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Analysis of the Viscous Force on Pressure Gradient in MHD Flow Through an Underground Pipe in the Presence of an Inclined Magnetic Field

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Abstract

In this study, MHD viscous flow through an underground pipe in the presence of inclined magnetic field is investigated. The pipe is considered to be of infinite length placed at different angles. The distance between the walls is taken to be large enough that all the dimensions are significant.Three dimensional cylindrical coordinates system is used. The governing partial differential equations are reduced to ordinary differential equations (ODEs). The ODEs are non dimensionalized and solved analytically using power series method with appropriate boundary conditions. The effect of angle of inclination,velocity of the flow, Reynolds number, Hartmann number of magnetic field on pressure gradient are shown graphically. The results show that when the angle of inclination is increasing the pressure gradient increases as well, but when Reynolds number and Hartmann number increase, pressure gradient decreases. On contrary the velocity profiles increase as the pressure gradient increases.

Keywords: Magnetohydrodynamic, Pipeline, Pressure drop, Velocity, Viscosity, Laminar flow.

NOMENCLATURE

$\overrightarrow{B}$	*Magnetic field vector*	σ	*Electric conductivity*
$\overrightarrow{E}$	*Electric field vector*	η	*Fluid viscosity*
$\overrightarrow{H}$	*Magnetic field vector*	$\triangle P$	*Pressure difference*
M	*Hartmann Number*	L	*Characteristic Length*
μ_e	*Magnetic permeability*	Re	*Reynolds number*
μ	*Dynamic viscosity*	F_z	*Lorentz force*
$\overrightarrow{V}(u_r, u_\theta, u_z)$	*Velocity components*	$\overrightarrow{J}$	*Current Density*

1. INTRODUCTION

Magnetohydrodynamics or MHD is the study of electrically conducting fluids under the influence of Magnetic field. It describes the movement of electrically conducting fluids which occur in nature and those made in the laboratory. When this fluid comes across a magnetic field a current is induced (M. Faraday, 1832)[1]. This current produced its own electric field which in turn interact with magnetic field to produce a body force on the fluid which is called electromotive force or simply Lorentz force (Richie, 1832)[2].The Lorentz force is very significant to the study of MHD for it has considerable effect on the fluid flow. MHD principally deals with electrically conducting liquids and fairly dense ionized gases. That is why the abbreviation MHD stands for Magneto (Magnetic), Hydro (liquids/water) and Dynamics (movement by force). Thus MHD has been relatively new branch of fluid dynamics which stem from Magnetofluiddynamics (MFD)which embraces both Magnetohydrodynamics (MHD) and Magnetogasdynamics (MGD).

The study of MHD therefore brings together in a unified form fluid dynamic theories, electromagnetic theories and some knowledge about plasma physics. The study of MHD became so popular in the 19^{th} century especially after the great work done by Michael Faraday and Richie in 1832 [2]. The problem of MHD flows continued under thorough investigation which lead to a deeper advancement. Both steady and unsteady, compressible and uncompressible MHD flows were been studied by different innovators such as William F. Hughes, John A. Brighton amongst others.

The idea of Faraday later was applied to a number of fields of knowledge such as in Astrophysics, especially when Bigalow (1899) discovered that the universe is full of magnetic fields, in Geophysics when the conducting fluid interact with magnetic fields that are present in and around heavenly bodies, in power conversion where electrical energy was extracted from MHD generators, in petroleum industries, crude oil purification, polymer technology and many more.

Young, Gerrard and Jevons [3] were the first in 1920 to study tidal motions with an induced voltage device, a technique which has been used widely in oceanography since then. Further advancement were made by Hartmann and Lazarus who like Faraday, studied channel flow of mercury [4]. In 1950's the study of MHD became explosive where extensive literature in the journals and reports grew up. It is during this time most of the practical work were done especially in nuclear engineering, high temperature gases and space technology amongst others.

2. LITERATURE REVIEW

The study of steady MHD flow under the inclined magnetic field has been done by many researchers. This section presents a few of them with their investigations in order to establish the similarities and differences to the current study.

Makinde, (2003)[5] studied about the stability of plane-poiseuille flow where he was able to come up with the fact that magnetic field has a stability effect on the MHD flow which is directly related to Hartmann number.Marzo, *et al* (2005), conducted a numerical simulation of three dimensional steady, Newtonian viscous flows through a collapsible tube[11]. Manyonge *et al* (2013) [6] conducted a study on two dimensional steady flow of a viscous electrically conducting incompressible fluid flow between two infinite plates one of which is porous under transverse magnetic field.Jhankal and Kumar (2014) did a study on Magnetohydrodynamics (MHD) plane Poiseuille Flow with variable viscosity and unequal wall temperatures in the presence of magnetic field[13]. Also Mburu *et al* (2016) [7] studied MHD flow between two parallel infinite plates, of which one is porous, subjected to inclined magnetic field under pressure gradient. Through analytical method she was able to establish that,the velocity profiles were inversely related to the angle of inclination and Hartmann number. Samim *et al*(2017) [8] analyzed friction loss in local pipes, where pressure drop was vividly noticed even in short length pipe of 5 metre. Malia, (2018) [9] studied about the effects of applying varibale pressure gradient to unsteady MHD flow between two parallel plates , one porous and the other held stationary. Chepkonga, *et al* (2019), conducted a study on Fluid flow and heat transfer through a vertical cylindrical collapsible tube in presence of magnetic field and an obstacle[12]. A.Sharma and Dubewar (2019) ,considered MHD Flow between Two Parallel Plates, where one is moving while the other is stationary, under the Influence of Inclined Magnetic Field and at constant pressure gradient by using Finite Difference Method[10].

3. MOTIVATION

In the study of fluid transportation viscosity is a phenomenon that cannot be overlooked at all. It exist in all types of fluids but in different magnitude of internal friction. The media through which the fluid is transported have a certain degree of viscosity which add effect to the flow as well. Most often people concentrate on the minor losses in pipes which are due to eddy formation along the channel, for instance, sudden enlargement, sudden contraction, pipe bend, pipe fitting and other obstacles. However little attention has been put to the major losses which results from the wall frictions, magnetic influence, molecular properties of fluid and others. More so none of the previous studies whose review are presented here tackled the problem of viscosity in underground pipelines as the major course of pressure drop in MHD flows. This study intends to bring on board the effects of viscous forces on pressure gradient of an eclectically conducting fluid under the influence of inclined magnetic field. We shall model an infinite pipe of length (L) with parallel walls placed at a distance (d) where $r = \pm R$ in the radial direction. The axis of the pipe is at $r = 0$. The flow is considered to be unidirectional towards z-direction but depends on r only.

4. GEOMETRIC CONFIGURATION OF THE PROBLEM

The study considers three dimensional incompressible, Newtonian and steady flow passing through underground pipelines. The pipe is placed horizontally such that the velocity of the fluid is in the axial of the main flow such that all the physical variables except pressure depend on r only. Therefore the gravitational force is considered negligible leaving alone the pressure gradient as the driving force of the flow.

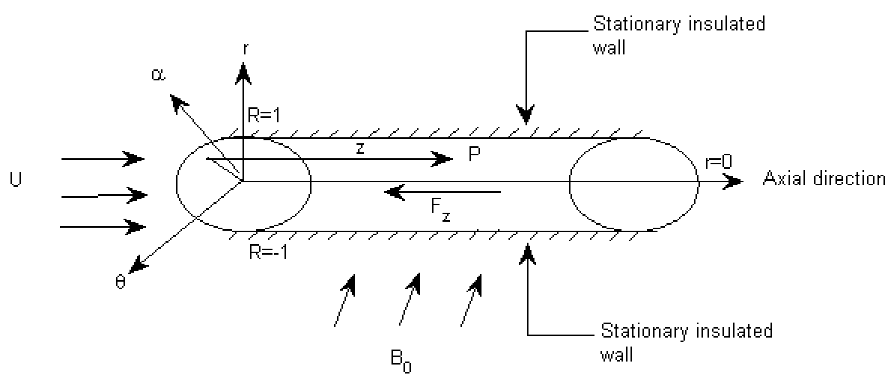


Figure 1: The Geometrical configuration

where F = Lorentz force, P = pressure, α = the angle of inclination U = velocity of the fluid, B_0 = Applied Magnetic field

5. MATHEMATICAL FORMULATION

Since the flow is steady and along the $z - axis$ depending on r only then we can obtain both velocity field and magnetic field vectors respectively. Therefore velocity field vector $\overrightarrow{V}$ is given by $\overrightarrow{V}(0, 0, u(r))$ and the magnetic field vector as $\overrightarrow{B}(B_0 Sin\alpha, 0, B_0 Cos\alpha)$ According to Michael Faraday when magnetic field comes across an electrically conducting fluid a current is induced. Now since the fluid under study is moving at a constant velocity $\overrightarrow{V}$,(from the continuity equation) across the magnetic field $\overrightarrow{B}$ an electric field $\overrightarrow{E}$ perpendicular to both $\overrightarrow{V}$ and $\overrightarrow{B}$ is produced. Mathematically this is represented as

$$\overrightarrow{V} \times \overrightarrow{B} = \overrightarrow{E} \tag{1}$$

The density of the induced current is given by

$$\overrightarrow{J} = \sigma\left(\overrightarrow{V} \times \overrightarrow{B}\right) \tag{2}$$

The electromotive force produced which is perpendicular to the electric field and magnetic field is given by

$$F_z = \overrightarrow{J} \times \overrightarrow{B} \tag{3}$$

This force is parallel to the velocity of the fluid but in opposite direction, see figure 1

5.1. Modeling the Electromotive force

Consider the following figure

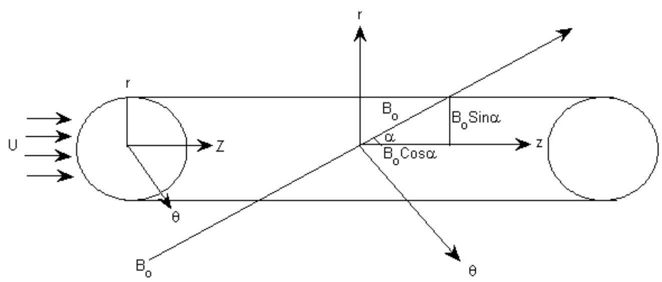


Figure 2: Diagram showing Magnetic field Vector

Where B_o is the inclined magnetic field at an angle α and U is the velocity towards z direction and $B = B_o Sin\alpha\,\tilde{i}$ +0$\tilde{j}$+ $B_o Cos\alpha\,\tilde{k}$

Using equation 1 and 2

$$\left(\overrightarrow{V}\times\overrightarrow{B}\right)=\begin{bmatrix}\tilde{i} & \tilde{j} & \tilde{k}\\ 0 & 0 & u_z\\ B_0Sin\alpha & 0 & B_0Cos\alpha\end{bmatrix}=\tilde{i}\,(0)+\tilde{j}\,(u_zB_0Sin\alpha)+\tilde{k}\,(0)$$

$$\overrightarrow{J}=\sigma\left(\overrightarrow{V}\times\overrightarrow{B}\right)=\sigma u_zB_0Sin\alpha\tilde{j}$$

The Lorentz force produced which is perpendicular to the electric field and magnetic field is given by

$$F_z=\overrightarrow{J}\times\overrightarrow{B}$$

$$F_z=\begin{bmatrix}\tilde{i} & \tilde{j} & \tilde{k}\\ 0 & \sigma u_zB_0Sin\alpha & 0\\ B_0Sin\alpha & 0 & B_0Cos\alpha\end{bmatrix}=\tilde{i}\left(-\sigma u_zB_0^2Sin\alpha Cos\alpha\right)-\tilde{j}\,(0)+\tilde{k}\left(\sigma u_zB_0^2Sin^2\alpha\right)$$

$$\therefore F_z=-\sigma u_zB_0^2Sin^2\alpha\tilde{k} \tag{4}$$

5.2. Governing Equations

The governing equations shall be the continuity equation and the Navier stokes equations. Let us consider a cylindrical coordinate system (r,α,z) where α is the azimuthal angle, r is the radial distance and z is the axial distance. u_r, u_α and u_z are velocity components along r, α and z directions respectively. The continuity equation is therefore given by

$$\frac{\partial\left(u_r\right)}{\partial r}+\frac{1}{r}\frac{\partial\left(u_\alpha\right)}{\partial\alpha}+\frac{\partial u_z}{\partial z}=0 \tag{5}$$

and the momentum equations

In r- direction will be

$$\rho\left(\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\alpha}{r}\frac{\partial u_r}{\partial\alpha}-\frac{u_\alpha^2}{r}+u_z\frac{\partial u_r}{\partial z}\right)=$$

$$-\frac{\partial P}{\partial r}+\rho g_r+\overrightarrow{J}\times\overrightarrow{B}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_r}{\partial r}\right)-\frac{u_r}{r^2}+\frac{1}{r^2}\frac{\partial^2u_r}{\partial\theta^2}-\frac{2}{r^2}\frac{\partial u_\alpha}{\partial\alpha}+\frac{\partial^2u_r}{\partial z^2}\right] \tag{6}$$

In α- direction will be

$$\rho\left(\frac{\partial u_\alpha}{\partial t}+u_r\frac{\partial u_\alpha}{\partial r}+\frac{u_\alpha}{r}\frac{\partial u_\theta}{\partial\alpha}+\frac{u_ru_\alpha}{r}+u_z\frac{\partial u_\alpha}{\partial z}\right)$$

$$=-\frac{1}{r}\frac{\partial P}{\partial\alpha}+\rho g_\alpha+\overrightarrow{J}\times\overrightarrow{B}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_\alpha}{\partial r}\right)-\frac{u_\theta}{r^2}+\frac{1}{r^2}\frac{\partial^2u_\alpha}{\partial\alpha^2}+\frac{2}{r^2}\frac{\partial u_r}{\partial\alpha}+\frac{\partial^2u_\alpha}{\partial z^2}\right] \tag{7}$$

In z- direction will be

$$\rho\left(\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+\frac{u_\alpha}{r}\frac{\partial u_z}{\partial \alpha}+u_z\frac{\partial u_z}{\partial z}\right)=$$

$$-\frac{\partial P}{\partial z}+\rho g_z+\overrightarrow{J}\times\overrightarrow{B}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_z}{\partial \alpha^2}+\frac{\partial^2 u_z}{\partial z^2}\right] \tag{8}$$

where $\overrightarrow{J}\times\overrightarrow{B}$ is the Lorentz force, P is pressure, ρ is the density, g is gravity.

The flow is along the z-direction such that the velocity profile along r and α directions becomes zero. Thus equation (5) reduces to

$$\frac{\partial u_z}{\partial z}=0 \tag{9}$$

Again, the flow is steady, i.e does not depend on time, and since $r=0$ and α=0, using equations (4) and (5), equation (8) becomes

$$\frac{\partial^2 u_z}{\partial r^2}+\frac{1}{r}\frac{\partial u_z}{\partial r}-\frac{\sigma u_z B^2 sin^2\alpha}{\mu}-\frac{1}{\mu}\frac{\partial P}{\partial z}=0 \tag{10}$$

where μ is the viscosity of the fluid and P is the pressure acting on the fluid.

5.3. Non-dimensionalization

Using the following non-dimensional quantities

$$z^*=\frac{z}{L},\qquad r^*=\frac{r}{L},\qquad u^*=\frac{u}{U},\qquad p^*=\frac{P}{\rho U^2}, \tag{11}$$

$$Re=\frac{\rho UL}{\mu},\qquad M=\overrightarrow{B}L\sqrt{\frac{\sigma}{\mu}}$$

where L is characteristic length, U is the stream velocity, ρ is density, Re is the Reynolds number which is the ratio between inertia force and viscous force. and M is the Hartmann number which is the ratio between electromagnetic force to viscous force, equation 10 becomes

$$\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{\sigma L^2 B^2 Sin^2\alpha}{\mu}u-\frac{\rho LU}{\mu}\frac{\partial p}{\partial z}=0 \tag{12}$$

Let the pressure gradient $-\frac{\partial p}{\partial z}=P$ such that pressure becomes a function of $z-$ only and the velocity of the flow is a function of $r-$ only, the partial derivatives becomes the total derivatives. Thus equation 8 becomes

$$\frac{d^2u}{dr^2}+\frac{1}{r}\frac{du}{dr}-M^2Sin^2\alpha u=-ReP \tag{13}$$

rewriting equation 13

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} - Au = B \tag{14}$$

where $A = M^2Sin^2\alpha$ and $B = -ReP$

To allow changes of the notation from $u(r)$ to $y(x)$ the r coordinates can be stretched so as to absorb the separation constant A as

$$x = \sqrt{A}r \qquad and \qquad u = y \tag{15}$$

Therefore

$$\frac{du}{dr} = \frac{dy}{dx} \cdot \frac{dx}{dr} = \frac{dy}{dx}\frac{d}{dr}(\sqrt{A}r) = \sqrt{A}\frac{dy}{dx} \tag{16}$$

Similarly

$$\frac{d^2u}{dr^2} = A\frac{d^2u}{dx^2} \tag{17}$$

substituting 17 and 16 in 14

$$A\frac{d^2y}{dx^2} + \frac{\sqrt{A}}{x} \cdot \sqrt{A}\frac{dy}{dx} - Ay = B$$

$$\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} - y = \frac{B}{A} \tag{18}$$

5.4. Solution Method

To solve equation 18 it is convenient to take a power series expansion. We assume a solution of the form

$$y = \sum_{n=0}^{\infty} a_n x^{c+n} \tag{19}$$

The first and second derivatives of equation 19 are

$$y^{'} = \sum_{n=0}^{\infty} a_n(c+n)x^{c+n-1} \tag{20}$$

$$y^{''} = \sum_{n=0}^{\infty} a_n(c+n)(c+n-1)x^{c+n-2} \tag{21}$$

Substituting equations 19, 20 and 21 in 18

$$\sum_{n=0}^{\infty} a_n(c+n)(c+n-1)x^{c+n-2} + x^{-1}\sum_{n=0}^{\infty} a_n(c+n)x^{c+n-1} - \sum_{n=0}^{\infty} a_n x^{c+n} = \frac{B}{A} \tag{22}$$

By equating the lowest power of x to zero and consider only the coefficients we obtain the indicial equation from where a recurrence relation is derived.

$$\begin{aligned} a_{n+2}(c+n)(c+n-1) + a_{n+2}(c+n) - a_n &= 0 \\ a_{n+2}(c+n+2)(c+n+1) + a_{n+2}(c+n+2) - a_n &= 0 \\ \left[(c+n+2)(c+n+1) + (c+n+2)\right] a_{n+2} - a_n &= 0 \\ \left[(c+n+2)(c+n+1) + 1\right] a_{n+2} &= a_n \\ (c+n+2)^2 a_{n+2} &= a_n \end{aligned}$$

$$a_{n+2} = \frac{a_n}{(c+n+2)^2} \tag{23}$$

where $n = 0, 1, 2, 3, 4.$

We can now determine the value of the coefficients a_1, a_2, a_3, a_4,....a_n in order that the power series will satisfy the given differential equation. The solution series is therefore given by

$$y = \frac{B}{A}\left(1 - \frac{x^2}{4} + \frac{x^4}{64} - \frac{x^6}{2304} + \frac{x^8}{147456} - ...\right) \tag{24}$$

The function in the brackets can be summarized as

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^{2n}(n!)^2} \tag{25}$$

for n =0,1,2,3,4,...

The function generated is usually symbolized as J_o which is a function of x. Therefore

$$J_0(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^{2n}(n!)^2} \tag{26}$$

where $J_0(x)$ is recognized as a bessel function of order zero of first kind whenever $c = 0$.

The general solutions to equation (14) is obtained from **Frobenius Method**.

$$y = C_1 \frac{B}{A} J_0(x) + C_2 g(x) \quad or \quad y = C_1 \frac{B}{A} J_0(x) + C_2 Y_0(x) \tag{27}$$

where

$$J_0(x) = \sum_{n=0}^{\infty} \frac{(-1)^n \, x^{2n}}{2^{2n}(n!)^2} \tag{28}$$

$J_0(x)$ can be approximated as

$$\sqrt{\frac{2}{\pi x}} cos(x - \frac{\pi}{4})$$

This asymptotic approximation applies as $x \longrightarrow \infty$
Returning to the original independent variable r (from equation 15), the general solution of equation 14 is

$$u(r) = C_1 \frac{B}{A} J_0(\sqrt{A}r) + C_2 Y_0(\sqrt{A}r) \tag{29}$$

The presence of the logarithmic term in Y_0 will produce infinite value of u (velocity flow) at $r = 0$, hence it is appropriate to set $C_2 = 0$ to remove $Y_0(\sqrt{A}r)$. The boundary conditions

$$R = \pm r \quad u = o, \;\; R = 0 \quad u = U \tag{30}$$

require that

$$u(r) = C_1 \frac{B}{A} J_0(\sqrt{A}r) \tag{31}$$

Taking the condition $u = 0$ when $r = R$

$$u(R) = C_1 \frac{B}{A} J_O(\sqrt{A}R) = 0 \tag{32}$$

There is infinite sequence of A values, A_i each of which satisfies equation 31. These may be found using tables of the zeros of $J_0(x)$. n^{th} zero of $J_0(x)$ is represented by $\mathit{\Gamma} n$. The first five zeros of the $J_0(x)$ are as follows,
$r_1 = 2.4048, \quad r_2 = 5.5201, \quad r_3 = 8.6537, \quad r_4 = 11.1915, \quad r_5 = 14.9309$
The corresponding values of A are of the form

$$A_n = \frac{(\mathit{\Gamma} n)^2}{R^2}$$

$A_1 = \frac{5.7831}{R^2}, \quad A_2 = \frac{30.4715}{R^2}, \quad A_3 = \frac{74.8865}{R^2}, \quad A_4 = \frac{139.039}{R^2}, \quad A_5 = \frac{222.932}{R^2}$
Therefore the corresponding U-eigen functions are

$$U_n(r) = \frac{B}{A} J_0(\sqrt{A}r) = \frac{B}{A}(r_n \frac{r}{R}) \tag{33}$$

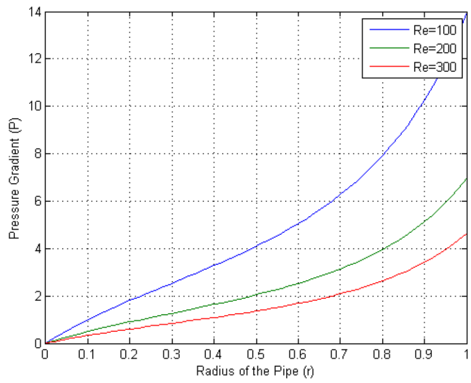
Therefore

$$U_n(r) = \frac{-ReP}{M^2 Sin^2\alpha}\left[\frac{2}{\pi(\sqrt{M^2 Sin^2\alpha})r} Cos(r\sqrt{M^2 Sin^2\alpha} - \frac{\pi}{4})\right] \tag{34}$$

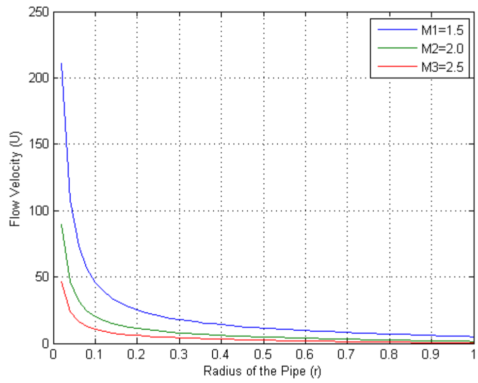
6. RESULTS AND DISCUSSION

Using equation 32 we analyse the effects of different parameters on pressure gradient by using computer codes in MATLAB version $(8.3.0.532)$ computer program. We were able to obtain the following results.

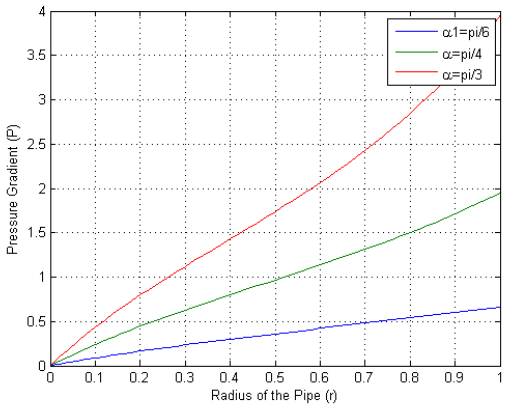
In Figure (a) it is clearly shown that when the Reynolds number increases pressure gradient decreases and vice versa. This depict an inverse relationship between Reynolds



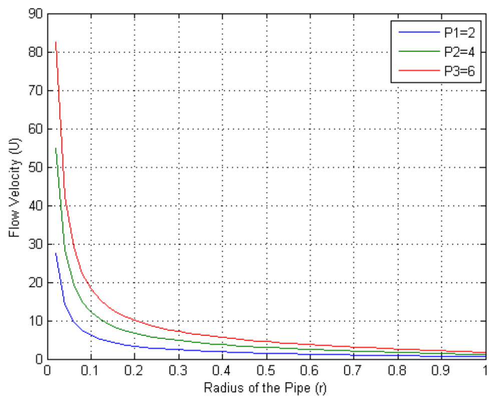
(a) Pressure Gradient for Different Values of Reynolds Number



(b) Flow Velocity for Different Values of Hartmann Number



(c) Pressure Gradient for Different Angles of Inclination



(d) Velocity Profile for Different Values of Pressure Gradient

number and pressure gradient. With high Reynolds number it implies that the fluid is less viscous hence low pressure is required. But when the Reynolds number is low it means that the viscous force is higher than the inertia force hence more pressure is required to pump the fluid. This is because the Reynolds number is the ratio of inertia force to the viscous force.

In figure (b) it is observed that the increase in Hartmann number leads to a decrease in velocity. This is because the Hartmann number is the ratio of electromagnetic force to the viscous force and so the larger the Hartmann number the stronger the magnetic force and the smaller the viscous force. So it is clear from the figure that increasing the Hartmann number reduces the velocity due to the effect of Lorentz force.

In figure (c) it is seen that when the angle of inclination of magnetic field is varied the pressure gradient increases. The strength of the magnetic field reaches maximum at

90^0 when the magnetic field transverses the flow. This is because the magnetic strength slower the flow velocity and in so doing the pressure increases since they are inversely related. When the angle of inclination is very big, and the pressure gradient as well, implies that the viscosity is very high. So for a better transportation of MHD fluid across areas with magnetic influence it is better to fix the pipe in such a way that the angle of inclination of the magnetic field is smaller.

In figure (d) we observe that the velocity profile is affected by pressure gradient. The higher the pressure the higher the velocity of the flow. This is because the pressure gradient helps in pushing the velocity of fluid. It is also observed that at the stationary wall the flow takes the velocity of the wall but gradually increases as you move away from the wall and reaches maximum at the free stream region which is at the center of the pipe. It is again observed from the figure that the graphs are asymptotic along the vertical axis (U). This is because according to our model $r \neq 0$ unless there will be an infinite values of velocity.

7. CONCLUSION

It has been observed that when the Reynolds number is high the pressure decreases and vice versa. Since we are dealing with laminar flow whose Reynolds number is small, then it is practical to apply more pressure when pumping oil through pipe. Again, the increase in Hartmann number leads to the decrease in flow velocity. So for practical application the engineers would always want to maximize the flow velocity to ensure effective oil transportation. In that case the electromagnetic force should be very low. Similarly when varying the angle of inclination of magnetic field the pressure gradient increases. Practically, the more the strength of magnetic field the low the velocity of the fluid, henceforth more pressure will be required to pump the fluid. More so, the increase in pressure gradient leads to the increase in velocity profile. Hence the engineers are advised to add more pressure to the flow provided other factors are considered.

Finally we recommend that a similar study can be done on the same problem but this time considering unsteady MHD boundary layer flow of fluid through pipelines with porous material that are conductive.

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