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Non-Existence of Solution of Rotation Flow in N-S Equs

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Abstract

Non-existence of solution of rotation flow in N-S equations is proved by decomposition method.

Keywords: Navier-Stockes equations, outer product, rotation flow, Eigen value equation, decomposition method.

1. INTRODUCTION

The Navier–Stokes equations are a set of differential equations which describe the motion of viscous fluid substances[1]. They have wide uses in scientific and engineering, They may be used to model the weather, ocean currents, water flow in a pipe, airflow around a wing, etc. They coupled with Maxwell's equations can be used to study magneto-hydrodynamics. They are also of great interest in mathematics. Existence and smoothness of the N-S equations has been concluded in one of the seven "Millennium Prize Problem"[2]by Claymath. "Who proved the N-S equations ?"[3] Jean Leray in 1934 proved the existence of the so called weak solution s to the N-S equations satisfying the equations in mean value, not point wise. Terence Tao in 2016 published a finite time blow result for an averaged version of the 3-D N-S equations. "Who recently solved the N-S equations? "[3] there were three answers: there is no proof. Since they are not a mathematics fact. N-S equations are a model for streaming fluid [3] and 2016-01-13 Agostino Prastaro added an answer by algebraic-topologic proof[4]..

This paper proves non-existence of solution of rotation flow in N-S equations by decomposition method.

2. NAVIER-STOCKES EQUATIONS

In the following, **Bold** face denotes **vector, matrix, tensor,** Non-bold face represents scalars.

The general form of the N-S equations of fluid motion is [5]:

$$\rho\left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V}\cdot\nabla\mathbf{V}\right) = -\nabla p + \nabla\cdot\mathbf{T} + \mathbf{f}, \tag{2-1}$$

where $\mathbf{V}$ is the flow velocity; ρ is the fluid density; p is the pressure; $\mathbf{T}$ is the (deviatoric) stress tensor; $\mathbf{f}$ represents the body force per unit volume acting on the fluid; ∇ is the del operator.

$\mathbf{V}\cdot\nabla\mathbf{V}$ which can be interpretation as [5]

$$\mathbf{V}\cdot\nabla\mathbf{V} = \nabla\left(\frac{\|\mathbf{V}\|^2}{2}\right) + (\nabla\times\mathbf{V})\times\mathbf{V}, \tag{2-2}$$

The last term 0f (2-2) represents the rotation flow; $(\nabla\times\mathbf{V}) = \mathrm{rot}\,\mathbf{V}$ represents the curl of the velocity (called vorticity); $\times$ is the outer product of vector, defined [6] by

Outer product:

$$\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = (a_y b_z - a_z b_y)\mathbf{i} - (a_x b_z - a_z b_x)\mathbf{j} + (a_x b_y - a_y b_x)\mathbf{k}, \tag{2-3}$$

$\mathbf{a} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}$, $\mathbf{b} = b_x\mathbf{i} + b_y\mathbf{j} + b_z\mathbf{k}$, $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are unit vectors in Cartesian coordinates. $|A| = \det A$ is the determinant.

$\nabla = \partial_x\mathbf{i} + \partial_y\mathbf{j} + \partial_z\mathbf{k}$, denotes the del operator, $\partial_x = \partial/\partial x$, $\partial_y = \partial/\partial y$, $\partial_z = \partial/\partial z$.

$$\nabla\times\mathbf{V} = \mathrm{rot}\,\mathbf{V} = \mathbf{D} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial_x & \partial_y & \partial_z \\ V_x & V_y & V_z \end{vmatrix} = (\partial_y V_z - \partial_z V_y)\mathbf{i}$$

$$-(\partial_x V_z - \partial_z V_x)\mathbf{j} + (\partial_x V_y - \partial_y V_x)\mathbf{k} = D_x\mathbf{i} + D_y\mathbf{j} + D_z\mathbf{k}, \tag{2-4}$$

Where $D_x = (\partial_y V_z - \partial_z V_y)$, $D_y = (\partial_x V_z - \partial_z V_x)$, $D_z = (\partial_x V_y - \partial_y V_x)$.

3. HOW TO PROVE THE EXISTENCE OF SOLUTION OF N-S EQUATIONS?

The N-S equations are a set of PDE with great difficult to prove its existence. We need the following steps to do it.

The proof of existence of solution of N-S equations needs to prove all unknown functions $\mathbf{V}$, p, $\mathbf{T}$, $\mathbf{f}$ etc. satisfy all requirements of N-S equations. However, the proof of non-existence of solution of N-S equations is easier and just needs a given counter example, or a special case. Therefore,

Step 1, To prove the non-existence of solution of N-S equations instead of to prove the existence of solution of N-S equations.

Here, we prove a special case , by decomposition method.

The decomposition method

The decomposition method is a method for lowering down high order D.E., by Eigen value equation and has been used to solve dimpling and bucking of spherical crust [7]. Where the Eigen value equation with $\boldsymbol{\nabla}$ operators have been solved.

The Eigen value problem related to D.E. and vibration problem and have been systemically studied in literatures, e.g., [8]. For example,

$y'' + \lambda^2 y = 0$, describes longitudinal vibration of elastic bar and torsional vibration; while four degree equation represents literal vibration of elastic bar. However, Eigen value problem relating to curl operator is seldom found, except [9] by numerical calculation based on experiment.

Step 2, How to select special case?

The special case should be simple to solve and has not been solved yet.

Here, we prove non-existence of solution of rotation flow in N-S equations of the form:

$$(\boldsymbol{\nabla} \times \mathbf{V}) \times \mathbf{V} = -\lambda^2 \mathbf{V}, \qquad (3\text{-}1)$$

Where the first term of (2-1) is absence (a time independent flow), the right hand side of (2-1) represents the forced terms of a non-homogeneous equation. We just concern with homogeneous equation shown in (3-1) and did not care on the forced terms. λ^2 is an Eigen value and (3-1) is a homogeneous equation, if $\lambda^2 \neq 0$, then, (3-1) has non-zero solution; otherwise, (3-1) has zero solution $\mathbf{V} = \mathbf{0}$.

Step 3, To prove $\lambda^2 = 0$. **($\mathbf{V} = \mathbf{0}$).**

Choosing a vector **U,** such that $\mathbf{V} \times \mathbf{U} = \mathbf{I}$, i.e.,

$$\mathbf{V} \times \mathbf{U} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ V_x & V_y & V_z \\ U_x & U_y & U_z \end{vmatrix} = (V_yU_z - V_z\mathbf{U}_y)\mathbf{i} - (V_xU_z - V_zU_x)\mathbf{j} + (V_xU_y - V_yU_x)\mathbf{k}, \tag{3-2}$$

Multiplying Outer-product **U** to both sides of (3-1), we have:

$$(\nabla \times \mathbf{V}) \times \mathbf{V} \times \mathbf{U} = (\nabla \times \mathbf{V}) \times \mathbf{I} = \mathbf{D} \times \mathbf{I} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ D_x & D_y & D_z \\ \mathbf{i} & \mathbf{j} & \mathbf{k} \end{vmatrix} = \mathbf{0} = -\lambda^2(\mathbf{V} \times \mathbf{U}) = -\lambda^2\mathbf{I}, \tag{3-3}$$

where two rows are the same in determinant , therefore,

$$\lambda^2 = 0, \text{ and } \mathbf{V} = \mathbf{0}, \tag{3-4}$$ ☐

4. CONCLUSION

Non-existence of solution of rotation flow of N-S equations does not mean that the fact of rotation flow does not exist, but the description by N-S model in rotation flow has no solution.

Any numerical or approximate calculation on rotation flow seems meaningless since no solution can be used for comparing, except other references (e.g., experiment) are used.

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