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Further Results on Even Sum Labeling of Graphs

Vinodray J. Kaneria[1] **and Pritesh P. Andharia**[2*]

[1,2]*Department of Mathematics, Saurashtra University, Rajkot-360005, Gujarat, India.*
[1]*Email: kaneriavinodray@gmail.com,* [2]*priteshandharia@gmail.com*

Abstract

In this paper, the even sum labeling of Jelly Fish graph $J_{n,n}$, splitting graph of a star graph $K_{1,n}$, degree splitting graph of a star graph $K_{1,n}$, splitting graph of $K_{2,n}$ and splitting graph of $K_{1,n,n}$ are presented.
Keywords : Labeling; even sum labeling; even sum graph; splitting graph; degree splitting graph.
AMS subject classification(2010) : 05C78.

1. INTRODUCTION

In this article, a word 'graph' is used for a finite simple and undirected graph with vertex set V, edge set E, order p and size q. For graph labeling related terminology and notations, we follow Gallian [1]. The concept of odd sum labeling was introduced by Arockiaraj and Mahalakshmi [2]. The odd sum labeling of various types of graph are presented in [2 - 5]. In [6], Monika and Murugan introduce the concept of odd-even sum labeling. The odd-even sum labeling of some graphs are presented in [7]. In [8], Andharia and Kaneria introduce the new concept of labeling called even sum labeling. Kaneria and Andharia [9] have presented the even sum labeling of various graphs. In this article we have presented even sum labeling of Jelly Fish graph $J_{n,n}$, splitting graph of a star $K_{1,n}$, degree splitting graph of a star $K_{1,n}$, splitting graph of $K_{2,n}$ and splitting graph of $K_{1,n,n}$.

Definition 1. An assignment of integers to the vertices and/or edges of a graph subject to certain conditions is called a graph labeling.

Definition 2. An injective function $f : V(G) \to \{0, \pm 2, \pm 4, \ldots, \pm 2q\}$ is called an even sum labeling of a graph G if the induced mapping $f^* : E(G) \to \{2, 4, \ldots, 2q\}$ defined by $f^*((u, v)) = f(u) + f(v), \forall (u, v) \in E(G)$ is bijective. A graph which admits even sum labeling is called an even sum graph.

Definition 3. For integers $m, n \geq 0$, we consider the graph Jelly Fish $J_{m,n}$ or $J(m, n)$ with the vertex set $V(J(m, n)) = \{x, y, u, v\} \cup \{u_1, u_2, \ldots, u_m\} \cup \{v_1, v_2, \ldots, v_n\}$ and the edge set $E(J(m, n)) = \{(x, u), (x, v), (x, y), (y, u), (y, v)\} \cup \{(u, u_i)/1 \leq i \leq m\} \cup \{(v, v_j)/1 \leq j \leq n\}$.

Definition 4. For a graph G, the Splitting Graph $S'(G)$ is a graph obtained by adding a new vertex v' in G corresponding to each vertex v of G and joining v' to all the vertices of G adjacent to v. [10]

Definition 5. Let G be a graph with $V(G) = S_1 \cup S_2 \cup \ldots \cup S_t \cup T$ where each S_i is a set of vertices having at least two vertices and having the same degree. The degree splitting graph of G denoted by $DS(G)$ is the graph obtained from G by adding vertices $w_1, w_2, \ldots w_t$ and joining w_i to each vertex of $S_i (1 \leq i \leq t)$. [11]

2. MAIN RESULTS

Theorem 2.1. *The Jelly Fish graph $J_{n,n}$ is even sum graph.*

Proof. Consider a Jelly Fish graph $J_{n,n}$ with vertices x, y, u, u_i, v, v_i, where $i = 1, 2, \ldots, n$ and edges $(x, y), (x, u), (x, v), (y, u), (y, v), (u, u_i), (v, v_i)$ where $i = 1, 2, \ldots, n$.

It is clear that $| V(J_{n,n}) |= p = 2n + 4$ and $| E(J_{n,n}) |= q = 2n + 5$.

Now, we define the vertex labeling function $f : V(J_{n,n}) \to \{0, \pm 2, \pm 4, \ldots, \pm 2q\}$ as

$$\begin{aligned} f(x) &= 0 \\ f(y) &= 2 \\ f(u) &= 4n + 4 \\ f(v) &= 4n + 8 \\ f(u_i) &= -2i, \forall i = 1, 2, \ldots, n \\ f(v_i) &= -2n - 4 - 2i, \forall i = 1, 2, \ldots, n. \end{aligned}$$

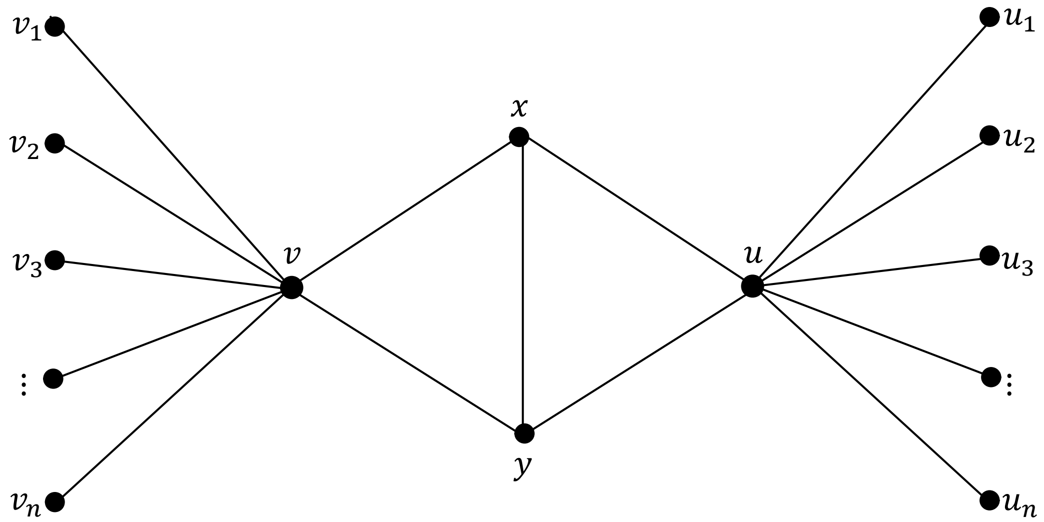

Figure 1: Ordinary labeling of a Jelly Fish graph $J_{n,n}$

The induced edge labeling function $f^* : E(J_{n,n}) \to \{2, 4, \ldots, 2q\}$ is given by

$$f^*((u, v)) = f(u) + f(v), \forall (u, v) \in E(J_{n,n}).$$

The above defined labeling pattern give rise to even sum labeling of $J_{n,n}$.
Hence the Jelly Fish graph $J_{n,n}$ is even sum graph.

□

Example 2.2. An even sum graph $J_{5,5}$ is shown in Figure 2.

Theorem 2.3. *$S'(K_{1,n})$, the splitting graph of star $K_{1,n}$ is even sum graph.*

Proof. Let $u_i, i = 1, 2, \ldots, n$ be the pendant vertices and v_1 be the apex vertex of a star graph $K_{1,n}$. Let u_i' be the added vertices corresponding to each u_i and v_1' be the vertex added corresponding to v_1. In order to obtain $S'(K_{1,n})$, each u_i' is joined to v_1 and each u_i is joined to v_1' by an edge as shown in Figure 3.

Thus, $V(S'(K_{1,n})) = \{u_i, v_1, u_i', v_1'; i = 1, 2, \ldots, n\}$ and
$E(S'(K_{1,n})) = \{(u_i, v_1), (v_1, u_i'), (u_i, v_1'); i = 1, 2, \ldots, n\}$.

It is clear that $| V(S'(K_{1,n})) |= 2n + 2$ and $| E(S'(K_{1,n})) |= 3n$.

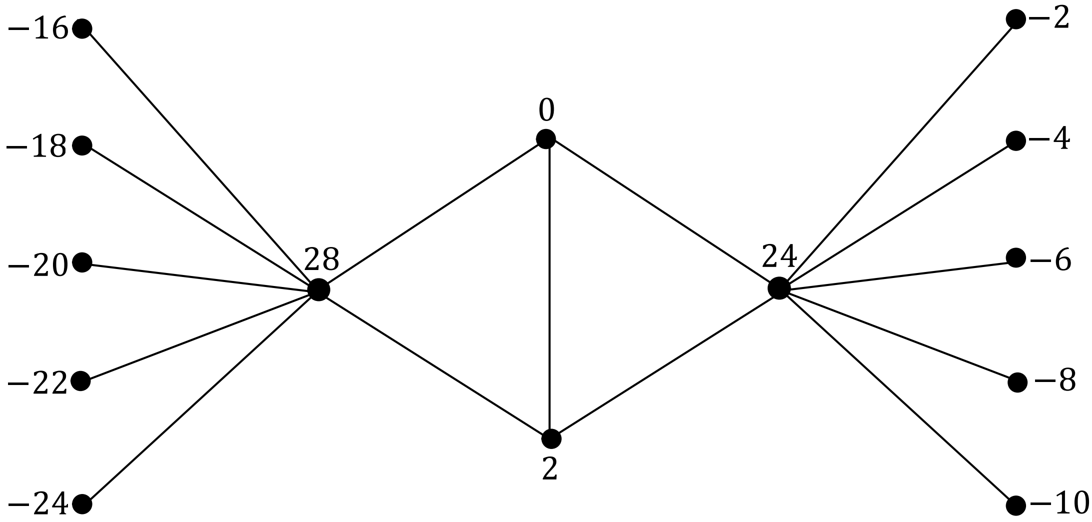

Figure 2: Jelly Fish graph $J_{5,5}$ with its even sum labeling

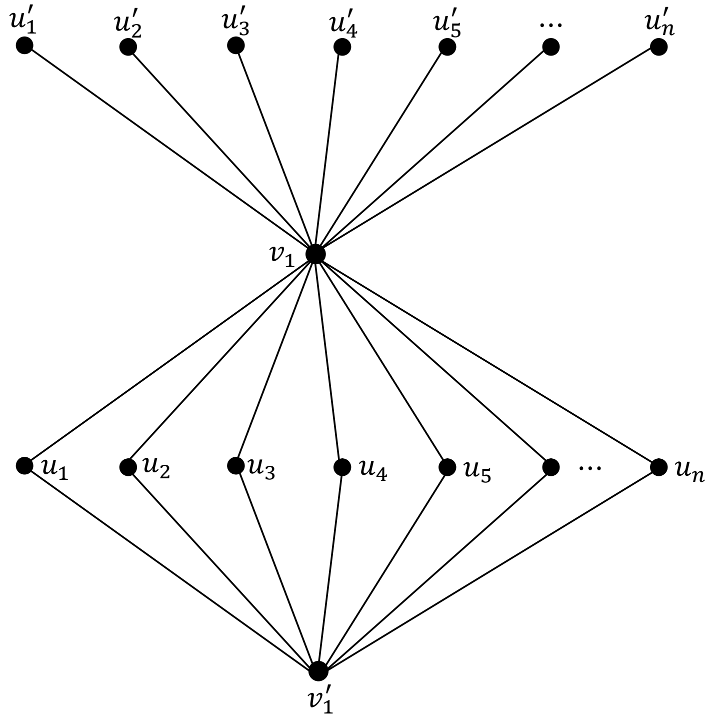

Figure 3: Splitting graph of star $K_{1,n}$ and its ordinary labeling

So $p = 2(n + 1)$ and $q = 3n$.

The vertex labeling of the graph $S'(K_{1,n})$ is defined as

$$f(u_i) = 4 - 2n - 2i, \forall i = 1, 2, \dots, n$$
$$f(v_1) = 6n - 2$$
$$f(u'_i) = 4 - 2i, \forall i = 1, 2, \dots, n$$
$$f(v'_1) = 4n - 2.$$

The induced edge labeling function f^* for the graph $S'(K_{1,n})$ is given by

$$f^*((u, v)) = f(u) + f(v), \forall (u, v) \in E(S'(K_{1,n})).$$

The labeling pattern defined above give rise to even sum labeling of $S'(K_{1,n})$. Hence $S'(K_{1,n})$, the splitting graph of star $K_{1,n}$ is even sum graph.

□

Example 2.4. The even sum graph $S'(K_{1,7})$ is shown in Figure 4.

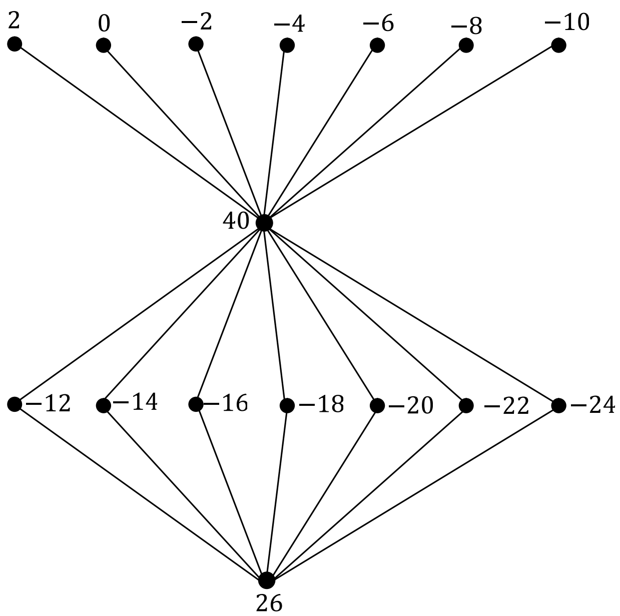

Figure 4: $S'(K_{1,7})$ with its even sum labeling

Theorem 2.5. *Degree Splitting graph of a star graph $DS(K_{1,n})$ is even sum graph.*

Proof. Consider a star graph $K_{1,n}$ with an apex vertex u and pendant vertices $v_i, i = 1, 2, \ldots, n$. Take $S_1 = \{v_i : 1 \leq i \leq n\}$ and $T = \{u\}$. In order to obtain $DS(K_{1,n})$, we add a new vertex w_1 corresponding to S_1 and join w_1 with each vertex of S_1 by an edge.

Thus, we have $| V(DS(K_{1,n})) |= p = n + 2$ and
$| E(DS(K_{1,n})) |= q = 2n$.

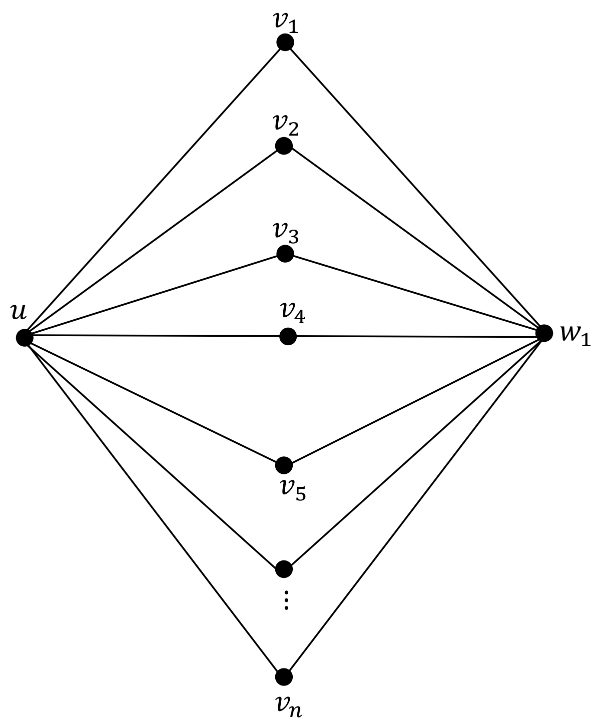

Figure 5: Ordinary vertex labeling of $DS(K_{1,n})$

Now, we define the vertex labeling function $f : V(DS(K_{1,n})) \to \{0, \pm 2, \pm 4, \ldots, \pm 2q\}$ as follow:

$$f(w_1) = 4n - 2$$
$$f(u) = 2n - 2$$
$$f(v_i) = 4 - 2i, \forall i = 1, 2, \ldots, n.$$

The induced edge labeling function f^* for the graph $DS(K_{1,n})$ is given by

$$f^*((u,v)) = f(u) + f(v), \forall (u,v) \in E(DS(K_{1,n})).$$

The labeling pattern defined above give rise to even sum labeling of $DS(K_{1,n})$. Hence $DS(K_{1,n})$, the degree splitting graph of star $K_{1,n}$ is even sum graph. □

Example 2.6. Even sum labeling of a graph $DS(K_{1,8})$ is shown in Figure 6.

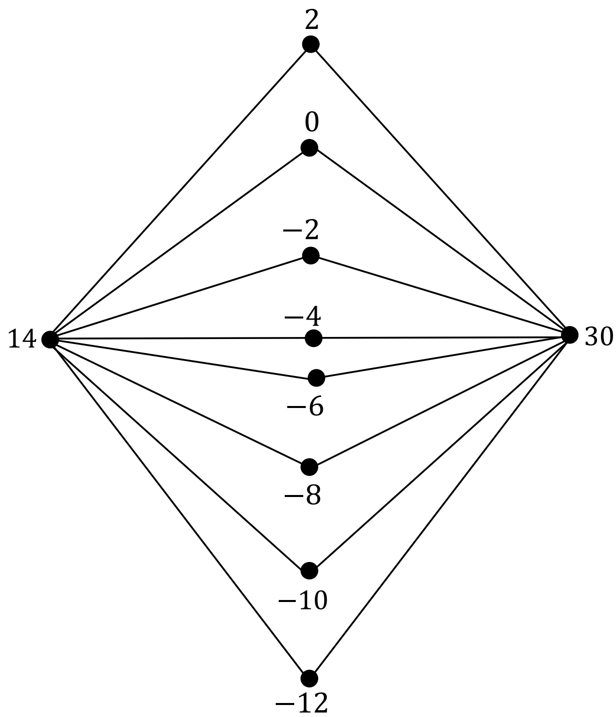

Figure 6: Even sum labeling of $DS(K_{1,8})$

Theorem 2.7. *Splitting graph of $K_{2,n}$ is even sum graph.*

Proof. Let $V(K_{2,n}) = \{x_1, x_2, v_i; 1 \leq i \leq n\}$ and $E(K_{2,n}) = \{(x_1, v_i), (x_2, v_i); 1 \leq i \leq n\}$. In order to obtain $S'(K_{2,n})$, we add vertices $x'_1, x'_2, v'_i; 1 \leq i \leq n$ such that x'_1 and x'_2 are adjacent to every $v_i; 1 \leq i \leq n$ and every $v'_i; 1 \leq i \leq n$ are adjacent to x_1 and x_2.

It is observed that
$V(S'(K_{2,n})) = \{x_1, x_2, v_i, x'_1, x'_2, v'_i; 1 \leq i \leq n\}$ and

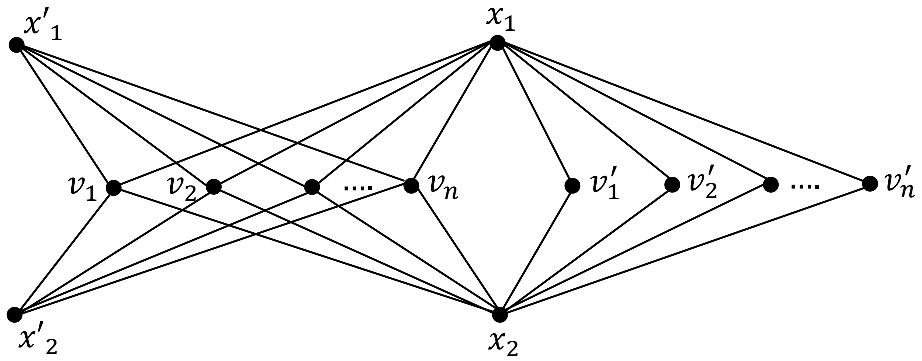

Figure 7: Ordinary labeling of $S'(K_{2,n})$

$$E(S'(K_{2,n})) = \{(x_1, v_i), (x_2, v_i), (x'_1, v_i), (x'_2, v_i), (x_1, v'_i), (x_2, v'_i); 1 \leq i \leq n\}.$$

So for the graph $S'(K_{2,n})$, the number of vertices $p = 2(n+2)$ and the number of edges $q = 6n$.

We shall define the vertex labeling function f for the graph $S'(K_{2,n})$ as follow:

$$\begin{aligned} f(x_1) &= 12n - 6 \\ f(x_2) &= 12n - 4 \\ f(x'_1) &= 8n - 6 \\ f(x'_2) &= 8n - 4 \\ f(v_i) &= 4(i + 1 - 2n); i = 1, 2, \dots, n \\ f(v'_i) &= 4(2 - i); i = 1, 2, \dots, n. \end{aligned}$$

Further its edge induced function is

$$f^* : E(S'(K_{2,n})) \to \{2, 4, 6, \dots, 12n\}$$

defined as

$$f^*((u, v)) = f(u) + f(v), \forall (u, v) \in E(S'(K_{2,n})).$$

Above defined pattern of labeling give rise to even sum labeling of the graph $S'(K_{2,n})$. So $S'(K_{2,n})$, the splitting graph of $K_{2,n}$ is even sum graph.

□

Example 2.8. Even sum labeling of a splitting graph of $K_{2,4}$ is shown in Figure 8.

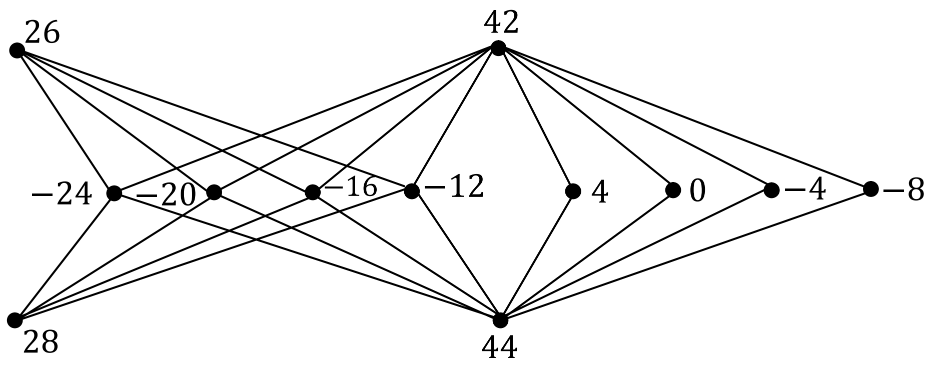

Figure 8: $S'(K_{2,4})$ with its even sum labeling

Theorem 2.9. *$S'(K_{1,n,n})$, the splitting graph of $K_{1,n,n}$ admits even sum labeling.*

Proof. Consider a graph $K_{1,n,n}$ with vertices $x_1, u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_n$.

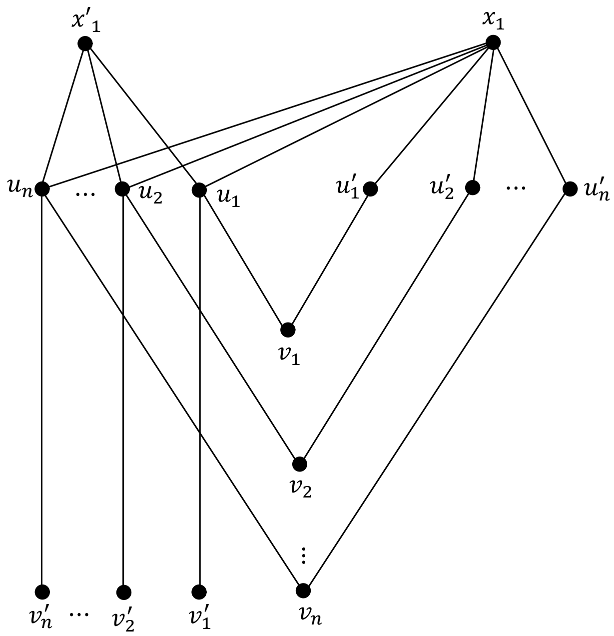

Figure 9: $S'(K_{1,n,n})$ with its vertex labeling

The splitting graph of $K_{1,n,n}$ is obtained by adding to each vertex v of $K_{1,n,n}$, by a new vertex such that it is adjacent to those vertices of $K_{1,n,n}$, which are adjacent to v in

$K_{1,n,n}$. The resulting graph $G = S'(K_{1,n,n})$ is shown in Figure 9.

It is observed that

$V(G) = \{x_1, u_i, v_i, x'_1, u'_i, v'_i; 1 \leq i \leq n\}$ and

$E(G) = \{(x_1, u_i), (u_i, v_i), (x'_1, u_i), (x_1, u'_i), (u'_i, v_i), (u_i, v'_i); 1 \leq i \leq n\}$.

So for the graph G, the number of vertices $p = 2(2n + 1)$ and the number of edges $q = 6n$.

We shall define the vertex labeling function f for the graph G as follow:

$$\begin{aligned} f(x_1) &= 12n \\ f(x'_1) &= 4(3n - 1) \\ f(u_i) &= 6(1 - i) \\ f(u'_i) &= 2(2 - 3i) \\ f(v_i) &= 6n + 2(i - 1) \\ f(v'_i) &= 2n + 4(i - 1); \forall i = 1, 2, \ldots, n. \end{aligned}$$

Further its edge induced function

$f^* : E(G) \to \{2, 4, 6, \ldots, 12n\}$

is defined by

$f^*((u, v)) = f(u) + f(v), \forall (u, v) \in E(G)$.

Above defined pattern of labeling give rise to even sum labeling of the graph G. So $S'(K_{1,n,n})$, the splitting graph of $K_{1,n,n}$ is even sum graph.

□

Example 2.10. Even sum labeling of the graph $S'(K_{1,3,3})$ is shown in Figure 10.

3. CONCLUSION

In this article, we have discussed even sum labeling property of Jelly Fish graph, splitting graph of a star graph $K_{1,n}$, degree splitting graph of a star graph $K_{1,n}$ and splitting graph of graphs $K_{2,n}$ and $K_{1,n,n}$.

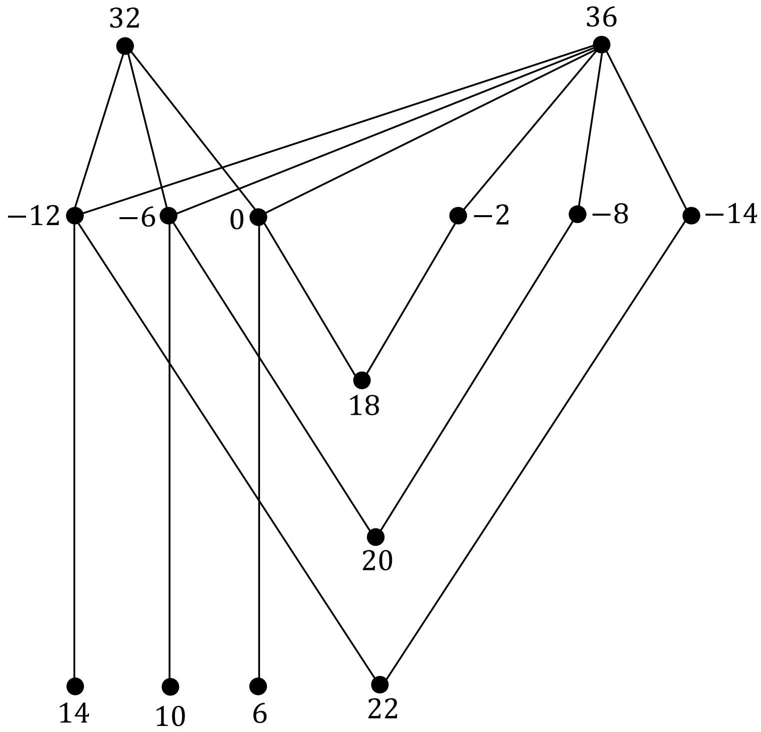

Figure 10: $S'(K_{1,3,3})$ and its even sum labeling

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