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Study of Numerical Accuracy in Different Spline Interpolation Techniques

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Abstract

In this paper, we introduced the accuracy of results using different spline functions. Some examples are also discussed. This work presents a numerical accuracy of results by different spline functions. Results are analyzed by comparing the actual sampled values with the values obtained by linear, quadratic and cubic spline interpolation.

Keywords: Interpolation, Linear Spline, Quadratic Spline, Cubic Spline, Trigonometric function.

INTRODUCTION:

Numerical analysis is the area of mathematics and computer sciences that creates analyzes and implements algorithms for solving numerically the problems of continuous mathematics.[1] In the mathematical field of numerical analysis, interpolation is a method of constructing new data points within the range of a discrete set of known data points. It is often required to interpolate, i.e., estimate the value of that function for an intermediate value of the independent variable. We have n + 1 distinct nodal point $x_0, x_1, \ldots, x_n$ and we want to determine an interpolating polynomial. The interpolating polynomial is linear in each subinterval (x_{i-1}, x_i) and it agrees with the function f(x) at the n+1 nodal points. The subintervals or the line segment are called finite elements in one space dimension and the nodal points are called knots. [2] A low-order polynomial approximation in each subinterval provides a better approximation to the tabulated function than fitting a single high-order polynomial to the entire interval. Cubic spline interpolation is a special case for spline interpolation that is used very often to avoid the problem of Runge's phenomenon. This method gives an interpolating polynomial that is smoother and has smaller error than some other interpolating polynomials such as Lagrange polynomial and Newton polynomial.

ANALYSIS AND IMPLEMENTATION:

LINEAR SPLINE INTERPOLATION:

Let the given data points be

$$(x_i, y_i) \quad i = 0, 1, 2, ...n$$

where,

$$a = x_0 < x_1 < x_2 < ... < x_n = b$$

and let,

$$h_i = (x_i - x_{i-1}) \quad i = 1, 2, 3, ..., n$$

Further let $s_i(x)$ be the spline of degree one defined in the interval $[x_{i-1}, x_i]$. Obviously, $s_i(x)$ represents a straight line joining the points (x_{i-1}, y_{i-1}) and (x_i, y_i). Hence, we write

$$s_i(x) = y_{i-1} + m_i(x - x_{i-1})$$

Where,

$$m_i = \frac{(y_i - y_{i-1})}{(x_i - x_{i-1})}.$$

QUADRATIC SPLINE INTERPOLATION:

Let the given data points be

$$(x_i, y_i), \quad i = 0, 1, 2, ..., n$$

Where

$$a = x_0 < x_1 < x_2 < ... < x_n = b$$

and let

$$h_i = x_i - x_(i - 1), \quad \imath = 1, 2, ..., n$$

Let $s_i(x)$ and $s_i^{'}(x)$ be continuous in $[x_0, x_n]$ and let

$$s_i x_i = y_i, \quad i = 0, 1, 2, ..., n.$$

Since $s_i(x)$ is a quadratic in the interval $[x_{i-1}, x_i]$.

$$s_i(x) = \frac{1}{h_i}[-\frac{(x_i - x)^2}{2}m_{i-1} + \frac{(x - x_{i-1})^2}{2}m_i] + y_{i-1} + \frac{h_i}{2}m_{i-1}.$$

CUBIC SPLINE INTERPOLATION:

Let the given data points be

$$(x_i, y_i), \quad i = 0, 1, 2, ..., n$$

Where

$$a = x_0 < x_1 < x_2 < ... < x_n = b$$

and let

$$h_i = x_i - x_{i-1}, \quad i = 1, 2, ..., n$$

Let $s_i(x)$ be the cubic spline defined in the interval $[x_{i-1}, x_i]$. The conditions for the natural cubic spline are,

1. $s_i(x)$ is atmost a cubic in each subinterval $[x_{i-1}, x_i], \quad i = 1, 2, 3, ..., n$
2. $s_i x_i = y_i, \quad i = 0, 1, 2, ..., n$
3. $s_i(x)$ and $s_i^{'}(x)$ and $s_i^{''}(x)$ are continuous in $[x_0, x_n]$ and
4. $s_i^{''}(x_0) = s_i^{''}(x_n) = 0.$

$$s_i(x) = \frac{1}{h_i}[\frac{(x_i - x)^3}{6}M_{i-1} + \frac{(x - x_{i-1})^3}{6}M_i$$
$$+(y_{i-1} - \frac{(h_i^2)}{6}M_{i-1})(x_i - x) + (y_i - \frac{(h_i^2)}{6}M_i)(x - x_{i-1})]$$

NUMERICAL EXAMPLES:

EXAMPLE:1

x	1	2	4	8
f(x)	3	7	21	73

Table 1: Example:1

In the interval [1, 2], we have

$$L_1(x) = \frac{(x-2)}{(-1)}(3) + (x-1)(7) = 4x - 1$$

In the interval [2, 4], we have

$$L_2(x) = \frac{(x-4)}{(-2)}(7) + \frac{(x-2)}{2}(21) = 7x - 7$$

In the interval [4, 8], we have

$$L_3(x) = \frac{(x-8)}{(-4)}(21) + \frac{((x-4)}{4}(73) = 13x - 31$$

Hence the linear interpolating polynomials are:

$$L_1(x) = 4x\text{–}1. \quad 1 \le x \le 2$$

$$L_2(x) = 7x\text{–}7. \quad 2 \le x \le 4$$

$$L_3(x) = 13x\text{–}31. \quad 4 \le x \le 8$$

EXAMPLE 2:

x	1	2	3
f(x)	-8	-1	18

Table 2: Example:2

$$L_1(x) = 7x\text{–}15. \quad Ininterval[1,2]$$

$$L_2(x) = 19x\text{–}39. \quad Ininterval[2,3]$$

EXAMPLE:3

x	1	2	3
f(x)	-1	-8	18

Table 3: Example:3

$$Q(x) = 12x^2\text{–}41x + 33$$

.

EXAMPLE:4

x	-3	-2	-1	1	3	6	7
f(x)	369	222	171	165	207	990	1779

Table 4: Example:4

$$Q_1(x) = 48x^2 + 93x+$$

$$Q_2(x) = 6x^2 – 3x + 162.$$

$$Q_3(x) = 132x^2 – 927x + 1800.$$

EXMPLE:5

x	0	$\pi/2$	π
f(x)	0	1	0

Table 5: Example:5

In the interval $[0, \pi/2]$, the natural cubic spline is given by

$$c_1(x) = 2/\pi[-(2x^3)/\pi^2 + 3x/2]$$

$$y(\pi/6) = c_1(\pi/6) = 2/\pi(-\pi/108 + \pi/4) = 0.4815$$

In the interval $[0, \pi/4]$, the natural cubic spline is given by

$$c_1(x) = 4/\pi(-0.1240x^3 + 0.7836x)$$

$$y(\pi/6) = c_1(\pi/6) = 0.4998$$

COMPARISON OF RESULTS:

From above examples we see that when we half the interval then cubic spline has produced a better approximation.The cubic spline values together with the exact values are given in the following table.

$$y = \sin x$$

X(in degree)	Cubic spline values	Exact values
5	0.087155743	0.087155530
15	0.258819045	0.258818415
25	0.422618262	0.422617233
35	0.573576436	0.573575040
45	0.707106781	0.707105059

Table 6: Example:6

CONCLUSION:

In this paper, accordingly to the analysis the performance of function is presented. Experimental result shows that the cubic spline has produced a better approximation when the interval is halved. We finally consider values of $y = \sin x$ in intervals of 100 from $x = 0$ to π and then interpolate for $x = 50, 150, 250, 350$ and $450,$ using the natural cubic spline.

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