x, \upsilon^{\alpha}, \beta)$ is Sadik transform of $\varphi(x,t)$ and $\varphi_t(x,t)$ is first partial derivative of $\varphi(x,t)$ with respect to variable t then

$S[\varphi_t(x,t)] = \upsilon^{\alpha} G(x, \upsilon^{\alpha}, \beta) - \upsilon^{-\beta}\varphi(x,0)$
also
$S[\varphi_{tt}(x,t)] = \upsilon^{2\alpha} G(x, \upsilon^{\alpha}, \beta) - \upsilon^{\alpha-\beta}\varphi(x,0) - \upsilon^{-\beta}\varphi_t(x,0)$

3. ADOMIAN DECOMPOSITION METHOD

Consider the nonlinear equation in the form $Lu + Ru + Nu = g(t)$
Where L is easily invertible differential operator, R is a remainder linear differential operator, N is an analytic nonlinear terms and g is a known function.
Taking the inverse linear operator $L^{-1}(.)$ to both sides of above Equation we get.

$$u = c(t) - L^{-1}(Ru) - L^{-1}(Nu) + L^{-1}(g)$$

Where $c(t)$ represents the terms arising from using the given conditions. The Adomian decomposition method introduces the solution by decomposing $u(t)$ to an infinite series $u(t) = \sum_{n=0}^{\infty} u_n(t)$ and the nonlinear term Nu by the infinite series $Nu = \sum_{n=0}^{\infty} A_n$ where A_n are the Adomian polynomials which are generated for each nonlinearity and can be found by the formula

$$A_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n} N\left(\sum_{i=0}^{\infty}\lambda^i u_i\right)\right]_{\lambda=0}, n = 0, 1, 2, ...$$

4. MAIN RESULTS

General form of two dimensional non homogeneous, non linear, partial Differential equation is given as
$Lu(x,t) + Ru(x,t) + Nu(x,t) = h(x,t) ... (1)$
With initial condition $u(x,0) = f(x), u_t(x,0) = g(x), x\epsilon[a,b]$
Where $L = \frac{\partial^2}{\partial t^2}$ is partial derivative of order two w.r.t. t (linear differential operator) R is the remaining linear operator of order less than L and $Nu(x,t)$ represents the general non linear differential operator with $h(x,t)$ as a non homogeneous term.
Let S be the Sadik transform, applying Sadik transform on (1)

$$S\left[Lu\left(x,t\right)\right]+S\left[Ru\left(x,t\right)\right]+S\left[Nu\left(x,t\right)\right]=S\left[h\left(x,t\right)\right]...(2)$$

$$\upsilon^{2\alpha}S\left[u\left(x,t\right)\right]-\upsilon^{\alpha-\beta}u\left(x,0\right)-\upsilon^{-\beta}u_t\left(x,0\right)+S\left[Ru\left(x,t\right)\right] + S\left[Nu\left(x,t\right)\right]=S\left[h\left(x,t\right)\right].......(3)$$

$$S\left[u\left(x,t\right)\right]=\frac{\upsilon^{\alpha-\beta}}{\upsilon^{2\alpha}}u\left(x,0\right)+\frac{\upsilon^{-\beta}}{\upsilon^{2\alpha}}u_t\left(x,0\right)-\frac{1}{\upsilon^{2\alpha}}S\left[Ru\left(x,t\right)\right] - \frac{1}{\upsilon^{2\alpha}}S\left[Nu\left(x,t\right)\right]+\frac{1}{\upsilon^{2\alpha}}S\left[h\left(x,t\right)\right]........(4)$$

$$S^{-1}\left[\frac{\upsilon^{\alpha-\beta}}{\upsilon^{2\alpha}}\right]=S^{-1}\left[\frac{1}{\upsilon^{\alpha+\beta}}\right]=1 \text{ and } S^{-1}\left[\frac{\upsilon^{-\beta}}{\upsilon^{2\alpha}}\right] =S^{-1}\left[\frac{1}{\upsilon^{\alpha+(\alpha+\beta)}}\right]=t\ ....(5)$$

If we apply inverse transform we get the solution but for non – linear partial diffuential equation we apply Adomian polynomial method for which we represent solution $u\left(x,t\right)$ by infinit series

$u\left(x,t\right)=\sum_{n=0}^{\infty}u_i\left(x,t\right)$ and $Nu\left(x,t\right)=\sum_{n=0}^{\infty}A_i ...(6)$

Where A_i are Adomian polynomial given as
$A_n=\frac{1}{n!}\left[\frac{d_n}{d\lambda^n}N\left(\sum_{i=0}^{\infty}\lambda^i u_i\right)\right]_\lambda=0, n=0,1,2,.......(7)$

Substituting (6) and initial condition in (4)

$$S\left[\sum_{i=0}^{\infty}u_i\left(x,t\right)\right]=\frac{1}{\upsilon^{\alpha+\beta}}f\left(x\right)+\frac{1}{\upsilon^{\alpha+(\alpha+\beta)}}g\left(x\right) +\frac{1}{\upsilon^{2\alpha}}S\left[h\left(x,t\right)\right]-\frac{1}{\upsilon^{2\alpha}}S\left[R\sum_{i=0}^{\infty}u_i\left(x,t\right)\right]-\frac{1}{\upsilon^{2\alpha}}S\left[N\sum_{i=0}^{\infty}u_i\left(x,t\right)\right]$$

Taking inverse Sadik Transform we get

$$\sum_{i=0}^{\infty}u_i\left(x,t\right)=f\left(x\right)+t.g\left(x\right)+S^{-1}\left\{\frac{1}{\upsilon^{2\alpha}}S\left[h\left(x,t\right)\right]\right\} -S^{-1}\left\{\frac{1}{\upsilon^{2\alpha}}S\left[R\sum_{i=0}^{\infty}u_i\left(x,t\right)+\sum_{i=0}^{\infty}A_i\right]\right\}....(8)$$

comparing both sides of (8) we get

$$u_0(x,t) = f(x) + t.g(x) + S^{-1}\left\{\frac{1}{\upsilon^{2\alpha}}S[h(x,t)]\right\}$$

Therefor (8) is given by

$$\sum_{i=1}^{\infty} u_i(x,t) = -S^{-1}\left\{\frac{1}{\upsilon^{2\alpha}}S\left[R\sum_{i=0}^{\infty} u_i(x,t) + \sum_{i=0}^{\infty} A_i\right]\right\} \text{.... } (9)$$

comparing $(n+1)^{th}$ term we get

$$u_{n+1}(x,t) = -S^{-1}\left\{\frac{-1}{\upsilon^{\alpha}}S[u_{xx}]_n + \frac{1}{\upsilon^{\alpha}}S[u.u_x]_n\right\} \text{.... } (10)$$

Therefor General solution is given by

$$u_0(x,t) = f(x) + t.g(x) + S^{-1}\left\{\frac{1}{\upsilon^{2\alpha}}S[h(x,t)]\right\}, n = 0$$

$$u_{n+1}(x,t) = -S^{-1}\left\{\frac{1}{\upsilon^{2\alpha}}S[Ru_n(x,t) + A_n]\right\}, n > 0$$

5. APPLICATION OF THIS METHOD

By applying Sadik Transform based on adomian decomposition we will solve non linear partial differential equations with initial (value problem) condition like Burgers equation, non linear wave equation, nonlinear heat equation etc.

Example

Consider non linear P.D.E. $u_t + u.u_x = u_{xx}$........ (1)

with initial condition $u(x,0) = 2x, t > 0$........ (2)

Solution: Applying Sadik transform with ADM we get the solution of (1) as $u_0(x,t) = u(x,0) = 2x$

$u_{n+1}(x,t) = -S^{-1}\left\{\frac{-1}{\upsilon^{\alpha}}S[u_{xx}]_n + \frac{1}{\upsilon^{\alpha}}S[u.u_x]_n\right\}$

$u_{n+1}(x,t) = S^{1}\left\{\frac{1}{\upsilon^{\alpha}}S[u_{xx}]_n\right\} - S^{-1}\left\{\frac{1}{\upsilon^{\alpha}}S[A_n]\right\}$ .... (3)

Where $A_n = (u.u_x)_n$

$$A = (u.u_x)$$

$$A_0 = (u_0.(u_0)_x)$$

$$A_0 = ((2x).(2x)_x) = 4x$$

$$u_1(x,t) = -S^{-1}\left\{\frac{-1}{\upsilon^\alpha}S[u_0]_{xx} + \frac{1}{\upsilon^\alpha}S[A_0]\right\}$$

$$u_1(x,t) = -S^{-1}\left\{\frac{-1}{\upsilon^\alpha}S[0] + \frac{1}{\upsilon^\alpha}S[4x]\right\}$$

$$u_1(x,t) = -S^{-1}\left\{0 + \frac{4x}{\upsilon^\alpha}S[1]\right\}$$

$$u_1(x,t) = -4xS^{-1}\left\{\frac{1}{\upsilon^{\alpha+(\alpha+\beta)}}\right\} = -4xt$$

$$A_1 = \frac{d}{d\lambda}[N(u_0+\lambda u_1)] = \frac{d}{d\lambda}(u_0+\lambda u_1)(u_0+\lambda u_1)_x$$

$$A_1 = (u_0+\lambda u_1)(u_1)_x + ((u_0)_x + \lambda(u_1)_x)(u_1)$$

$$A_1 = u_0(u_1)_x + (u_0)_x u_1$$

$$A_1 = (2x)(-4xt)_x + (2x)_x(-4xt) = -16xt$$

$$u_2(x,t) = -S^{-1}\left\{\frac{-1}{\upsilon^\alpha}S(u_1)_{xx} + \frac{1}{\upsilon^\alpha}S(A_1)\right\}$$

$$u_2(x,t) = -S^{-1}\left\{0 + \frac{1}{\upsilon^\alpha}S(-16xt)\right\}$$

$$u_2(x,t) = -S^{-1}\left\{-\frac{16x}{\upsilon^\alpha}S(t)\right\} = -S^{-1}\left\{-\frac{16x}{\upsilon^\alpha}\left(\frac{1}{\upsilon^{\alpha+(\alpha+\beta)}}\right)\right\}$$

$$u_2(x,t) = -S^{-1}\left\{\frac{-16x}{\upsilon^{2\alpha+\alpha+\beta}}\right\}$$

$$u_2(x,t) = 8x.S^{-1}\left\{\frac{2}{\upsilon^{2\alpha+\alpha+\beta}}\right\} = 8xt^2$$

$$A_2 = \frac{1}{2!}\frac{d^2}{d\lambda^2}[N(u_0+\lambda u_1+\lambda^2 u_2)]$$

$$A_2 = \frac{1}{2!}\frac{d^2}{d\lambda^2}[(u_0+\lambda u_1+\lambda^2 u_2)(u_0+\lambda u_1+\lambda^2 u_2)_x]$$

$$A_2 = u_0(u_2)_x + u_1(u_1)_x + u_2(u_0)_x$$

$$A_2 = (2x)(8xt^2)_x + (-4xt)(-4xt)_x + (8xt^2)(2x)_x$$

$$A_2 = 48xt^2$$

$$u_3(x,t) = -S^{-1}\left\{\frac{-1}{\upsilon^\alpha}S(u_2)_{xx} + \frac{1}{\upsilon^\alpha}S(A_2)\right\}$$

$$u_3(x,t) = -S^{-1}\left\{0 + \frac{1}{\upsilon^\alpha}S(48xt^2)\right\}$$

$$u_3(x,t) = -S^{-1}\left\{\frac{48x}{\upsilon^\alpha}S(t^2)\right\} = -S^{-1}\left\{\frac{48x}{\upsilon^\alpha}\frac{2!}{\upsilon^{2\alpha+(\alpha+\beta)}}\right\}$$

$$u_3(x,t) = -S^{-1}\left\{\frac{48x}{\upsilon^{3\alpha+(\alpha+\beta)}}2\right\} = -16xS^{-1}\left\{\frac{3!}{\upsilon^{3\alpha+(\alpha+\beta)}}\right\} = -16xt^3$$

Similarly, A_3, A_4..... Can be found

Therefor the Solution is given by $u(x,t) = \sum_{i=0}^{\infty} u_i(x,t)$

$u(x,t) = 2x - 4xt + 8xt^2 - 16xt^3......$

$u(x,t) = 2x\left[1 - (2t) + (2t)^2 - (2t)^3 + ...\right] = 2x(1+2t)^{-1}$

Hence

$u(x,t) = \frac{2x}{1+2t}$

Example: Consider non linear P.D.E. $u_{tt} + uu_x = -sint.....(1)$

With initial condition $u(x,0) = 0$ and $u_t(x,0) = 1.....(2)$

Solution: Applying Sadik Transform with ADM we get

$u_0(x,t) = u(x,0) + tu_t(x,0) + S^{-1}\left[\frac{1}{\upsilon^{2\alpha}} S(-sint)\right]$

$u_0(x,t) = 0 + t + S^{-1}\left[\frac{-1}{\upsilon^{2\alpha}} \frac{\upsilon^{-\beta}}{\upsilon^{2\alpha}+1}\right]$

$u_0(x,t) = t + S^{-1}\left[-\upsilon^{-\beta}\left(\frac{1}{\upsilon^{2\alpha}(\upsilon^{2\alpha}+1)}\right)\right] = t - S^{-1}\left[\upsilon^{-\beta}\left(\frac{\upsilon^{2\alpha}+1-\upsilon^{2\alpha}}{\upsilon^{2\alpha}(\upsilon^{2\alpha}+1)}\right)\right]$

$u_0(x,t) = t - S^{-1}\left[\upsilon^{-\beta}\left(\frac{1}{\upsilon^{2\alpha}} - \frac{1}{\upsilon^{2\alpha}+1}\right)\right] = t - S^{-1}\left[\frac{1}{\upsilon^{\alpha+\alpha+\beta}} - \frac{\upsilon^{-\beta}}{\upsilon^{2\alpha}+1}\right]$

$u_0(x,t) = t - (t - sint) = sint$

$A_0 = u_0 u_{0x} = 0$

$u_{n+1}(x,t) = -S^{-1}\left[\frac{1}{\upsilon^{2\alpha}} S\{A_n\}\right]$

Now If $n = 0, u_1 = -S^{-1}\left[\frac{1}{\upsilon^{2\alpha}} S(0)\right] = 0$

From previous example

$A_1 = u_0(u_1)_x + (u_0)_x u_1 = 0$

$u_2(x,t) = -S^{-1}\left[\frac{1}{\upsilon^{2\alpha}} S(A_1)\right] = 0$

$A_2 = u_0(u_2)_x + u_1(u_1)_x u_2(u_0)_x = 0$

Now Similarly $u_3(x,t) = u_4(x,t) = u_5(x,t) = 0$

Therefor $u(x,t) = \sum_{i=0}^{\infty} u_i(x,t) = sint + 0 + 0 + 0... = sint$

6. CONCLUSIONS

In this paper a new method has been introduced for solving two dimensional Non-Homogeneous non-linear Partial Differential Equation by using Sadik transform combine with Adomian Decomposition Method, through this method we can easily solve these types of Partial Differential Equation.

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