Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 16, Number 3 (2020), pp. 515-528

http://www.ripublication.com/gjpam.htm

Some New Improved Classes of Estimators Using Multiple Auxiliary Information

Shashi Bhushan[1], Raksoni Gupta[2], Saurabh Singh[1] and Anoop Kumar[1]

[1.] *Department of Mathematics and Statistics, Dr. Shakuntala Misra National Rehabilitation University, Lucknow, U.P, India.*

[2.] *Department of Statistics, National Post Graduate College, Lucknow, U.P., India.*

Abstract

In this article, three classes of estimators are suggested to estimate the population mean for the study variable using two auxiliary variables. Some theoretical comparisons are made with commonly used estimators. The extensions of the proposed classes of estimators are proposed if more information like coefficient of variation, standard deviation, skewness, kurtosis, etc. about the two auxiliary variables is available. These classes are also extended to multiple auxiliary variable case. Finally, an empirical study is included as an illustration.

Keywords: Classes of estimators, multiple auxiliary information

1. INTRODUCTION

It is a common practice to use supplementary information provided by the auxiliary variables in survey sampling. This information can be used to improve the precision of the estimators. The literature describes a variety of techniques for using auxiliary information (for eg. Cochan 1977, Murthy 1967, Singh 1986, 2003, Bhushan 2013, etc.). The use of auxiliary information can increase the precision of an estimator when study variable y is highly correlated with the auxiliary variable x. It is a well-known fact that by suitably incorporating the auxiliary information, the related inferences became more and more precise and hence provide better estimation of the parameter under investigation. In large scale sample surveys, we often collect data on more than one auxiliary character and some of these may be correlated with the study variable. Olkin (1958) has used a linear combinations of estimators based on several auxiliary characters. The coefficients of the linear combination were so obtained, as to minimize the variance of the estimator. However, some authors including Rao and

Mudholkar (1967), Raj (1965), Shukla (1966), Singh (1967), Srivastava (1965) had made the use of linear combinations of several estimators based on each auxiliary character separately.

In this paper, three classes of estimators using two auxiliary variables are considered in section 2. Section 3 consists of the extension of these estimators to the case of multiple auxiliary variables. Section 4 provides a comparative study and the section 5 proposes the extensions of such estimators. Finally, section 6 gives an empirical study to show the superiority of the proposed estimators.

2. ESTIMATORS BASED ON TWO AUXILIARY VARIABLES

Consider a finite population of size *N* from which a sample of size *n* is drawn with the help of simple random sampling without replacement. Let *y* denotes the variable under study whose mean is to be estimated making the use of two auxiliary variables. It is to be assumed that the population mean is known. In this paper, we propose the following classes of estimators of population mean given by

$$T_1 = \alpha_1 \bar{y} + \beta_1\left(\bar{x}_1 - \bar{X}_1\right) + \gamma_1\left(\bar{x}_2 - \bar{X}_2\right) \tag{2.1}$$

$$T_2 = \alpha_2 \bar{y}\left(\frac{\bar{x}_1}{\bar{X}_1}\right)^{\beta_2}\left(\frac{\bar{x}_2}{\bar{X}_2}\right)^{\gamma_2} \tag{2.2}$$

$$T_3 = \alpha_3 \bar{y}\left(\frac{\bar{X}_1}{\bar{X}_1 + \beta_3\left(\bar{x}_1 - \bar{X}_1\right)}\right)\left(\frac{\bar{X}_2}{\bar{X}_2 + \gamma_3\left(\bar{x}_2 - \bar{X}_2\right)}\right) \tag{2.3}$$

where $\alpha_i, \beta_i, \gamma_i$ are the characterising scalars, $\bar{y}$ denotes the sample mean of the study variable and $\bar{x}_1$, $\bar{x}_2$ denotes the sample means for the auxiliary variables x_i respectively.*(i=1,2)* . Further, it is noteworthy that these classes not only extend the work of Olkin (1958), Srivastava (1965), etc but also provides the improvement by using Searle's approach.

Theorem 2.1

The bias and the mean square error of the estimator is considered up to the terms of order n^{-1}

$$Bias\left(T_1\right) = \bar{Y}\left(\alpha_1 - 1\right) \tag{2.4}$$

$$MSE\left(T_1\right) = \left\{\begin{array}{l} \bar{Y}^2\left(\alpha_1^2\lambda C_y^2 + \alpha_1^2 + 1 - 2\alpha_1\right) + \beta_1^2\bar{X}_1^2\lambda C_{x_1}^2 + \gamma_1^2\bar{X}_2^2\lambda C_{x_2}^2 \\ +2\beta_1\bar{X}_1\bar{Y}\alpha_1\lambda\rho_{yx_1}C_yC_{x_1} + 2\gamma_1\bar{X}_2\bar{Y}\alpha_1\lambda\rho_{yx_2}C_yC_{x_2} + 2\beta_1\gamma_1\bar{X}_1\bar{X}_2\lambda\rho_{x_1x_2}C_{x_1}C_{x_2} \end{array}\right\} \tag{2.5}$$

Corollary 2.2

The optimum values minimising the *MSE* are given by,

$$\alpha_{1(opt)}=\frac{1}{1+\lambda C_y^2\left(1-R_{y.12}^2\right)}$$

$$\beta_{1(opt)}=\frac{-\bar{Y}C_y b_{y1.2}}{\left[1+\lambda C_y^2\left(1-R_{y.12}^2\right)\right]\overline{X_1}C_{x_1}}$$

$$\gamma_{1(opt)}=\frac{-\bar{Y}C_y b_{y2.1}}{\left[1+\lambda C_y^2\left(1-R_{y.12}^2\right)\right]\overline{X_2}C_{x_2}} \tag{2.6}$$

and the minimum value of *MSE* is given by,

$$\min MSE\left(T_1\right)=\frac{\lambda\bar{Y}^2C_y^2\left(1-R_{y.12}^2\right)}{1+\lambda C_y^2\left(1-R_{y.12}^2\right)}=\bar{Y}^2\left(1-\alpha_{1(opt)}\right) \tag{2.7}$$

where $b_{y1.2}$ and $b_{y2.1}$ are the partial regression coefficients

and $R_{y.12}^2$ is the multiple correlation coefficient of y on x_1 and x_2.

Theorem 2.3

Bias and MSE are given by

$$Bias\left(T_2\right)=\bar{Y}.\left\{\alpha_2\begin{pmatrix}1+\beta_2\lambda\rho_{yx_1}C_yC_{x_1}+\gamma_2\lambda\rho_{yx_2}C_yC_{x_2}+\beta_2\gamma_2\lambda\rho_{x_1x_2}C_{x_1}C_{x_2}\\+\frac{\beta_2\left(\beta_2-1\right)}{2}\lambda C_{x_1}^2+\frac{\gamma_2\left(\gamma_2-1\right)}{2}\lambda C_{x_2}^2\end{pmatrix}-1\right\} \tag{2.8}$$

and

$$MSE\left(T_2\right)=\bar{Y}^2.\left\{\begin{array}{l}\alpha_2^2\begin{pmatrix}1+\lambda C_y^2+\beta_2^2\lambda C_{x_1}^2+\gamma_2^2\lambda C_{x_2}^2+4\beta_2\lambda\rho_{yx_1}C_yC_{x_1}+4\gamma_2\lambda\rho_{yx_2}C_yC_{x_2}+4\beta_2\gamma_2\lambda\rho_{x_1x_2}C_{x_1}C_{x_2}\\+\beta_2\left(\beta_2-1\right)\lambda C_{x_1}^2+\gamma_2\left(\gamma_2-1\right)\lambda C_{x_2}^2\end{pmatrix}\\-2\alpha_2\begin{pmatrix}1+\beta_2\lambda\rho_{yx_1}C_yC_{x_1}+\gamma_2\lambda\rho_{yx_2}C_yC_{x_2}+\beta_2\gamma_2\lambda\rho_{x_1x_2}C_{x_1}C_{x_2}\\+\frac{\beta_2\left(\beta_2-1\right)}{2}\lambda C_{x_1}^2+\frac{\gamma_2\left(\gamma_2-1\right)}{2}\lambda C_{x_2}^2\end{pmatrix}+1\end{array}\right\}$$

$$=\bar{Y}^2.\left\{\alpha_2^2.A_2-2\alpha_2.B_2+1\right\} \tag{2.9}$$

where,

$$A_2=\begin{pmatrix}1+\lambda C_y^2+\beta_2^2\lambda C_{x_1}^2+\gamma_2^2\lambda C_{x_2}^2+4\beta_2\lambda\rho_{yx_1}C_yC_{x_1}+4\gamma_2\lambda\rho_{yx_2}C_yC_{x_2}\\+4\beta_2\gamma_2\lambda\rho_{x_1x_2}C_{x_1}C_{x_2}+\beta_2\left(\beta_2-1\right)\lambda C_{x_1}^2+\gamma_2\left(\gamma_2-1\right)\lambda C_{x_2}^2\end{pmatrix}$$

$$B_2 = \begin{pmatrix} 1+\beta_2\lambda\rho_{yx_1}C_yC_{x_1} + \gamma_2\lambda\rho_{yx_2}C_yC_{x_2} + \beta_2\gamma_2\lambda\rho_{x_1x_2}C_{x_1}C_{x_2} \\ +\dfrac{\beta_2(\beta_2-1)}{2}\lambda C_{x_1}^2 + \dfrac{\gamma_2(\gamma_2-1)}{2}\lambda C_{x_2}^2 \end{pmatrix} \quad (2.10)$$

Corollary 2.4

Minimum value of MSE is obtained by optimising α_2. The optimum value of α_2 is obtained as

$$\alpha_{2(opt)} = \frac{B_2}{A_2} \quad (2.11)$$

and the minimum value of MSE is given by:

$$MSE_{\min}(T_2) = \bar{Y}^2\left(1-\frac{B_2^2}{A_2}\right) \quad (2.12)$$

Theorem 2.5

Bias and MSE are given by

$$Bias(T_3) = E(T_3) - \bar{Y}$$

$$= \bar{Y}\left\{\alpha_3.\left(1-\beta_3\lambda\rho_{yx_1}C_yC_{x_1} - \gamma_3\lambda\rho_{yx_2}C_yC_{x_2} + \beta_3\gamma_3\lambda\rho_{x_1x_2}C_{x_1}C_{x_2} + \beta_3^2\lambda C_{x_1}^2 + \gamma_3^2\lambda C_{x_2}^2\right) - 1\right\} \quad (2.13)$$

$$MSE(T_3) = \bar{Y}^2.\left\{\begin{matrix} \alpha_3^2\begin{pmatrix} 1+\lambda C_y^2 + \beta_3^2\lambda C_{x_1}^2 + \gamma_3^2\lambda C_{x_2}^2 - 4\beta_3\lambda\rho_{yx_1}C_yC_{x_1} - 4\gamma_3\lambda\rho_{yx_2}C_yC_{x_2} \\ +4\beta_3\gamma_3\lambda\rho_{x_1x_2}C_{x_1}C_{x_2} + 2\beta_3^2\lambda C_{x_1}^2 + 2\gamma_3^2\lambda C_{x_2}^2 \end{pmatrix} \\ -2\alpha_3\begin{pmatrix} 1-\beta_3\lambda\rho_{yx_1}C_yC_{x_1} - \gamma_3\lambda\rho_{yx_2}C_yC_{x_2} \\ +\beta_3\gamma_3\lambda\rho_{x_1x_2}C_{x_1}C_{x_2} + \beta_3^2\lambda C_{x_1}^2 + \gamma_3^2\lambda C_{x_2}^2 \end{pmatrix} + 1 \end{matrix}\right\}$$

$$= \bar{Y}^2.\left\{\alpha_3^2.A_3 - 2\alpha_3.B_3 + 1\right\} \quad (2.14)$$

where

$$A_3 = \begin{pmatrix} 1+\lambda C_y^2 + \beta_3^2\lambda C_{x_1}^2 + \gamma_3^2\lambda C_{x_2}^2 - 4\beta_3\lambda\rho_{yx_1}C_yC_{x_1} - 4\gamma_3\lambda\rho_{yx_2}C_yC_{x_2} \\ +4\beta_3\gamma_3\lambda\rho_{x_1x_2}C_{x_1}C_{x_2} + 2\beta_3^2\lambda C_{x_1}^2 + 2\gamma_3^2\lambda C_{x_2}^2 \end{pmatrix}$$

$$B_3 = \left(1-\beta_3\lambda\rho_{yx_1}C_yC_{x_1} - \gamma_3\lambda\rho_{yx_2}C_yC_{x_2} + \beta_3\gamma_3\lambda\rho_{x_1x_2}C_{x_1}C_{x_2} + \beta_3^2\lambda C_{x_1}^2 + \gamma_3^2\lambda C_{x_2}^2\right) \quad (2.15)$$

Corollary 2.6

Optimum value of α_3 is obtained by minimising MSE with respect to α_3 as

$$\alpha_{3(opt)} = \frac{B_3}{A_3} \quad (2.16)$$

And the minimum MSE is given by

$$MSE_{\min}\left(T_3\right)=\overline{Y}^2\left(1-\frac{B_3^2}{A_3}\right) \tag{2.17}$$

3. MULTIVARIATE EXTENSIONS FOR MORE THAN TWO AUXILIARY VARIABLES

Let there are k auxiliary variables then we can use the variables by taking a linear combination of these k estimators of the form (), calculated for every auxiliary variable separately, for estimating the population mean. Then the estimators for population mean will be defined as,

$$T_1^*=\alpha_1\overline{y}+\sum_{i=1}^{k}\beta_{1i}\left(\overline{x}_i-\overline{X}_i\right) \tag{3.1}$$

$$T_2^*=\alpha_2\overline{y}\prod_{i=1}^{k}\left(\frac{\overline{x}_i}{\overline{X}_i}\right)^{\beta_{2i}} \tag{3.2}$$

$$T_3^*=\alpha_3\overline{y}\prod_{i=1}^{k}\left(\frac{\overline{X}_i}{\overline{X}_i+\beta_{3i}\left(\overline{x}_i-\overline{X}_i\right)}\right) \tag{3.3}$$

where $\alpha_j,\beta_{ji}\left(i=1,2,...,k\ ;j=1,2,3\right)$ are the characterising scalars.

Theorem 3.1

$$Bias\left(T_1^*\right)=\overline{Y}\left(\alpha_1-1\right) \tag{3.4}$$

$$MSE\left(T_1^*\right)=\left\{\begin{array}{l}\overline{Y}^2\left[\alpha_1^2+\alpha_1^2\lambda C_y^2+1-2\alpha\right]+\lambda\sum_{i=1}^{k}\beta_{1i}^2\overline{X}_i^2C_{x_i}^2\\+2\lambda\alpha_1\overline{Y}\sum_{i=1}^{k}\beta_{1i}\overline{X}_i\rho_{yx_i}C_yC_{x_i}+2\lambda\sum_{i\neq j=1}^{k}\beta_{1i}\beta_{1j}\overline{X}_i\overline{X}_j\rho_{x_ix_j}C_{x_i}C_{x_j}\end{array}\right\} \tag{3.5}$$

Corollary 3.2

Minimum value of *MSE* is obtained by optimising the characterising scalars. The optimum values are given by $\alpha_{1(opt)}=\dfrac{1}{1+\lambda C_y^2\left(1-R_{y.12...k}^2\right)}$

$$\beta_{1i(opt)}=\frac{-\overline{Y}C_yb_{yi.12...(i-1)}}{\overline{X}_1C_{x_1}\left[1+\lambda C_y^2\left(1-R_{y.12...k}^2\right)\right]} \tag{3.6}$$

and min *MSE* is obtained as

$$MSE_{\min}\left(T_1^*\right)=\frac{\lambda\overline{Y}^2C_y^2\left(1-R_{y.12...k}^2\right)}{\left[1+\lambda C_y^2\left(1-R_{y.12...k}^2\right)\right]}=\overline{Y}^2\left(1-\alpha_{1(opt)}\right) \tag{3.7}$$

Theorem 3.3

Bias and MSE of the proposed estimator are given by

$$Bias\left(T_2^*\right)=\overline{Y}\left\{\alpha_2\left(1+\lambda\sum_{i=1}^{k}\beta_{2i}\rho_{yx_i}C_yC_{x_i}+\lambda\sum_{i=1}^{k}\frac{\beta_{2i}\left(\beta_{2i}-1\right)}{2}C_{x_i}^2+\lambda\sum_{i\neq j=1}^{k}\beta_{2i}\beta_{2j}\rho_{x_ix_j}C_{x_i}C_{x_j}\right)-1\right\} \tag{3.8}$$

$$MSE\left(T_2^*\right)=\overline{Y}^2\left\{\begin{array}{l}\alpha_2^2\left(1+\lambda C_y^2+\lambda\sum_{i=1}^{k}\beta_{2i}^2C_{x_i}^2+4\lambda\sum_{i=1}^{k}\beta_{2i}\rho_{yx_i}C_yC_{x_i}+\lambda\sum_{i=1}^{k}\beta_{2i}\left(\beta_{2i}-1\right)C_{x_i}^2+4\lambda\sum_{i\neq j=1}^{k}\beta_{2i}\beta_{2j}\rho_{x_ix_j}C_{x_i}C_{x_j}\right)\\-2\alpha_2\left(1+\lambda\sum_{i=1}^{k}\beta_{2i}\rho_{yx_i}C_yC_{x_i}+\sum_{i=1}^{k}\frac{\beta_{2i}\left(\beta_{2i}-1\right)}{2}C_{x_i}^2+\sum_{i\neq j=1}^{k}\beta_{2i}\beta_{2j}\rho_{x_ix_j}C_{x_i}C_{x_j}\right)+1\end{array}\right\}$$

$$=\overline{Y}^2\left\{\alpha_2^2.A_2-2\alpha_2.B_2+1\right\} \tag{3.9}$$

where

$$A_2=\left(1+\lambda C_y^2+\lambda\sum_{i=1}^{k}\beta_{2i}^2C_{x_i}^2+4\lambda\sum_{i=1}^{k}\beta_{2i}\rho_{yx_i}C_yC_{x_i}+\lambda\sum_{i=1}^{k}\beta_{2i}\left(\beta_{2i}-1\right)C_{x_i}^2+4\lambda\sum_{i\neq j=1}^{k}\beta_{2i}\beta_{2j}\rho_{x_ix_j}C_{x_i}C_{x_j}\right)$$

$$B_2=\left(1+\lambda\sum_{i=1}^{k}\beta_{2i}\rho_{yx_i}C_yC_{x_i}+\sum_{i=1}^{k}\frac{\beta_{2i}\left(\beta_{2i}-1\right)}{2}\lambda C_{x_i}^2+\lambda\sum_{i\neq j=1}^{k}\beta_{2i}\beta_{2j}\rho_{x_ix_j}C_{x_i}C_{x_j}\right) \tag{3.10}$$

Corollary 3.4

An optimising value of the scalar minimizing the MSE is

$$\alpha_{2(opt)}=\frac{B_2}{A_2} \tag{3.11}$$

and the minimum value of MSE is given by

$$MSE_{\min}\left(T_2^*\right)=\overline{Y}^2\left(1-\frac{B_2^2}{A_2}\right) \tag{3.12}$$

Theorem 3.5

Bias and MSE of the proposed estimator are given by

$$Bias\left(T_3^*\right)=\overline{Y}\left\{\alpha_3\left(1-\lambda\sum_{i=1}^{k}\beta_{3i}\rho_{yx_i}C_yC_{x_i}+\lambda\sum_{i=1}^{k}\beta_{3i}^2C_{x_i}^2+\lambda\sum_{i\neq j=1}^{k}\beta_{3i}\beta_{3j}\rho_{x_ix_j}C_{x_i}C_{x_j}\right)-1\right\} \tag{3.13}$$

$$MSE\left(T_3^*\right)=\overline{Y}^2\left\{\begin{array}{l}\alpha_3^2\left(1+\lambda C_y^2+3\lambda\sum_{i=1}^{k}\beta_{3i}^2C_{x_i}^2-4\lambda\sum_{i=1}^{k}\beta_{3i}\rho_{yx_i}C_yC_{x_i}+4\lambda\sum_{i\neq j=1}^{k}\beta_{3i}\beta_{3j}\rho_{x_ix_j}C_{x_i}C_{x_j}\right)\\-2\alpha_3\left(1-\lambda\sum_{i=1}^{k}\beta_{3i}\rho_{yx_i}C_yC_{x_i}+\lambda\sum_{i=1}^{k}\beta_{3i}^2C_{x_i}^2+\lambda\sum_{i\neq j=1}^{k}\beta_{3i}\beta_{3j}\rho_{x_ix_j}C_{x_i}C_{x_j}\right)+1\end{array}\right\}$$

$$=\overline{Y}^2\left\{\alpha_3^2.A_3-2\alpha_3.B_3+1\right\} \tag{3.14}$$

where,

$$A_3 = \left(1 + \lambda C_y^2 + 3\lambda \sum_{i=1}^{k} \beta_{3i}^2 C_{x_i}^2 - 4\lambda \sum_{i=1}^{k} \beta_{3i} \rho_{yx_i} C_y C_{x_i} + 4\lambda \sum_{i \neq j=1}^{k} \beta_{3i} \beta_{3j} \rho_{x_i x_j} C_{x_i} C_{x_j}\right)$$

$$B_3 = \left(1 - \lambda \sum_{i=1}^{k} \beta_{3i} \rho_{yx_i} C_y C_{x_i} + \lambda \sum_{i=1}^{k} \beta_{3i}^2 C_{x_i}^2 + \lambda \sum_{i \neq j=1}^{k} \beta_{3i} \beta_{3j} \rho_{x_i x_j} C_{x_i} C_{x_j}\right) \qquad (3.15)$$

Corollary 3.6

MSE will be minimum for the optimum value of the characterising scalar

$$\alpha_{3(opt)} = \frac{B_3}{A_3} \qquad (3.16)$$

and the minimum value of the MSE of the proposed estimator is given by

$$MSE_{\min}\left(T_3^*\right) = \bar{Y}^2 \left(1 - \frac{B_3^2}{A_3}\right) \qquad (3.17)$$

4. Efficiency comparison theorem:

In this section a comparison of the proposed classes of estimators with some of the known estimators in terms of biases and mean square error up to order of is given. We consider the following estimators:

(a). Mean per unit estimator

It is an unbiased estimator of population mean and its variance is given by,

$$Var\left(\bar{y}\right) = \lambda \bar{Y}^2 C_y^2 \qquad (4.1)$$

(b). Ratio estimator

$$\bar{y}_R = \bar{y} \frac{\bar{X}}{\bar{x}}$$

where x may be chosen as x_1 or x_2

$$Bias\left(\bar{y}_R\right) = \lambda \bar{Y} C_x \left(C_x - \rho_{yx} C_y\right) \qquad (4.2)$$

$$MSE\left(\bar{y}_R\right) = \lambda \bar{Y}^2 \left(C_y^2 + C_x^2 - 2\rho_{yx} C_y C_x\right) \qquad (4.3)$$

(c). Product estimator

$$\bar{y}_P = \bar{y} \frac{\bar{x}}{\bar{X}}$$

$$Bias\left(\bar{y}_P\right)=\lambda\bar{Y}\rho_{yx}C_yC_x \tag{4.4}$$

$$MSE\left(\bar{y}_P\right)=\lambda\bar{Y}^2\left(C_y^2+C_x^2+2\rho_{yx}C_yC_x\right) \tag{4.5}$$

(d). If $\alpha_1=\alpha_2=\alpha_3=1$ then the proposed classes of estimators become:

(i). $T_1^{\#}=\bar{y}+\beta_1\left(\bar{x}_1-\bar{X}_1\right)+\gamma_1\left(\bar{x}_2-\bar{X}_2\right)$

(ii). $T_2^{\#}=\bar{y}\left(\dfrac{\bar{x}_1}{\bar{X}_1}\right)^{\beta_2}\left(\dfrac{\bar{x}_2}{\bar{X}_2}\right)^{\gamma_2}$

(iii). $T_3^{\#}=\bar{y}\left(\dfrac{\bar{X}_1}{\bar{X}_1+\beta_3\left(\bar{x}_1-\bar{X}_1\right)}\right)\left(\dfrac{\bar{X}_2}{\bar{X}_2+\gamma_3\left(\bar{x}_2-\bar{X}_2\right)}\right)$

The biases and the mean square errors of the above estimators are given below:

(i). $Bias\left(T_1^{\#}\right)=0$ (4.6)

$$MSE\left(T_1^{\#}\right)=\lambda\bar{Y}^2C_y^2\left(1-R_{y.12}^2\right) \tag{4.7}$$

(ii). $Bias\left(T_2^{\#}\right)=\lambda\bar{Y}\left\{\begin{array}{l}\beta_2\rho_{yx_1}C_yC_{x_1}+\gamma_2\rho_{yx_2}C_yC_{x_2}+\beta_2\gamma_2\rho_{x_1x_2}C_{x_1}C_{x_2}\\+\dfrac{\beta_2\left(\beta_2-1\right)}{2}C_{x_1}^2+\dfrac{\gamma_2\left(\gamma_2-1\right)}{2}C_{x_2}^2\end{array}\right\}$ (4.8)

$$MSE\left(T_2^{\#}\right)=\lambda\bar{Y}^2C_y^2\left(1-R_{y.12}^2\right) \tag{4.9}$$

(iii). $Bias\left(T_3^{\#}\right)=\lambda\bar{Y}\left\{\beta_3^2C_{x_1}^2+\gamma_3^2C_{x_2}^2-\beta_3\rho_{yx_1}C_yC_{x_1}-\gamma_3\rho_{yx_2}C_yC_{x_2}+2\beta_3\gamma_3\rho_{x_1x_2}C_{x_1}C_{x_2}\right\}$ (4.10)

$$MSE\left(T_3^{\#}\right)=\lambda\bar{Y}^2C_y^2\left(1-R_{y.12}^2\right) \tag{4.11}$$

(e). The proposed classes of estimators are

(i). $T_1=\alpha_1\bar{y}+\beta_1\left(\bar{x}_1-\bar{X}_1\right)+\gamma_1\left(\bar{x}_2-\bar{X}_2\right)$

(ii). $T_2=\alpha_2\bar{y}\left(\dfrac{\bar{x}_1}{\bar{X}_1}\right)^{\beta_2}\left(\dfrac{\bar{x}_2}{\bar{X}_2}\right)^{\gamma_2}$

(iii). $T_3=\alpha_3\bar{y}\left(\dfrac{\bar{X}_1}{\bar{X}_1+\beta_3\left(\bar{x}_1-\bar{X}_1\right)}\right)\left(\dfrac{\bar{X}_2}{\bar{X}_2+\gamma_3\left(\bar{x}_2-\bar{X}_2\right)}\right)$

Biases and MSEs can be seen from equations (2.4), (2.7), (2.8), (2.12), (2.13) and (2.17)

5. MULTIVARIATE EXTENSIONS OF THE PROPOSED ESTIMATORS

(i). $T_1^{**} = \alpha_1 \bar{y} + \sum_{i=1}^{k} \beta_{1i} \left(\bar{x}_i^* - \bar{X}_i^* \right)$ (5.1)

(ii). $T_2^{**} = \alpha_2 \bar{y} \prod_{i=1}^{k} \left(\frac{\bar{x}_i^*}{\bar{X}_i^*} \right)^{\beta_{2i}}$ (5.2)

(iii). $T_3^{**} = \alpha_3 \bar{y} \prod_{i=1}^{k} \left(\frac{\bar{X}_i^*}{\bar{X}_i^* + \beta_{3i} \left(\bar{x}_i^* - \bar{X}_i^* \right)} \right)$ (5.3)

where $\bar{X}_i^* = a_i \bar{X}_i + b_i$; $\bar{x}_i^* = a_i \bar{x}_i + b_i$

and $\upsilon_i = \frac{a_i \bar{X}_i}{a_i \bar{X}_i + b_i}$; for $i = 1, 2, ..., k$

such that $a_i (\neq 0), b_i$ which are either known values or functions of the known parameters of the auxiliary variables x_i such as the standard deviations S_{x_i} , coefficient of variation C_{x_i} , coefficient of kurtosis $\beta_2(x_i)$ and correlation coefficient ρ_{yx_i} of the population. $\alpha_j, \beta_{ji} \left(i = 1, 2, ..., k \; ; j = 1, 2, 3 \right)$ are the characterising scalars.

The respective biases and mean square errors of the above proposed class of estimators can be obtained on the similar lines as it have been obtained for the estimators proposed in section 3.

5.1. Some Generalized members of the proposed class of estimators T_2^{**}

In this sub-section, some members of the suggested classes of estimators are given for the case of two auxiliary variables only. As an illustration the members belonging to the classes (5.1), (5.2) and (5.3), we describe the members of the class (5.2) only. The members of the remaining (5.1) and (5.3) can be written on similar lines.

Table 1

estimators T_2^{**}	***a1***	***b1***	***a2***	***b2***
$T_{2_1}^{**} = \alpha_2 \bar{y} \left(\frac{\bar{x}_1}{\bar{X}_1} \right)^{\beta_{21}} \left(\frac{\bar{x}_2}{\bar{X}_2} \right)^{\beta_{22}}$	1	0	1	0
$T_{2_2}^{**} = \alpha_2 \bar{y} \left(\frac{\bar{x}_1 + C_{x_1}}{\bar{X}_1 + C_{x_1}} \right)^{\beta_{21}} \left(\frac{\bar{x}_2 + C_{x_2}}{\bar{X}_2 + C_{x_2}} \right)^{\beta_{22}}$	1	C_{x_1}	1	C_{x_2}

estimators T_2^{**}	***a₁***	***b₁***	***a₂***	***b₂***
$T_{2_3}^{**}=\alpha_2\bar{y}\left(\frac{\beta_2(x_1)\bar{x}_1+C_{x_1}}{\beta_2(x_1)\bar{X}_1+C_{x_1}}\right)^{\beta_{21}}\left(\frac{\beta_2(x_2)\bar{x}_2+C_{x_2}}{\beta_2(x_2)\bar{X}_2+C_{x_2}}\right)^{\beta_{22}}$	$\beta_2(x_1)$	C_{x_1}	$\beta_2(x_2)$	C_{x_2}
$T_{2_4}^{**}=\alpha_2\bar{y}\left(\frac{C_{x_1}\bar{x}_1+\beta_2(x_1)}{C_{x_1}\bar{X}_1+\beta_2(x_1)}\right)^{\beta_{21}}\left(\frac{C_{x_2}\bar{x}_2+\beta_2(x_2)}{C_{x_2}\bar{X}_2+\beta_2(x_2)}\right)^{\beta_{22}}$	C_x	$\beta_2(x_1)$	C_{x_2}	$\beta_2(x_2)$
$T_{2_5}^{**}=\alpha_2\bar{y}\left(\frac{\bar{x}_1+S_{x_1}}{\bar{X}_1+S_{x_1}}\right)^{\beta_{21}}\left(\frac{\bar{x}_2+S_{x_2}}{\bar{X}_2+S_{x_2}}\right)^{\beta_{22}}$	1	S_{x_1}	1	S_{x_2}
$T_{2_6}^{**}=\alpha_2\bar{y}\left(\frac{\beta_1(x_1)\bar{x}_1+S_{x_1}}{\beta_1(x_1)\bar{X}_1+S_{x_1}}\right)^{\beta_{21}}\left(\frac{\beta_1(x_2)\bar{x}_2+S_{x_2}}{\beta_1(x_2)\bar{X}_2+S_{x_2}}\right)^{\beta_{22}}$	$\beta_1(x)$	S_{x_1}	$\beta_1(x_2)$	S_{x_2}
$T_{2_7}^{**}=\alpha_2\bar{y}\left(\frac{\beta_2(x_1)\bar{x}_1+S_{x_1}}{\beta_2(x_1)\bar{X}_1+S_{x_1}}\right)^{\beta_{21}}\left(\frac{\beta_2(x_2)\bar{x}_2+S_{x_2}}{\beta_2(x_2)\bar{X}_2+S_{x_2}}\right)^{\beta_{22}}$	$\beta_2(x_1)$	S_{x_1}	$\beta_2(x_2)$	S_{x_2}
$T_{2_8}^{**}=\alpha_2\bar{y}\left(\frac{\bar{x}_1+\rho}{\bar{X}_1+\rho}\right)^{\beta_{21}}\left(\frac{\bar{x}_2+\rho}{\bar{X}_2+\rho}\right)^{\beta_{22}}$	1	ρ	1	ρ
$T_{2_9}^{**}=\alpha_2\bar{y}\left(\frac{\bar{x}_1+\beta_2(x_1)}{\bar{X}_1+\beta_2(x_1)}\right)^{\beta_{21}}\left(\frac{\bar{x}_2+\beta_2(x_2)}{\bar{X}_2+\beta_2(x_2)}\right)^{\beta_{22}}$	1	$\beta_2(x_1)$	1	$\beta_2(x_2)$
$T_{2_{10}}^{**}=\alpha_2\bar{y}\left(\frac{C_{x_1}\bar{x}_1+\rho}{C_{x_1}\bar{X}_1+\rho}\right)^{\beta_{21}}\left(\frac{C_{x_2}\bar{x}_2+\rho}{C_{x_2}\bar{X}_2+\rho}\right)^{\beta_{22}}$	C_{x_1}	ρ	C_{x_2}	ρ
$T_{2_{11}}^{**}=\alpha_2\bar{y}\left(\frac{\rho\bar{x}_1+C_{x_1}}{\rho\bar{X}_1+C_{x_1}}\right)^{\beta_{21}}\left(\frac{\rho\bar{x}_2+C_{x_2}}{\rho\bar{X}_2+C_{x_2}}\right)^{\beta_{22}}$	ρ	C_{x_1}	ρ	C_{x_2}
$T_{2_{12}}^{**}=\alpha_2\bar{y}\left(\frac{\beta_2(x_1)\bar{x}_1+\rho}{\beta_2(x_1)\bar{X}_1+\rho}\right)^{\beta_{21}}\left(\frac{\beta_2(x_2)\bar{x}_2+\rho}{\beta_2(x_2)\bar{X}_2+\rho}\right)^{\beta_{22}}$	$\beta_2(x_1)$	ρ	$\beta_2(x_2)$	ρ
$T_{2_{13}}^{**}=\alpha_2\bar{y}\left(\frac{\rho\bar{x}_1+\beta_2(x_1)}{\rho\bar{X}_1+\beta_2(x_1)}\right)^{\beta_{21}}\left(\frac{\rho\bar{x}_2+\beta_2(x_2)}{\rho\bar{X}_2+\beta_2(x_2)}\right)^{\beta_{22}}$	ρ	$\beta_2(x_1)$	ρ	$\beta_2(x_2)$

6. EMPIRICAL STUDY

The comparison among these estimators is given in this section using a real data set. The data for this study is taken from [1], District Handbook of Aligarh, India. The population contains 332 villages.a simple random sample 80 villages is taken for the study. We consider the variables Y, X_1 and X_2 as the number of cultivators, area of the village and number of household in the village respectively. We compute the bias and the MSE for all estimators. The following values were obtained using the whole data sets:

$$\bar{Y} = 1093.1,\ \ \bar{X}_1 = 181.57,\ \ \bar{X}_2 = 143.31$$

$$C_y = 0.7626,\ \ C_{x_1} = 0.7684,\ \ C_{x_2} = 0.7616$$

$$\rho_{yx_1} = 0.973,\ \ \rho_{yx_2} = 0.862,\ \ \rho_{x_1x_2} = 0.842$$

Using the above results we have calculated the MSE and PRE for all the estimators in section 2. The PRE for each estimator with respect to the sample mean of a SRS is defined as follows:

$$e\left(y^{'}\right) = \left[\frac{MSE\left(\bar{y}\right)}{MSE\left(y^{'}\right)}\right]*100$$

where $MSE\left(y^{'}\right)$ is the mean square error for each estimator suggested in Section 2 and $MSE\left(\bar{y}\right) = Var\left(\bar{y}\right)$ for a sample of size 80.

Estimators	Auxiliary variables	MSE	Percent Relative Efficiency (PRE)
$\bar{y}$	None	6593.042276	100
$\bar{y}_R$	x_1	359.1134	1835.922
$\bar{y}_R$	x_2	1817.305	362.7923
$\bar{y}_P$	x_1	26214.39	25.15047
$\bar{y}_P$	x_2	24520.31	26.88809
T_1	x_1, x_2	**309.7676**	**2128.384**
T_2	x_1, x_2	**309.6608**	**2129.117**
T_3	x_1, x_2	**309.7253**	**2128.674**
$T_1^{\#}$	x_1, x_2	**309.8479**	**2127.832**
$T_2^{\#}$	x_1, x_2	**309.8479**	**2127.832**
$T_3^{\#}$	x_1, x_2	**309.8479**	**2127.832**

REFERENCES

[1] Bhushan, S. and Gupta, R.(2019): "A Class of Log-Type Estimators for Population Mean Using Auxiliary Information on an Attribute and a Variable Using Double Sampling Technique", International Journal of Computational and Applied Mathematics, Volume 14, Number 1(2019) , pp. 1-12

[2] Bhushan, S. and Gupta, R.(2019): " Searls' Ratio Product Type Estimators ", International Journal of Statistics and Systems, Volume 14, Number 1 (2019), pp. 29-37

[3] Bhushan, S. and Gupta, R.(2019): " Some Log-Type Classes of Estimators Using Auxiliary Attribute", Advances in Computational Sciences and Technology, Volume 12, Number 2 (2019) pp. 99-108

[4] Bhushan, S. and Gupta, R.(2019): " Some New Log Type Class of Double Sampling Estimators", International Journal of Applied Agricultural Research, Volume 14, Number 1 (2019) pp. 31-40

[5] Bhushan, S. et al. (2013): Improved sampling strategies in finite population, Scholar Press, Germany.

[6] Cramer, H.(1945): Mathematical Methods of Statistics, Princeton: N. J.: Princeton Uniiversity Press.

[7] Murthy, M. N.(1967): Sampling Theory and Methods, Calcutta: Statistical Publishing Society, 1967.

[8] Olkin, I.(1958): "Multivariate Ratio Estimation for Finite Populations," Biometrika, 45, 154-65.

[9] Raj, D. (1965): "On a Method of Using Multiauxiliary Information in Sample Surveys," Journal of the American Statistical Association, 60, 270-7.

[10] Rao, P. S. R. S. and Mudholkar, G. S. (1967): "Genieralized Multi- variate Estimator for the Mean of Finite Populations," Journal of the American Statistical Association 62, 1009-12.

[11] Shukla, G. K.(1966): "An Alternative Multivariate Ratio Estimate for Finite Population," Calcutta Statistical Association Bulletin 15, 127-34.

[12] Singh, M. P.(1967): "Ratio cum Product Method of Estimation," Metrika, 12 (No. 1), 34-42.

[13] Singh, M. P.(1967): "Multivariate Product Method of Estimation for Finite Populations," Journal of the Indian Society of Agri- cultural Statistics, 19, 1-10.

[14] Smith, T. M. F.(1966): "Ratio of Ratios and Their Applications," Journal of the Royal Statistical Society, Ser. A, 129 (Part 4), 531-3.

[15] Srivastava, S. K.(1965): "An Estimate of the Mean of a Finite Population Using Several Auxiliary Variables," Journal of the Indian Statistical Association, 3, 189-94.

[16] Srivastava, S. K.(1967): "An Estimator Using Auxiliary Information in Sample Surveys," Calcutta Statistical Association Bulletin, 16, 121-32.

[17] Searls, D. T. (1964): The Utilization of a Known Coefficient of Variation in the Estimation Procedure. Journal of the American Statistical Association. 59: 1225-1226.

[18] Bahl, S. And Tuteja, R.K. (1991): Ratio and product type exponential estimator, Information and Optimization Sciences, Vol.XII, I, 159-163.

[19] Bhushan, S. (2013): Improved sampling strategies in finite population, Scholars Press, Germany, 2013.

[20] Bhushan, S. and Gupta, R.(2015): An unbiased class of log-type estimators for population mean using auxiliary information on an attribute and a variable, Mathematical Sciences International Research Journal, 4(1), 143-147.

[21] Bhushan S., Misra P.K. and Yadav S.K. (2017): On the class of double sampling exponential ratio type estimator using auxiliary information on an attribute and an auxiliary variable, International Journal of Computational and Applied Mathematics, 12(1), 1-10.

[22] Bhushan S., Misra P.K. and Yadav S.K. (2017): A generalized class of unbiased estimators for population mean using auxiliary information on an attribute and an auxiliary variable, International Journal of Computational and Applied Mathematics, 12(1), 11-28.

[23] Bhushan S., Misra P.K. and Yadav S.K. (2017): On the class of double sampling ratio estimators using auxiliary information on an attribute and an auxiliary variable, International Journal of Statistics and Systems, 12(1), 15-23.

[24] Bhushan S., Misra P.K. and Yadav S.K. (2017): A generalized class of double sampling estimators using auxiliary information on an attribute and an auxiliary variable, International Journal of Computational Intelligence Research, 13(1), 23-33.

[25] Bhushan S., Misra P.K. and Yadav S.K. (2017): On unbiased class of ratio estimator for population mean using auxiliary information on an attribute and a variable, International Journal of Statistics and Systems, 12(1), 25-32.

[26] Bhushan S., Misra P.K. and Yadav S.K. (2017): An unbiased class of ratio type estimator for population mean using an attribute and a variable, Advances in Computational Sciences and Technology, 10(1), 139-146.

[27] Bhushan S., Misra P.K. and Yadav S.K. (2017): An unbiased estimator for population mean using an attribute and an auxiliary variable, Advances in Dynamical Systems and Applications, 12(1), 29-39.

[28] Bhushan S. and Misra P.K. (2017): An improved class of unbiased separate regression type estimator under stratified random sampling, International Journal of Computational Intelligence Research, 13(1), 35-44.

[29] Bhushan S. and Misra P.K. (2017): On efficient class of estimators for population mean, International Journal of Applied Agricultural Research, 12(1), 13-20.

[30] Bhushan S. and Misra P.K. (2017): A family of unbiased estimators of

population mean using an auxiliary variable, Advances in Computational Sciences and Technology, 10(1), 129-137.

[31] Bhushan S. and Misra P.K. (2017): On an unbiased jack-knife regression type estimator, Advances in Fuzzy Mathematics, 12(1), 147-155.

[32] Bhushan S. and Misra P.K. (2017): On efficient class of estimators for population mean, Global Journal of Pure and Applied Mathematics, 13(4), 1209-1216.

[33] Bhushan S. and Misra P.K. (2017): A generalised class of unbiased separate regression type estimator under stratified random sampling, International Journal of Statistics and Analysis, 7(1), 1-8.

[34] Cochran, W. G. (1977): Sampling Techniques, 3rd ed., John Wiley and Sons, New York.

[35] Gray, H.L and Schucancy, W. R. (1972): The generalized jack-knife statistic, Marcel Dekker, New York.

[36] Hidiroglou,M.A., and Sarndal,C.E. (1998): Use of auxiliary information for two-phase sampling , Survey Methodology, 24(1): 11-20.

[37] Kalidar, C. and Cingi, H. (2006): An improvement in estimating the population mean by using the correlation coefficient. Hacett. J. Math. Statist. 35(1): 103-109.

[38] Naik, V. D. and Gupta. P.C. (1996): A note on estimation of mean with known population proportion of an auxiliary character. Jour. Ind. Soc. Agr. Stat., 48(2), 151-158.

[39] Neyman, J. (1938): Contribution to the theory of sampling human populations, J. Amer. Statist. Assoc., 33: 101-116.

[40] Singh, H. P., Tailor, R., Kakran, M. S. (2004): An estimator of population mean using power transformation. J. Ind. Soc. Agri. Statist. 58(2): 223-230.

[41] Singh, H. P., Tailor, R., Singh, S., Kim, J. M. (2008): A modified estimator of population mean using power transformation. Statist. Pap. 49: 37-58.

[42] Sisodia, B. V. S., Dwivedi, V. K. (1981): A modified ratio estimator using coefficient of variation. J. Ind. Soc. Agri. Statist. 33: 13-18.

[43] Sukhatme, P. V and Sukhatme, B. V (1970): Sampling Theory of Surveys with Applications, Iowa State University Press, Ames, USA.

[44] Srivastava, S. K. (1967): An estimator using Auxiliary information. Calcutta Statistical Association, Bulletin, 16, 121-132.

[45] Upadhayaya, L. N. and Singh, H. P. (1999): Use of transformed auxiliary variable in estimating the finite population mean. Biometrical J. 41: 27-36.